PROBLEM SOLVING IN MATHEMATICS TEXTBOOKS

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Problem solving in mathematics textbooks

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The Swedish name of the graduate school Developing Mathematics Education is “Att utveckla undervisning och didaktik i matematik”.

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Abstract

The aims of this thesis are: 1) to analyse how mathematical problem solving is represented in mathematics textbooks for Swedish upper secondary school, 2) to introduce an analytical tool to categorise tasks as being a mathematical problem or an exercise. The thesis is a widening and deepening of the two papers it builds on, and also connects the two papers to each other. Paper I is a textbook analysis of how mathematical problem solving is represented in textbooks. The analysis covers three predominant Swedish textbook series. Based on an analysis of 5,722 tasks, the conclusion drawn here is that the textbooks themselves contain very few tasks that may be defined as mathematical problems. The ones which are mathematical problems are found at the end of a chapter, at the most difficult level and are produced in pure mathematical context. The analytical tool to separate tasks into mathematical problems and exercises, developed in Paper I, is then elaborated on in Paper II, where the rationale for and the underlying perspectives of the tool are highlighted and evaluated. Thus, the result of Paper II is an analytical schema in conjunction with the argumentation for and the supplement to the schema. The tool contributes to the research field of mathematics education as it is supposed to be applicable to any textbook worldwide. Further, the result from Paper I serves the research field of mathematics education as one piece of knowledge for e.g. future comparative studies. Contributions to ‘the practice’ are mainly to teachers and textbook writers, for whom the result from Paper I is informative and the result from Paper II may serve as a guideline for interpreting mathematical problem solving in textbooks. Further plausible contributions are discussed.

Keywords: Mathematics textbooks, problem solving, textbook analysis, upper secondary school
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List of Papers

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CHAPTER 1

Introduction

The textbook is the most important artefact in mathematics education (Fan, Zhu & Miao, 2013; Love & Pimm, 1996; Pepin & Haggarty, 2003; Thomson & Fleming, 2004), the major resource for teacher planning and student practice (Boesen et al., 2014; Jablonka & Johansson, 2010; Pepin & Haggarty, 2003; Stein, Remillard & Smith, 2007), operationalizes the steering documents (Boesen et al., 2014) and often defines, to the students, what (school) mathematics is (Johansson, 2003). The textbook has even been described as the curriculum (Budiansky, 2001) and the importance of textbooks in mathematics education is found worldwide (e.g. Li, Chen & An, 2009; Van Stiphout, 2011; Vincent & Stacey, 2008). The importance of the textbook is highlighted in recent research on classroom practice in Sweden which finds that student’s work in the classroom is dominated by solving tasks in textbooks (Boesen et al., 2014). Also, the content and goal of textbooks internationally focuses more on procedural skills than on conceptual understanding and mathematical problem solving (abbreviated MPS) (e.g. Vincent & Stacey, 2008).

However, although textbooks do not seem to promote conceptual understanding and MPS, such competencies are central in many countries (e.g. MOE, 2007; NCTM, 2000). In Sweden, in the new syllabus released in 2011, MPS is stressed as a central competence for the student to develop (Skolverket, 2011). Further, MPS is mentioned as one of the main goals in school mathematics (ibid.) and is also included in the seven competencies defined for students to develop and for teachers to assess (ibid.). As the new Swedish steering documents were introduced, commercially managed publishers started to rework old textbooks and produce new textbooks.

To sum up, the importance of the textbook seems indisputable and earlier research points out how the content of textbooks focuses on procedural skills rather than MPS. At the same time the importance of MPS is stressed internationally in various steering documents. These two contradictory facts motivate research on mathematics textbooks with respect to how MPS is represented.

