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The Diamond–Dybvig model of bank runs as a coordination game

by

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Abstract

A bank run occurs when a large number of customers withdraw their deposits from a financial institution at the same time. This can destabilise the bank to the point where it runs out of cash and thus faces sudden bankruptcy. As more people withdraw their deposits, the likelihood of bankruptcy increases, thus triggering further withdrawals. In game theory this type of situation can be modelled as a "coordination game", that is, a game with two pure equilibria: If sufficiently many people keep their money in the bank, then it will not default and it is rational for everyone to keep their money in the bank. On the other hand, if sufficiently many people withdraw their deposits the bank will default and it is then rational for everyone to try to withdraw their deposits.

The overall objective of this study is to explain the phenomenon of bank runs by introducing the Diamond–Dybvig model. This model assumes that the function of a bank is to offer both long-term loans for investments and relatively short-term deposit service. Bank runs comes out as one of two equilibria when too many withdraw early before the long-term loans is paid back. Our task is to find out the condition that can lead to bank runs and more importantly, we will suggest two ways to address the problem of bank runs.

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List of Notations

Type 1: depositors who always withdraw early.
Type 2: depositors who always withdraw late.
t: the share of deposits who are type 1.
f: the share of depositors who withdraw early, $t \leq f \leq 1$.
f_j: the share of depositors who withdraw early and before player j.
r_1: represent how much money a depositor withdraws early.
fr_1: the share of all money withdraw early. When $fr_1 > 1$, it means that all money is withdrew early.
f^*: the break point frequency, where type 2's earn equally much from early or late withdrawal.
$\frac{1}{r_1}$: the frequency at which all the money runs out in the bank.
V_1: The payoff when a depositor withdraw at time $T = 1$. This payoff varies based on the place of the depositor in line.
V_2: The payoff when a depositor withdraw at time $T = 2$.

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Chapter 1

Introduction

Bank runs are a behaviour where a massive amount of money is withdrawn simultaneously by its depositors. It is a real life phenomenon that happens every now and then when bank credit is decreased due to rumours of bankruptcy and other reasons, for instance, recession in the economy. As more customers withdraw from the bank, the danger of the occurrence of bankruptcy increases. This situation can be modelled mathematically as a coordination game which is a type of game that has multiple equilibriums. Precise definitions of the concept of such equilibria and other theoretic concepts of a game is presented later in Chapter 2.

The purpose of this thesis is to do a literature study and of the famous Diamond–Dybvig model and investigate the occurrence of bank runs with the help of the model. How is it related to a coordination game? Any solution or remedy can be conducted to avoid bank runs?

1.1 Background

Bank runs frequently happened in the United states before the establishment of the Federal Deposit Insurance Corporation in 1933. The construction of it has largely reduced the occurrence of bank runs. This corporation is created by the U.S. government and it saves banks from bank run through the enactment of the deposit insurance program. But this could not stop a new wave of bank runs happening because of the economic crisis. Examples include the runs on Northern Rock in 2007, on Bear Stearns in 2008 and on IndyMac in 2008, see Arifovic et al. (2013).

The theoretical framework of bank runs is mostly built on a paper by Diamond and Dybvig (1983). They pointed out that coordination failure between the bank's assets and liabilities opens the gate to bankruptcy. The paper regards banks as intermediaries between savers who prefer to deposit in liquid accounts and borrowers who prefer to take out long-maturity loans. The most important assumption in Diamond–Dybvig model about how banking system works is that depositors' unpredictable needs for cash are unlikely to occur at the same time in real life. Therefore, since depositors' needs reflect their individual circumstances, by accepting deposits from many different sources, the bank expects only a small fraction of withdrawals in the short term, even though all depositors have the right to take out their deposits at any time. Thus a bank can make loans over a long horizon, while keeping only relatively small amounts of cash on hand to pay any depositors that wish to make withdrawals.

In the game theory literature, coordination games refer to a type of games where there are more than one equilibria. The Diamond—Dybvig model provides an example of a game with more than one Nash equilibrium, see Carmichael (2005), which implies that the model can be regarded as a coordination game. If depositors expect most other depositors to withdraw only when they have real expenditure needs, then it is rational for all depositors to withdraw only when they have real expenditure needs. But if depositors expect most other depositors to rush quickly to close their accounts, then it is rational for all depositors to rush quickly to close their accounts. Obviously, the first equilibrium is better than the second. If depositors withdraw only when they have real expenditure needs, they all benefit from holding their savings in a liquid, interest-bearing account. If instead everyone rushes to close their accounts, then they all lose the interest they could have earned, and some of them lose all their savings.

Bank runs can be modelled as a game with imperfect information meaning that players do not have all the relevant information to make their decision. Therefore, the behaviour of players is decided by their own beliefs and higher order beliefs. Higher order beliefs basically means what you believe about the beliefs of others.