1 Throughout the thesis, MPS is short for Mathematical Problem Solving, MP is short for a Mathematical Problem (a task) and MPs is short for Mathematical Problems (tasks, plural).
CHAPTER 1

Introduction

The textbook is the most important artefact in mathematics education (Fan, Zhu & Miao, 2013; Love & Pimm, 1996; Pepin & Haggerty, 2003; Thomson & Fleming, 2004), the major resource for teacher planning and student practice (Boesen et al., 2014; Jablonka & Johansson, 2010; Pepin & Haggarty, 2003; Stein, Remillard & Smith, 2007), operationalizes the steering documents (Boesen et al., 2014) and often defines, to the students, what (school) mathematics is (Johansson, 2003). The textbook has even been described as the curriculum (Budiansky, 2001) and the importance of textbooks in mathematics education is found worldwide (e.g. Li, Chen & An, 2009; Van Stiphout, 2011; Vincent & Stacey, 2008). The importance of the textbook is highlighted in recent research on classroom practice in Sweden which finds that that student’s work in the classroom is dominated by solving tasks in textbooks (Boesen et al., 2014). Also, the content and goal of textbooks internationally focuses more on procedural skills than on conceptual understanding and mathematical problem solving (abbreviated MPS\(^1\)) (e.g. Vincent & Stacey, 2008).

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sented in them. Additional justification for this research is that work done by students in Swedish classrooms mainly consists of solving tasks in textbooks (Boesen et al., 2014) combined with the fact that Swedish textbooks are not under state control (Jablonka & Johansson, 2010). Also, textbooks for upper secondary school in Sweden tend not to convey the notion of modelling (Frejd, 2013), which may be seen as one part of a MPS process (Kongelf, 2011) or even as a definition of MPS (Lech & Zawojewski, 2007).

Therefore, the aims of this thesis are: 1) to examine how MPS is represented in the new mathematics textbooks, adapted to conform to the new steering documents for Swedish upper secondary school, and 2) to present an analytical tool to separate tasks in mathematics textbooks into MPs and exercises. The aim is operationalized through the research questions;

Q1 What characterises the distribution of MP tasks in three selected Swedish textbook series?

Q2 Which aspects are important to consider in designing an analytical tool to separate tasks into MPs and exercises in a mathematics textbook?

The first research question is operationalized through four analytical questions: What proportion of the tasks could be classified as MPs? For those which are classified as such, where in the textbook are they found, at what level of difficulty are they and in what context are they produced? Rationale for these analytical questions is elaborated on in section 3.2.2. According to Q1 the mathematical area calculus is examined according to its significance for future MP solvers. To answer Q1, via the analytical questions, a tool to analyse textbooks was developed in Paper I. It was not possible to fully describe the work of developing the analytical tool within Paper I. Thus, to answer Q2, Paper II elaborates on the analytical tool.

This thesis summarises the content of the two papers it builds on and connects them to each other. It also complements the papers by giving more detailed explanations of parts that, due to lack of space, are incompletely described in the individual papers. The thesis is intended to be read while the reader has access to the original papers, since reference to the papers is made in preference to repeating parts of them. Nevertheless, some crucial parts of

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2 Tasks in textbooks are divided into MPs and not MPs. Tasks that are not MPs are, throughout the thesis, called exercises following Schoenfeld (1985) “If one has ready access to a solution schema for a mathematical task, that task is an exercise and not a problem.” (p. 74).

3 This is further elaborated on in section 2.3 and 3.2.
the content of the papers, such as the analytical schema, are repeated in the thesis in order to make it intelligible.

The text is structured as: Chapter 2, literature review, summarises the literature studies applicable to the research conducted, and also serves as background reading to the issues of this thesis. The focus is on MPS, mathematics textbooks, and the mathematical area calculus; Chapter 3, methodology, consists of three subsections. The first presents the examined textbooks as the data source. In the second subsection, analytical approach, the headlines of the analytical approach of the two papers are described and in the final subsection, ethical considerations, limitations and trustworthiness of the study is discussed; In Chapter 4, summary of papers, the two papers are presented separately, focusing on the result of each study; In Chapter 5, conclusion and discussion, the results of the study are linked to the literature review to answer the research questions to create conclusions, which are discussed in relation to the literature. Chapter 5 ends with a discussion of contributions of the thesis and suggestions for further research.
CHAPTER 2

Literature review

This chapter is divided into three subchapters and presents literature of relevance to the thesis. The subchapters Mathematical problem solving and Mathematics textbooks, respond to the aim of the thesis and the subchapter calculus motivates the choice of mathematical area for the research, which is a methodological delimitation. The review is a widening (more to read) and deepening (more thorough descriptions) of the literature overview in Paper I, and the literature referred to in Paper II.

2.1 Mathematical problem solving

Problem solving (PS) is something we use regularly in everyday and professional life and is sometimes regarded as “the most important learning outcome of life” (Jonassen, 2000, p. 63). In educational terms, Gagné (1980, p.85) believes “the central point of education is to teach people to think, to use their rational powers, to become better problem solvers”. In trying to find reasons why teaching PS is important, Pehkonen (1997) finds no specific answer but rather a large and worldwide agreement on the importance of teaching MPS. Pehkonen’s research revealed that the two most frequent reasons for teaching MPS among teachers were i) Problem solving develops general cognitive skills and ii) Problem solving motivates pupils to learn mathematics. Thus, educators agree on the importance of developing students’ ability in MPS and this is reflected in the national steering documents of many countries (e.g. MOE, 2007; NCTM, 2000; Skolverket, 2012, a) which focus on MPS.

In different attempts to define MPS, three key features seem to be recurrent; 1) no solution schema for solving the task is at hand to the solver (Björkqvist, 2001; Blum & Niss, 1991; Lithner, 2008), 2) the solver needs to put some effort to come to a solution and that effort should be a cognitive challenge and not a computational one (Blum & Niss, 1991; Hagland et al., 2005; Lesh & Zawojewski, 2007) and 3) the problem should be meaningful or, for the solver, worth solving (Hagland, Hedrén & Taflin, 2005; Jonassen, 2000). In Schoenfeld (1985, p. 74) a MP is defined as;
“Being a ‘problem’ is not a property inherent in a mathematical task. Rather, it is a particular relationship between the individual and the task that makes the task a problem for that person. [...] If one has ready access to a solution schema for a mathematical task, that task is an exercise and not a problem.”

In the appendix to the Swedish steering documents for upper secondary school, MPS is defined as; “… a task that is not of standard character and can be solved by routine.” and “… every question where there, for the student, is no known solution method at hand may be seen as a problem” (Authors translation) (Skolverket, 2012, b, p. 2). Thus, the Swedish syllabi emphasise the first feature, and the last sentence in Schoenfeld’s definition of MPS, no solution schema is at hand to the solver for solving the task. Schoenfeld (1992) further states; “every study or discussion of problem solving (should) be accompanied by an operational definition of the term and examples of what the author means” (p. 364). Such an operational definition, along with operational definitions of words as they are to be understood within the thesis, is elaborated on within the methodology section.

Beyond defining MPS, Schoenfeld (1985) indicates four competencies that are needed for being a successful problem solver: resources, heuristics, control and beliefs. In short, resource concerns the mathematical tools the solver needs to solve the problem and heuristics concerns different strategies when solving a problem. Control concerns self-regulation and metacognition when the solver reflects upon his/her heuristics and resources. Beliefs concerns what preconceptions the solver has about mathematics and MPS. In this thesis the competence of resources, which refers to facts, procedures and skills and is called ‘mathematical skills’, or ‘knowledge base’ (Schoenfeld, 1992), is of particular relevance as the textbook is seen as the main artefact that transfers the knowledge base to the students.