1.2 Review of Literature

Since Diamond–Dybvig model will be the focus of our study, the original work investigated by Diamond and Dybvig must be mentioned here. In Diamond and Dybvig (1983), the bank is interpreted as a financial intermediary between long-term loans and demand deposits that might be withdrawn at any time. Thus this mismatch give rise to a risk that a large number of depositors might withdraw at once. They evaluated the strength of bank deposit contracts and other contracts that can be useful to prevent bank runs. They gave equilibrium analysis to the demand deposit contract and proved it to be feasible despite the danger of runs. But they also emphasised that this is not the only equilibrium in the game, meaning that the efficient allocation of no bank runs is not always applicable. The model is useful to explain the phenomenon of bank runs as well as other kinds of financial crises. Furthermore, they also suggest remedies to avoid such crises.

Two experimental studies based on Diamond–Dybvig model provided evidence of bank runs as coordination failure: Arifovic et al. (2013) pointed out that bank runs can happen due to the consequence of pure coordination failures. And these coordination failures are caused by the level of required coordination — if the level of coordination requirement is high enough, the coordination will fail and bank runs are around the corner. They found out this through an experimental method. To do this, the authors enroll some volunteers to act as depositors and simulate the game of saving and withdrawing behaviour for a repeated times. In this experiment, the coordination parameter is the only variation. This parameter which ranges from 0.1 to 0.9 measures the amount of coordination that is demanded for depositors who wait and thus get a higher payoff. It can be divided into three regions: the non-run region(≤ 0.5), the non-run or run region and the run region. The result of their experiment is that if the coordination parameter is high(≥ 0.8), then bank runs are most likely about to happen.

The second empirical study by Garratt and Keister (2009) focused on the effects of changing withdraw opportunities and adding uncertainty of the withdrawal demand. They found that coordination is sensitive to uncertainty. This uncertainty means the demand of fundamental withdrawal. If the uncertainty is accumulated, then it is more likely to give rise to bank runs. They also found that the depositors are more likely to withdraw when there are several opportunities to do so rather than a single opportunity.

Aggregate certainty under imperfect information have been investigated by Kinateder and Kiss (2014). They showed that the aggregate certainty can be regarded as a signal to tell the patient depositors to wait to maximise their payoff.

1.3 Method and Outline

The objective of this report is a thorough study in Diamond–Dybvig model as a coordination game.

This thesis has a chapter structure as follows:

Chapter 1 is the introduction part where I gave a description of the phenomenon of bank runs and develop the purpose of the thesis. In Chapter 2, fundamental concepts of game theory will be introduced and defined since this will be the foundation for the rest of the thesis. Chapter 3 constitute the core of this thesis in which we illustrate the Diamond–Dybvig model and present an mathematical analysis of the model. Chapter 4 have a focus on the prevention of bank runs. The last chapter fully summarie the content involved in this thesis.

Chapter 2

Fundamental Concepts of Game Theory

There are different types of games in the world of game theory. In this thesis we will focus on the type known as "Coordination Game". Before we define what a coordination game is, a few fundamental concepts related to game theory will be introduced with examples in the following part.

2.1 Game Theory

Decision theory is about making optimal decisions in general. Game theory is a special branch of decision theory which focuses on optimal decisions in situations where outcome depends also on the decision of others. In game theory, the most important and basic assumption is that all of the players wish to optimise their expected payoff.

A game cannot be described properly without the introduction of the following concepts:

1. *A set of players*. Game theory assumes that there is a set P of players. Let n denote the number of players, which can be any positive integer. (Most research in game theory deals with two-player games, but there is also research on general n-player games).

2. *A set of strategies for each player*. Each player has a set of strategies. Here we shall deal only with the *symmetric* case in which every player has the same strategy set S.

3. *A strategy profile*. A choice of a strategy for each player is called a strategy profile. Thus, for a two-player game where players has the same set S of strategies, a strategy profile is a pair $\pi(s_1, s_2) \in S \times S$, where s_i denotes the choice of player i.

4. *A payoff function*. When every player has chosen a strategy, they obtain a payoff according to a payoff function π which maps a strategy profile to a real-valued payoff for each player: $\pi(s_1, s_2) = (\pi_1, \pi_2) \in \mathbb{R} \times \mathbb{R}$, where π_i denotes the payoff to player i, $\mathbb{R}$ represent the set of real numbers.

5. *Nash equilibrium*. A strategy profile such that each player's strategy s_1^* is the best response to the other player's current strategies s_2^*, in other words, none of the players could increase their payoff by deviating from his or her strategy provided that

other players do not change their strategy, i.e., if $\pi_1(s_1^*, s_2^*) \geq \pi_1(s_1, s_2^*)$ for all s_1 and $\pi_2(s_1^*, s_2^*) \geq \pi_2(s_1^*, s_2)$ for all s_2, then (s_1^*, s_2^*) is a Nash equilibrium.

6. *Pareto efficiency (also known as Pareto optimality).* A strategy profile in which it is impossible to make any one individual better off without making at least one of other individual worse off, i.e., if $s_1^* > s_1$ for all s_1, $s_2^* > s_2$ for all s_2, or $\pi(s_1^*, s_2^*) > \pi(s_1, s_2)$ for all s_1 and s_2, then (s_1^*, s_2), (s_1, s_2^*) or (s_1^*, s_2^*) could be regarded as Pareto efficiency. In particular, it is interesting to see whether an Nash equilibrium is Pareto optimal.