Lech and Zawojevski (2007) introduce two perspectives on developing such MPS competencies; the traditional and the models-and-modelling perspective. The traditional perspective is described with four steps; 1) Master prerequisite ideas and skills, which corresponds to Schoenfeld’s competence resource, 2) Practise the new skills, which corresponds to Schoenfeld’s competences resources and practices, 3) Learn general problem solving processes and heuristics, which corresponds to Schoenfeld’s competences heuristics and control and 4) Learn to use 1, 2 and 3 when additional information is required, which corresponds to applying Schoenfeld’s competences heuristics and control. In this perspective, applied problem solving (step 4) is treated as a subset of traditional problem solving. The models-and-modelling perspective is described as “… mathematical ideas and problem-solving capabilities co-develop during the problem-solving process. The constructs, processes and abilities… are assumed to be at intermediate stages of development, rather than ‘mastered’ prior to engaging in problem solv-
ing.” (ibid, p. 783). In this view traditional problem solving is treated as a subset of applied problem solving, or in other words, start with step four (a real problem) and via that go through steps 1-3. Ryve (2006), inspired by Wyndhamn, Riesbeck and Schoultz (2000), expresses these two perspectives of MPS as learning mathematics for MPS (the traditional perspective) or through MPS (the models-and-modelling perspective).

Further Lech and Zawojewski (2007) argue that when the iterative cycles in understanding a problem and interpreting the situation mathematically, modelling, is done, the link to adequate procedures to solve the problem become almost trivial. Thus they claim that modelling is not only a part of, but also even a definition for MPS. According to the modelling aspect of MPS, Frejd (2013) examines how modelling is presented in Swedish textbooks for upper secondary school. He found that these textbooks, adapted for the new written curriculum in Sweden (Skolverket, 2011), had a lack of tasks according to the notion of mathematical modelling. These aspects, the modelling as a definition of MPS or a part of the process in MPS, combined with the lack of tasks according to modelling in Swedish textbooks, further motivate research on these textbooks with respect to MPS.

Continuing the Swedish case, MPS has been a recurrent notion in Swedish mathematics syllabuses. When comparing previous Swedish mathematics syllabuses Wyndhamn et al. (2000) point to a change regarding MPS and learning mathematics. Lgr 62; Lgr 69; Lgr 80; Lpo 94 are examined and for Lgr 62 and Lgr 69 MPS seems to be treated as something the student reaches after having learned some mathematical techniques, as MPS function as a summary of and application of what has been learned. In Lgr 80 MPS is a topic of its own to be learned. The emphasis in Lpf 94 is to use MPS to learn mathematics through. In brief, the treatment of MPS in previous Swedish written curricula goes from learning mathematics for MPS via about MPS to through MPS. In the new curriculum for upper secondary school, PS in general is mentioned in “aims for exam”4 for several national programmes, MPS as both “means and ends” in the description of the subject mathematics and as both a “central content”5 to learn and one of seven “competencies to assess”6 (even for the lowest rating) in the syllabus for mathematics (Skolverket, 2011). Thus, that description emphasises MPS as something to learn mathematics both through and for.

To what extent MPS is focused on the teaching of mathematics probably depends on how MPS is represented in the textbook used (as it is the major

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4 Authors’ translation.
5 Authors’ translation.
6 Authors’ translation.
resource for teachers´ planning and students´/teachers´ classroom practice?) and how the teacher perceives and interprets the term MPS. When Swedish teachers at upper secondary school interpret MPS as it is described in the mathematics syllabi, Berqvist and Bergqvist (2012) found that the most common answers were a task to solve (50.8%), word task (7.1%) and a task set into a context (6.3%). These results suggest that more than 60% of the Swedish upper secondary school teachers interpret MPS, which is an ability to assess, in a way that is not based on any definition of MPS.

Finally, the core definition of MPS in this thesis*: “If one has ready access to a solution schema for a mathematical task, that task is an exercise and not a problem” (Schoenfeld, 1985, p. 74), is also adopted by Lithner (2006 & 2008) in a conceptual framework for different types of mathematical reasoning*. Lithner distinguishes between creative and imitative reasoning and clarifies that creative reasoning occurs when no ready solution schema to a task is at hand. But he emphasises that solving a task by creative reasoning is not the same as MPS because creative reasoning does not have to be a challenge to the solver, which MPS has to be (Lithner, 2008). This implies that there are similarities between MPS and creative reasoning, but they differ depending on how MPS is defined. In the Swedish mathematics syllabi the definition of MPS only refers to no ready solution schema at hand and the notions of (cognitive) effort to solve it and interest to the solver are not mentioned. Thus, if the operational definition for a task to be considered as a MP only consists of a task that has no ready solution schema to the solver is a MP (as in the Swedish case and argued for in Paper II), and a task that has no ready solution schema demands creative reasoning to be solved, a task that needs creative reasoning to be solved is considered as a MP.

2.2 Mathematics textbooks

The textbook has a strong position in mathematics teaching internationally (e.g., Jablonka & Johansson 2010; Stein et al., 2007). That is, the textbook is often the major resource for teachers´ planning and students´/teachers´ classroom practice. As such researchers argue that the textbook often compensates, or even replaces, the national steering documents (e.g., Jablonka & Johansson, 2010; Stein & Kim, 2009; Stein et. al., 2007).

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* The impact of the textbook on mathematics teaching and learning is elaborated on in section 2.2.
* This core definition of what a MP is, is further elaborated on in both the papers connected to the thesis.
* The framework for different mathematical reasoning is described in detail in Paper II.
Studies suggest that the most common use of the textbook by teachers is to follow the content and order as presented in the book (Pepin & Haggerty, 2002 & 2003; Schmidt et al., 1996; Skolverket, 2008), relying on them to fulfil the written curriculum and ensure effective teaching (Thomson & Fleming, 2004; Vincent & Stacey, 2008). Reys, Reys and Chávez (2004, p. 1) claim “the choice of textbook often determines what teachers will teach, how they will teach it, and how their students will learn”. Also, Bierhoff (1996) and Fan and Kaeley (2000) argue that the teaching approaches the teacher uses in the classroom are highly similar to those presented in the textbook. Remillard (2005) offers a framework for how curriculum materials (CMs)\(^\text{10}\), such as textbooks, may be seen/used by the teacher. In outlining components in the relationship teacher-curriculum and different perspectives of understanding the relation teacher – CM – classroom practices, she emphasises that to be able to better understand teachers’ use of CMs it is essential to relate this use to affordances and constraints of CMs. A highly relevant framework to use in further research that may take its lead from earlier studies on textbook use by teachers.

Beyond teachers’ use of textbooks, Garner (1992) also found the textbook to be the primary tool for students to obtain knowledge and even points out that it can replace the teacher as the main source of information. A more recent study in the Swedish context (Boesen et al., 2014) shows that students' work in the mathematics classroom mainly consists of solving tasks in the textbook.

Also, according to several studies (e.g., Bruin Muurling, 2010; Li, Chen & An, 2009; Van Stiphout, 2011) the focus in the textbooks is on solution procedures and operations and considerably less attention is put on conceptual understanding and problem solving. Vincent and Stacey (2008) found that some of the best-selling books in Australia had an emphasis on memorisation and procedures without connections. Fan and Zhu (2007) studied textbooks from China, Singapore and USA and found a noticeable gap between curricular goals in each country and the mathematical content in the textbooks. In the Swedish context Lundberg (2011) found that most tasks in her analysis of books for Grade 10 required no more than imitating solved examples and Ahl (2014) concluded that textbooks for lower secondary school focuses on procedures rather than on the underlying mathematical structure\(^\text{11}\). Further, Brändström (2005) claims that the differentiation of tasks according to level of difficulty occurs, but on a low difficulty level. And also, as mentioned above, Frejd (2013) found that textbooks for upper secondary school, adapted for the new Swedish national curricula (Skolverket, 2011), lacked in tasks relevant to mathematical modelling.

\(^{10}\) The curriculum material focused on in this thesis is mathematics textbooks.

\(^{11}\) Both these studies are focused on proportional reasoning.

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Continuing the Swedish context, the textbook has a strong position in the mathematical classroom and Swedish mathematics teachers mainly use the textbook, and not the teacher guide, in their planning and classroom practice (Skolverket, 2008). Also, there has been no state control over curriculum materials in Sweden since 1991 (Jablonka & Johansson, 2010) and teachers are free to choose what curricular materials they want to use.