2.2 Games and Coordination Games

2.2.1 Prisoner's Dilemma

The concepts introduced above can be illustrated by means of a canonical example of game theory. It is called "Prisoner's dilemma".

In the prisoner's dilemma, there are two prisoners P_1 and P_2, so the number of players $n = 2$. Both are from the same criminal gang. Both players have a set S of two strategies to choose from: $\{\text{confess, deny}\} = S$. The rules of this game is as following: If both deny, P_1 and P_2 serves 1 year in prison each, that is: π(deny, deny)= $(-1, -1)$. If both confess, each serves 2 years in prison, i.e.π(confess, confess)= $(-2, -2)$. But if only P_1 confess, i.e. (confess, deny) then P_1 will set free from the prison and P_2 is sentenced to 3 years alone in the prison, i.e. π(confess, deny)= $(0, -3)$ and vice versa, i.e. π(deny, confess)= $(-3, 0)$. This game requires P_1 and P_2 to design the best strategy to decrease the number of years staying in the prison as many as possible. The number of years here measures the payoff π according to strategy picked by two players in this game. In other examples, the payoff could be measured in terms of a certain amount of money or the utility that one attains from the outcome, etc.

The following matrix is an illustration of the payoff function of this game:

Prisoner's Dilemma

	P1 deny	P1 confess
P2 deny	(-1,-1)	(-3,0)
P2 confess	(0,-3)	(-2,-2)

Table 2.1: Prisoner's Dilemma

Nash Equilibrium in Prisoner's Dilemma

To find the Nash equilibrium, we start from identifying P_2's best responses to P_1's strategies. If P_1 stays silent, then P_2's best response is to confess because the best response means the highest payoff and it is 0 here (0, -3). When P_1 chooses to confess, the best move of P_2 will also be to confess (-2,-2). Mark these two points and we will turn to identify P_1's best response to P_2's strategies.

If P_2 chooses to deny his or her offense then it is best for P_1 to confess (-3,0); if P_2 chooses to confess, then P_1 should confess too in order to avoid 3 years but two years in jail (-2,-2).

In the 2×2 payoff matrix above, if we scan all the best responses we mentioned before, we will see that the only strategy profile that is both players' best responses is the strategy profile π(confess, confess)$= (-2, -2)$. Since this is the best response of both players to the other player, thus this is the only Nash equilibrium in this game.

Pareto Efficiency in Prisoner's Dilemma

We can find the Pareto efficiency by comparing the four strategy profiles. When it is impossible to find another profile such that one player's payoff is better without someone else's payoff is being worse, this profile will said to be Pareto optimal.

We begin with discussing the two asymmetric strategy profiles. These two strategy profiles (P_1 confess, P_2 deny) and (P_1 deny, P_2 confess) is a combination of maximised payoff (0) and minimised payoff (-3) among the four outcomes. When one of the player choose to deny while another player choose to confess, the payoff of the one who confess is maximised. When this player's payoff is fixed at 0, it is impossible to improve the payoff of the other one who denied without worse off the payoff of the confessed player. So these two profiles are both Pareto optimal.

Then we move on to the strategy profile (P_1 deny, P_2 deny), when P_1 deny and get -1, payoff of P_2 cannot be better. Thus this profile is Pareto optimal.

At last, let us move on to the Nash equilibrium π(confess, confess)$= (-2, -2)$, where both P_1 and P_2 could get a higher payoff by moving to π(deny, deny)$= (-1, -1)$. Thus this profile is not Pareto optimal. It is interesting to see that the Nash equilibrium is not a Pareto efficiency in this example.

According to the analysis above, we can see that we have two objectives at the same time: maximise the payoff of P_1 and P_2, respectively. However, under the circumstances of multi-goal, a strategy profile can reach the criteria for Pareto optimal as long as one of the player's payoff is maximised. Because it is not allowed to increase other players' payoff while keeping the player's own payoff at the same level sometimes.

2.2.2 Stag Hunt

In this subsection, we are going to introduce the concept of coordination games through the illustration of a game — stag hunt.

This game is about a conflict between social cooperation and individualism. Suppose there are two hunters in the forest, the number of players $n = 2$. They can choose to hunt between two kinds of targets: stag and hare. Thus the set of strategies of each player consist of stag hunting and hare hunting $\{$stag, hare$\} = S$. The difference is that hares can be captured by one and serves to the hunter for 7 days while stags needs the cooperation of two hunters but it will be the food of 10 days for both hunters. The strategy profiles are thus (stag, hare), (hare, stag), (stag, stag) and (hare, hare). Therefore, the rules of this game is clear: if both hunters decide to get their own food without the help of others, they both get their food for the following 7 days respectively, i.e. π(hare, hare)$= (7, 7)$; if both want a stag, then they will cooperate and get their food that is enough to eat for 10 days, i.e. π(stag, stag)= $(10,10)$. However, a hunter might die of starving if this hunter tries to hunt the stag independently, i.e. π(hare, stag)= $(7,0)$ or π(stag, hare)= $(0,7)$.

The outcome matrix illustrate the payoff function is shown as follows:

	stag	hare
stag	(10,10)	(0,7)
hare	(7,0)	(7,7)

Table 2.2: Stag hunt

This example belongs to a type of game called "coordination game". A coordination game is a subclass of a game with multiple Nash equilibria.

Nash Equilibria in Stag Hunt

To find the Nash equilibria of this example, we follow the routine which we used before — find out the best response of each players to the strategies of others.

From the perspective of H_1, if H_2 hunts a stag, H_1's best strategy is to hunt a stag too (stag, stag). However, if H_2 chooses a hare, then H_1 should also hunt a hare (hare, hare).

From the perspective of H_2, if H_1 hunt a stag, H_2's best strategy is to hunt a stag too (stag, stag). However, if H_1 chooses a hare, then H_2 should also hunt hare (hare, hare).

Therefore, (stag, stag) and (hare, hare) are both the best responses to the other player which implies that we have two Nash equilibria in this game. And more than one Nash equilibrium is a important feature to classify this game as a coordination game.

Pareto Efficiency in Stag Hunt

We start from discussing two strategy profiles: (stag, hare) and (hare, stag). When one of the player get 7 hares, the other player get nothing to eat. It is possible to improve the other player's payoff if we compare these two with other profiles. Thus these two asymmetric profiles are both not Pareto optimal.

Then we turn to the profile (hare, hare), this profile still does not fulfill the requirement for Pareto efficiency because both player can get a higher payoff without hurting anyone's payoff.

At last, the profile (stag, stag) is the only outcome that is Pareto optimal because both players already get the highest payoff that they can get from this game.

Chapter 3

The Diamond–Dybvig Model

This chapter is the core of this report which explains real-world bank runs by introducing the Diamond–Dybvig model. The conclusion derived by Diamond–Dybvig model coincide the results getting from the equilibrium analysis of bank runs modelled as a coordination game.

3.1 Banks Runs Modelled as A Coordination Game

People deposit money in the bank and expect to be able to withdraw them again, with interest, at a later date. The bank uses the deposited money for various investments, such as lending money to others, on the expectation that people will generally not withdraw their deposits early. Thus banks may not have liquidity if many people want to withdraw earlier than the bank expected.

If people expect others to withdraw late, it will typically also be in their own interest to withdraw late because they can reap the full interest on their deposit. If, on the other hand, people expect others to withdraw early, then they may think that the bank is going to face problems with liquidity, which will make it risky to have deposits in the bank and therefore create an incentive for people to withdraw early.

To understand the Diamond–Dybvig model easier, suppose there are two depositors. So this is a game with 2 players: $n = 2$. Both of them have deposit in a bank. Each depositor is free to make choice about when to withdraw their money in the bank. If this is the case, this 2 depositors can only get their principal back if they are withdrawing early, i.e. π(early, early)= (a, a). Instead, if both 2 depositors are withdrawing late, they will both get a higher payoff — the total amount of deposited principal plus the interest will be the final payoff, that is: π(late, late)= $(a+b, a+b)$, where $a+b$ is the principal plus the interest.

Assume that there are only two possible strategies, namely to withdraw early or late. Further assume that if one player withdraws early then the other player should also withdraw early to avoid a loss, but if one player withdraws late then the other player should also withdraw late to maximise profit. This can be captured with the following payoff matrix, in which the parameters a and b are assumed to satisfy $a+b > a > b > 0$.

This is also a coordination game similar to Stag hunt in the previous chapter.

Using the same method to find the Nash equilibria, the “best responses” are (early, early)

	early	late
early	(a,a)	$(a,0)$
late	$(0,a)$	$(a+b,a+b)$

Table 3.1: The payoff matrix of bank runs

and (late, late).

But (late, late) Pareto dominates (early, early) because the payoff π(late, late)= $(a+b,a+b) > \pi$(early, early)= (a,a), i.e. this pair π means that both elements in the first bracket are larger than the elements in the second bracket, respectively.

If 2 depositors both choose different strategies (early, late) or (late, early), the payoff of one of the player turns out to be 0 while the other player could still obtain a positive payoff a. So these two strategy is harming one of the players' payoff who get 0.

Therefore, Pareto efficiency is reached at the profile (late, late).

3.2 The Diamond–Dybvig Model

Two well-known financial experts Douglas W. Diamond and Philip H. Dybvig put forward a far-reaching Bank run model in 1983 — the Diamond–Dybvig model. The paper by Diamond and Dybvig (1983) analysed the behaviour of panic and bank runs in an unique insight. The model deploys concise mathematical structure to analyse sophisticated economic phenomena and economic behaviour. It illustrates the causes of bank runs, process of bank run and also the result which has harmful effect on real economy.

The Diamond–Dybvig model states that banks are intermediaries between depositors and borrowers. On one hand, Borrowers are usually investing on business with long-term return. Therefore they need long-term loans from the bank. On the other hand, depositors of the bank might need cash at any time. This implies a contradiction between borrowers and depositors. However, the vital point that Diamond and Dybvig state is that the cash demands of depositors is unlikely to happen at the same time under ordinary economy circumstances. This is a precondition for the bank to exist and work normally.