These studies begin to paint a picture of: 1) a gap between the intended level of mathematical conceptual understanding and MPS as presented in steering documents, and the content of the textbook that shapes the mathematics classroom, and 2) the importance, and consequently the impact, of the textbook in the mathematical classroom. This thesis add to this research by: 1) mapping the Swedish context of upper secondary mathematics education by studying if and how MPS is constituted in dominant textbook series and 2) presenting an analytical tool for separating tasks into MPs and exercises in mathematics textbooks.

2.3 Calculus

Calculus, or the calculus of infinitesimals\(^{12}\), is the mathematical study of change. It incorporates differential calculus (rates of change and slopes of curves) and integral calculus (accumulation of quantities and the areas under and between curves). The choice of searching for MPs in the mathematical area calculus is due to the following reasons:

Calculus is a mathematical area of enormous significance in a wide range of disciplines (Steen, 1988). It works as a cornerstone in problem solving for engineers, economists, biologists, physicists, meteorologists etc. and “…engineering and scientific applications just cannot exist without the calculus” (White, R., p. 22. in Steen, 1988).

In Selden, Mason and Selden (1989) a lack of ability in solving non-routine problems by using calculus is reported among average calculus students. They claim that many calculus courses focus on solving as great a variety of tasks as possible but lack cognitive non-routine problems to be solved by students. In the wake of that, they call for the addition of more non-routine tasks to complement the books used, or the removal of explanations and examples for “converting existing exercises into non-routine problems” (ibid. p. 49). According to the end of section 2.1 this is a call for an increase in the number of tasks that demand creative reasoning to be solved.

\(^{12}\) Infinitesimal is the idea of objects so small that there is no way to see them or to measure them.
2.4 Summary of literature review

The central role and use of textbooks in mathematics education (e.g., Jablonka & Johansson, 2010; Stein & Kim, 2009; Stein et al., 2007), the emphasis on MPS in steering documents (e.g. MOE, 2007; Skolverket, 2011), and the indication of a focus on solution procedures in the textbooks (e.g., Bruin Muurling, 2010, Li, Chen & An, 2009, Stiphout, 2011) suggest that analyses of how MPS is represented in mathematics textbooks could render important results, which may be taken further by using the work of Remillard (2005) in future empirical studies.

Further, considering calculus when looking for MPs in the textbooks is not an aim in itself, but a methodological choice, because of the importance of that area for future MP solvers.

These literature studies serve as background knowledge and are used when forming the methods for this study. In the next section methods for analysing the textbooks is present and argued for, which is the subject of Paper I, and also describing/arguing for the creation of the analytical tool, which is the subject of Paper II.
CHAPTER 3

Methodology

This chapter consists of three sub-sections. The first one describes a summary of the data source, the textbooks from the presentation in Paper I. The second summarises the analytical approach and argues for the use of it. This sub-section mainly refers to descriptions in the two papers rather than repeating them. The chapter ends by highlighting limiting factors, ethical remarks and trustworthiness of the study.

Through the chapter, and as a basis for the study, Ernest’s (1998) definition of methodology is followed, described as “a theory of methods - the underlying theoretical framework … that determine a way of viewing the world and, hence, that underpin the choice of research methods” (p. 35). In this thesis the underlying theoretical framework is a conceptual framework describing different kinds of mathematical reasoning (Lithner 2006 & 2008) and the research methods emerges from that framework.

3.1 Data source: the textbooks

This section presents an description of the textbooks analysed in Paper I. Focus is on some benchmarking points from the complete description as presented in Paper I. The examined textbooks are the courses 3c, 4 and 5 for the textbook series Matematik 5000 (Alfredsson, L., Bråting, K., Erixon, P., Heikne, H., 2012), Origo (Szabo, A., Larson, N., Viklund, G., Dufåker, D., Marklund, M., 2012) and Matematik M (Sjunneson, J., Holmström, M., Smedhamre, E., 2012). As there are no statistics available about the use of textbooks in Swedish classrooms, the choice of textbooks to examine is guided by Frejd’s (2013) selection of textbooks. The choice is also based on the author’s 13 years of experience as a mathematics teacher in a Swedish upper secondary school, from which it is known that these textbook series cover a large part of the Swedish market. At the beginning of the data collec-