The model is characterised by three periods ($T = 0,\ 1,\ 2$) and a single homogeneous product. In period 0, only one unit is endowed, then the production could yield a payoff R (where $R > 1$) in period 2; if production is interrupted in period 1, then the output of production is just the initial investment(one unit). Thus there is a payoff decision to be made between $(1,0)$ and $(0,R)$ in period 1. Table 3.2 represents productive technology in the Diamond–Dybvig model. The unit which is endowed is regarded as investment and the units obtained after the investment are the units that can be used to consume.

	$T = 0$	$T = 1$	$T = 2$
Interruption	-1	1	0
No-interruption	-1	0	R

Table 3.2: A formal representation of consumption and investment in the Diamond-Dybvig model

All depositors are divided into two types of agents (impatient agent and patient agent).

- Impatient agent: consumer has liquidity needs and withdraws early at time $T = 1$.
- Patient agent: consumer has liquidity needs and withdraws late at time $T = 2$.

In period 0, all depositors are the same, everyone do not know his or her type. Each person is given an initial endowment of one unit, see Yi (2009). In period 1, consumers know their types, but the information is private information, i.e. the public was not observed. Impatient agent could obtain product and consume at period 1, while patient agent obtain products and consume at period 2.

Let c_k^i represent consumption at period k of agent who is type i, therefore, the agents obtain $c_1^1 = 1$, $c_2^1 = c_1^2 = 0$, and $c_2^2 = R$ according to the productive technology in the Diamond–Dybvig model.

When the type of each agent is publicly observable at period 1, agents could write a reasonable contract that could yield a appropriate output for different types of agents. The reasonable contract could offer an optimal consumption c_k^{i*}, the optimal consumption satisfies $R > c_2^{2*} > c_1^{1*} > 1$, see Diamond and Dybvig (1983).

3.2.1 Introduction to the demand deposit contract

In the Diamond–Dybvig model, a bank offers a demand deposit contract which ensure that each agent can obtain liquidity insurance. The contract requires each agent to deposit their endowment with bank at period 0. Then, the agent who withdraw at period 1 could obtain a payoff $r_1 > 1$. All agents are served sequentially in random order, until the bank will not have enough money. The contract satisfies a sequential service constraint(SSC), which specifies that only those agents who are early in line can get bank-promised payoff. The bank is mutually owned and pay the payoff for the patient agents in period 2. In period 2, each patient depositor could receive a pro–rated share of the bank's assets. Let V_1 and V_2 be the payoff to the agents who choose to withdraw in period 1 and withdraw in period 2, respectively. Then, the contract could be formulated by

$$V_1(f_j, r_1) = \begin{cases} r_1 & \text{if } f_j < r_1^{-1} \\ 0 & \text{if } f_j \geq r_1^{-1} \end{cases} \tag{3.1}$$

and

$$V_2(f, r_1) = \max\left\{R(1 - r_1 f)/(1 - f), 0\right\} \tag{3.2}$$

In the above equation, f is the share of depositors who withdraw early, f_j is the share of depositors who withdraw early and before player j. $r_1 f$ is the share of all money withdrawn early, when $r_1 f > 1$, in which case all money is withdrawn early because "1" here represent the total amount of money in the bank. Therefore, $f_j \cdot r_1$ represent the share of all money withdrawn early and before player j.

In the case of V_1, if $f_j \cdot r_1 < 1$, then each withdrawal get r_1. But if $f_j \geq {r_1}^{-1}$, i.e. $f_j \cdot r_1 \geq 1$, then all the money of the bank is withdrawn by early withdrawals, so the depositors after j get 0.

Since we introduced $r_1 f$ is the share of all money withdrawn early, thus $\max\{1-fr_1,\ 0\}$ is the share of money left in the bank for late withdrawals. R is the output yield by productive technology in period 2. So $(1-r_1 f)\cdot R$ is the share of all money that is available for late withdrawals. And this amount is shared by all late withdrawals, who each get:

$$\frac{\max\{1-fr_1,\ 0\}\cdot R}{(1-f)}. \tag{3.3}$$

3.2.2 Equilibrium Analysis

Based on the demand deposit contract introduced in Subsection 3.2.1, let us discuss the strategy of two types of agents (players). Since type 1 players have liquidity needs as time 1, whereas type 2 players have not, therefore type 1 players will always withdraw early and type 2 players will choose to withdraw early or late. Despite players' liquidity needs, their choice will also depend on whether they believe that many or few will withdraw early. In other words, players of type 2 are assumed to make their decisions in order to maximise their expected payoff, i.e., by comparing V_1 and V_2. The figure below illustrate the expected payoff of V_1 and V_2.

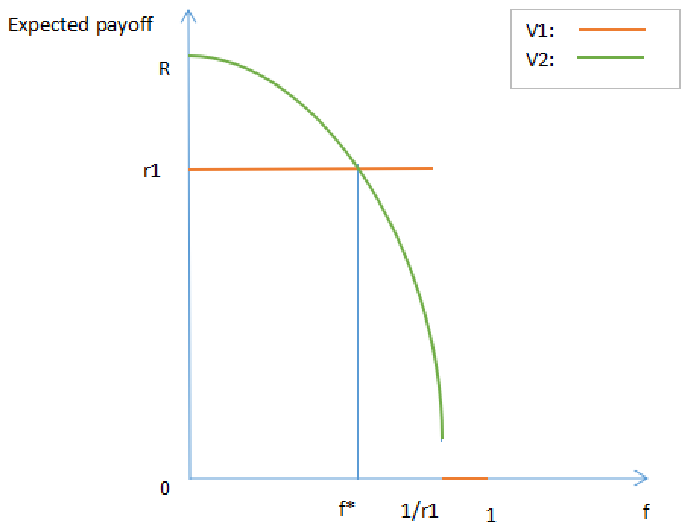

Figure 3.1: The expected values of V_1 and V_2 when $r_1 > 1$

The line of V_1(The payoff of those who choose to withdraw early) and V_2(withdraw late) have a crossing point named as the breaking point f^* in Figure 1. The players at f^* get exactly the same amount of money from the bank whether they withdraw early or late. Let t denote the frequency of type 1' s, i.e. the minimal frequency of early withdrawal. Since the longitudinal axis f denotes true frequency of early withdrawals($t \leq f \leq 1$), we would want to know the expression of f at this breaking point f^*:

$$r_1 = \frac{R\cdot(1-r_1 f^*)}{1-f^*},$$

solving for f^* yields $f^* = \frac{R - r_1}{r_1 \cdot (R-1)}$.

Equilibrium decision when $r_1 > 1$

Based on the value of f^* we can distinguish two cases:

For the first case, if $f^* < t$, which means that we must be to the right of the break-point in the figure, in which case $V_1 > V_2$ so the optimal decision for type 2 players is to withdraw early. Therefore only bank run is an equilibrium when $r_1 > 1$.

For the second case, if $f^* > t$, which means that we must be to the left of the break-point in the figure, in which case $V_2 > V_1$ so the best decision for type 2 players is to withdraw late in order to obtain a higher payoff. Thus there are two equilibria: bank run or players of type 2 withdraw late, when $f = t$.

Above is the case when $r_1 > 1$, which is susceptible to bank runs. But when $r_1 = 1$, bank is free from the threat of runs, this is illustrate by the following figure. The expression of expected value for $V_2(f, r_1)$ simplifies to the constant R. Therefore the previous graph will no longer apply and bank run will not be an equilibrium.

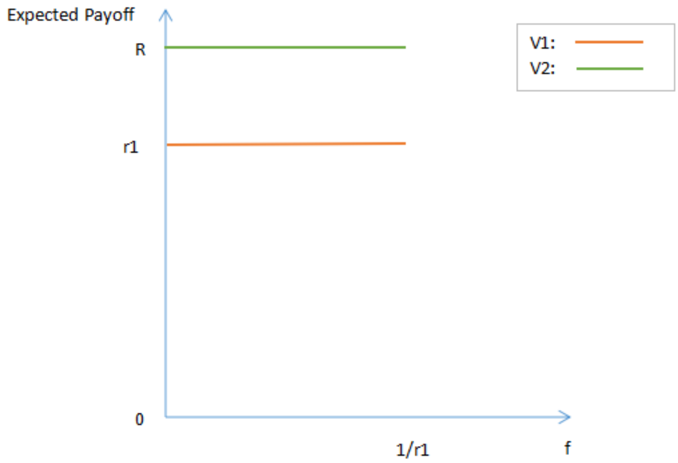

Figure 3.2: The expected values of V_1 and V_2 when $r_1 = 1$

Chapter 4

Prevention of Bank Runs

In this chapter, I will give a short exposition of the preventive action provided in Diamond and Dybvig's paper.

4.1 Suspension of Convertibility

Suspension of convertibility simply means the bank can suspend to cash in the deposits in order to prevent a bank run of a larger scale. The bank decides the timing of conducting such action. This timing is when a certain fraction f_j of depositors has withdrawn the deposits. The fraction f_j is not allowed to be larger than $\hat{f}$ which is the fraction of initial depositors withdraw at period 1(the fraction of impatient depositors.) As long as $f_j < \hat{f}$, the bank will work just fine. In addition, suppose one deposit x at time 0, it will be worth r_1 at period 1 and R at period 2. Therefore, the expected payoff of V_1 and V_2 will be

$$V_1(f_j, r_1) = \begin{cases} r_1 & \text{if } f_j \leqslant \hat{f} \\ 0 & \text{if } f_j > \hat{f} \end{cases} \tag{4.1}$$

and

$$V_2(f, r_1) = \max\left\{ \frac{(1 - f r_1)R}{1 - f}, \frac{(1 - \hat{f} r_1)R}{1 - \hat{f}} \right\} \tag{4.2}$$

This $\hat{f}$ could be regarded as f in Figure 1. When f_j is controlled to be equal or smaller than $1 - \frac{R}{r_1} + R \cdot f$, bank run will not occur no matter a depositor withdraws early or late. As we can find from the payoff equation above, a early withdraw V_1 will always get a payoff equals r_1 if f_j is controlled. In the case of V_2, $\hat{f}$ ensures that no one withdraw late should be worried about 0 payoff. Thus the objective of suspension of convertibility is achieved.

The advantage of this solution is that it stops not only a worse bank run but also stops the panic among depositors because the depositors know that they will not lose their deposits and they will be able to withdraw at a later time. The bank shows that the deposits are under good control by doing so.

However, such solution have its disadvantage too — it is harmful for those depositors who actually have liquidity needs to make consumptions. Most time it is fine if the bank doing so.

However, it is not acceptable by those depositors after the early withdraw j who really have to withdraw but facing suspension of convertibility at the same time.

To sum up, as long as the critical point is not reached, depositors could withdraw as they want. If the point is unfortunately reached, suspending the convertibility will still powerfully secure the payoff of depositors. Nevertheless, the bank should not take such actions frequently.

4.2 Deposit Insurance

It has been showed that suspension of convertibility has its own limitation. Therefore, another remedy called deposit insurance is worthy to mention. Actually, this solution is preferred in real life.

Deposit insurance is a solution provided and instituted by government. It is frequently conducted especially during situations like an economic crisis. No matter how many depositors is about to withdraw, the government make promise to offer the amount that depositor supposed to get.

A bank with deposit insurance is highly credible and fully confident to announce that bank runs will not exist. Government is able to provide such benefit and insurance because government have the authority of taxation. The share of the tax is as follows:

$$\tau(f) = \begin{cases} 1 - \frac{c_1^{1*}(f)}{r_1}, & \text{if } f \leq \bar{t} \\ 1 - \frac{1}{r_1}, & \text{if } f > \bar{t} \end{cases} \tag{4.3}$$

where $c_1^{1*}(f)$ and $c_2^{2*}(f)$ is the optimal consumption for a type 1 agent at period 1 and a type 2 agent at period 2 as we introduced in section 3.2. $\bar{t}$ is the frequency of type 1' s withdrawal.

The expected payoff of those who withdraw early was equal to r_1 when tax is not applied. But the payoff now is a fraction of r_1 with tax to ensure that everyone get at least 1 no matter they withdraw at any time. By doing this, bank run is avoided efficiently.

The proceeds that V_1 and V_2 can get after tax is therefore as follows:

$$V_1(f) = \begin{cases} c_1^{1*}(f), & \text{if } f \leq \bar{t} \\ 1, & \text{if } f > \bar{t} \end{cases} \tag{4.4}$$

and

$$V_2(f) = \begin{cases} \frac{R\{1-[c_1^{1*}(f)f]\}}{1-f} = c_2^{2*}(f), & \text{if } f \leq \bar{t} \\ \frac{R(1-f)}{1-f} = R, & \text{if } f > \bar{t}, \end{cases} \tag{4.5}$$

where $R > c_2^{2*} > c_1^{1*} > 1$.

If the fraction f is less or equal to $\bar{t}$, which means only type 1 withdraw in period 1. Then V_1 is maximised and equals to $c_1^{1*}(f)$.

For the case of the payoff function V_2, the share of the money withdrew early is equal to $c_1^{1*}(f) \times f$ when $f \leq \bar{t}$. Therefore $1 - \left[c_1^{1*}(f)f\right]$ represent the share of money left in the bank after early withdrawals. $\frac{R\{1-\left[c_1^{1^*}(f)f\right]\}}{1-f}$ thus represent the amount of money left in the bank that is shared by all late withdrawals in period 2.

Thus the optimal is achieved at:

$$V_1(f=t) = c_1^{1^*}(t),$$
$$V_2(f=t) = \frac{R\{1-\left[c_1^{1^*}(t)t\right]\}}{1-t} = c_2^{2^*}(t).$$

These two are optimal because both type 1 and type 2 depositors get a good payoff if only type 1 depositors withdraw early, i.e. $f \leq \bar{t}$.

While comparing deposit insurance and suspension of convertibility, it showed that the advantage of deposit insurance is that it is a law which commits the government to insure bank runs will not happen. For the case of suspension of convertibility, this is often a discretionary policy.

Chapter 5

Conclusions

In this thesis, we focus on discussing the Diamond–Dybvig model to explain the occurrence of bank runs in real life. This model is an influential model in banking theory. Because this model studied and proved three important points:

1. *The main function of a bank is to provide long-term loans to those who need to invest during a relative long period and at the same time attract deposits by offering more liquid than the original principal of depositors in return. By successfully attracted a large number of deposits, bank could share the risk among depositors since depositors' liquidity needs is not likely to arise simultaneously.*

2. *In the ideal situation, if depositors only withdraw when they have consumption needs, the banking system will work well, this is the case of a good equilibrium. However, the model also pointed out that a bad equilibrium might happen according to the theory of coordination games. Bank runs is one of the two outcome of this coordination game.*

3. *The Diamond-Dybvig model proved that the equilibrium of bank run is possible to occur at any bank based on various of reasons, e.g. rumors of bank runs, etc. Therefore, Diamond and Dybvig came up with two ways to prevent bankruptcy, the first one is the suspension of convertibility which is to threat that the bank is not allow to liquid a certain fraction of deposits. The second solution is to offer deposit insurance by the government.*

In chapters 3 and 4, I mainly studied the Diamond–Dybvig model by doing a literature review and attempted to write a mathematical exposition of the model. Concerning these two chapters, three questions are defined as follows to help us better understand this model. The answers of each question has been formulated in previous chapters. Here, we summarise the answers.

1. Under which conditions should we expect bank runs to occur?

Let's begin with the situation where bank runs is not a equilibrium. According to the deposit contract, we have shown that when $r_1 = 1$, bank runs will not occur because those who withdraw late can obtain a better payoff R than those who withdraw early and obtain r_1. The profit from withdraw late attract type 2 to stay and withdraw at a later time.

However, if r_1 is larger than 1, i.e. $\frac{1}{r_1} < 1$, when $t > f^*$, the share of type 1' s is too large that the money left in the bank become scarce, bank runs is around the corner. The later a depositor come after the point f^*, the less payoff he or she might get simply because the money left in the bank is insufficient. When a lot of type 2' s realise this, they will run to the bank and ask for withdraw. This is how bank runs occur.

2. How to model bank runs as a coordination game?

In our discussion about bank runs, there are two types of depositors: type 1 and type 2. These two types can be regarded as a game with two players. Each player have a set of two strategies: to withdraw early or late. Both players make their choice according to the expected payoff for themselves. This is a situation similar to the stag hunt game where both players tend to have the same decision: both withdraw late or both withdraw early. Bank runs is the case when both players withdraw early.

3. What can be done to avoid bank runs?

Two remedies was introduced in chapter 4, suspension of convertibility and government deposit insurance. Suspension of convertibility is a way to avoid bank runs by limit the number of withdrawals when the fraction of those who want to withdraw exceed a "safe" level which a bank have to keep this fraction below the safe level to ensure that the bank have money left to pay back other depositors later. In other words, when this safe level is reached, the bank will restrict withdrawals strictly to avoid bank runs. Even though suspension of convertibility is a efficient way to avoid runs, its biggest shortcoming is that it cause inconvenience to those who need to withdraw urgently.

The second method of offering deposit insurance is a better method to avoid bank runs. This government insurance is offered by levy a tax. Early withdrawals might include a higher number of tax than late. Thus more agents are willing to withdraw late to maximise their payoff except the situation when they really have consumption needs.

Chapter 6

Reflection of Objectives of This Thesis

Objective 1

For Bachelor degree, student should demonstrate knowledge and understanding in the major field of study, including knowledge of the field's scientific basis, knowledge of applicable methods in the field, specialisation in some part of the field and orientation in current research questions.

The major field of study is game theory and finding out the reasons of bank runs. Based on the aim of my thesis, I did a study on the Diamond–Dybvig model because their model together with the knowledge from game theory, especially the knowledge of coordination games, help me to gain a good understanding of the question that I want to solve in this thesis work. In chapter 2, I introduced the basis of game theory in detail with two classical examples: Prisoner's dilemma and Stag hunt. The comparison of these two games give prominence to the characteristics of coordination games. I start chapter 3 by introducing a bank runs model according to the basics introduced in chapter 2. Thus the whole structure of this thesis is build up step by step with related and necessary knowledge as the foundation.

Objective 2

For Bachelor degree, the student should demonstrate the ability to search, collect, evaluate and critically interpret relevant information in a problem formulation and to critically discuss phenomena, problem formulations and situations.

This thesis work was initialised by collecting and summarising the literature related to my major field of study — game theory and bank runs. Without the ability to searching, collecting and evaluating the useful and valuable materials that I have found for this thesis work, I would not be able to present this writing report because what I have wrote here relies on the information and knowledge that I obtained from the effort of literature review. Furthermore, I critically discussed the phenomenon of bank runs and the establishment of bank runs in chapter 1 and 3.

Objective 3

For Bachelor degree, the student should demonstrate the ability to independently identify, formulate and solve problems and to perform tasks within specified time frames.

The fulfillment of this requirement can be mainly demonstrated by the content in chapter

3. In section 3.1, the problem is identified, and then been formulated by introducing the Diamond–Dybvig model. The equilibrium analysis in section 3.2.2 solved the primary problem in this thesis which is the conditions to the occurrence of bank runs. The solutions to the occurrence of bank runs is also identified as an interesting and important question to discuss in this thesis. I gave answer to this question based on the method of a literature study in chapter 4.

Objective 4

For Bachelor degree, the student should demonstrate the ability to present orally and in writing and discuss information, problems and solutions in dialogue with different groups.

The thesis presentation is a chance to demonstrate the objective of one's thesis work and the answer to it. So the presentation will be divided into four parts: introduction, problem formulation, solution and conclusion part. This ability will later be demonstrated during the presentation.

Objective 6

For Bachelor degree, student should demonstrate ability in the major field of study make judgments with respect to scientific, societal and ethical aspects.

The reason of the occurrence bank runs can be complicated so the explanation of this phenomenon can involve different aspects like scientific, societal and ethical aspects. In the thesis, the introducing and apply of the Diamond–Dybvig model is a representation for scientific aspects. While the discussion of game theory can be regarded as a perspective from societal and ethical aspects. Game theory is a study about making rational choices which can help specialist to predict or analyse human behaviour like bank runs, i.e. the human behaviour when depositors rush to withdraw simultaneously. The choice made by a people usually involves largely on the societal and ethics. Therefore, the combination of chapter 2 and 3 construct a thesis with the full consideration of all three aspects in my thesis.

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