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13 Throughout this subchapter, all quotes are my translations from the original Swedish text.
14 After the release of a new syllabus in Sweden (Skolverket, 2011) Frejd (2013) examined new textbooks for Swedish upper secondary school, adapted for the new syllabi, with respect to modelling.
3.2 Analytical approach

In analysing prior research on mathematics textbooks, Fan, Zhu and Miao (2013) sorted the type of research into four categories; 1) Role of the textbook, 2) Textbook analysis and comparison, 3) Textbook use, and 4) Other areas. In this thesis, Paper I analyses three series of textbooks, which fit category 2), but the aim or intention is not to compare them. Paper II suggests an analytical tool adapted for textbook analysis, which also refers to category 2).

This section functions as a summary and a widening of the underlying thoughts of the analytical approach in the papers. The section ends with a widening of the rationale for the analytical questions adjacent to research question 1.
3.2.1 Overview of the analytical approach in the papers

The aim of Paper I is to analyse how MPS is represented in the new mathematics textbooks for Swedish upper secondary school. In doing so the aim is to clarify what proportion of the tasks in the textbook is MPs and for the ones defined as MPs, outline where in the textbook they appear, at what level of difficulty they are and in what context they are produced. These four aspects are raised as four analytical questions, which are elaborated on in section 3.2.2.

The rule of procedure is to separate the tasks into two groups, *MPs* or *exercises*, and then categorise the MPs into *placement*, *level of difficulty* and *context*. Consequently the unit of analysis is *the tasks* in the textbooks, and the first step, to classify all tasks as MPs or exercises, demands an analytical tool. The development and rationale for this tool is elaborated on in Paper II. In the development of the tool, which is a four-step schema, an operational definition for what a MP is, is developed. As Schoenfeld (1992) emphasises the need for an operational definition of what the author means, the presented operational definition, includes definitions of the words building it. The operational definition of a MP is here considered as a methodology to reach the analytical schema, which, in turn, is a result in Paper II. Thus, the analytical schema is presented as a result of Paper II in section 4 and the operational definition of a MP is presented in this section. Further, the resulting analytical schema is based on, and may be seen as a custom application of, the conceptual framework for different mathematical reasoning\(^\text{15}\) (Lithner, 2006 & 2008).

To analyse the tasks and classify them into *MPs* and *exercises*, Lithner’s framework lays the foundation of the analytical schema. The framework distinguishes between whether a task needs *creative reasoning* (CR) or *imitative reasoning* (IR) to be solved and adopts Schoenfeld’s (1985, p. 74) definition of PS, with “if one has ready access to a solution schema for a mathematical task, that task is an exercise and not a problem”, as a bearing phrase. Without such ready solution schema, possible to imitate and provided by the textbooks, it is not possible for the solver to use IR following what may be found in the textbook. According to Palm et al. (2011) it is reasonable for the student to use some IR if there is something present in the textbook that the student can imitate. An assumption that is supported by the fact that teachers follow the content and order as presented in the textbook (Thomson & Fleming, 2004; Vincent & Stacey, 2008) and students’ work is dominated by solving tasks in the textbook (Boesen et al., 2014).

\(^{15}\) The framework is further described both in Paper I and Paper II and not further described here. Imitative reasoning is abbreviated as IR and creative reasoning as CR.
The operational definition of a MP in Paper II is also based on accepted definitions of a MP\textsuperscript{16}. An argumentation regarding these definitions leads, in Paper II, to the conclusion that the aspect ‘No solution schema’ solely defines a task as a MP, which, in turn, leads to the idea that a MP may be defined as a task that needs CR (according to Lithner’s framework) to be solved. Lithner (2008) distinguishes between a task that needs CR to be solved and a MP due to the aspect difficulty; “C(M)R does not, as problem solving, have to be a challenge” (p. 266)\textsuperscript{17}. Thus, if the aspect difficulty is removed in the operational definition of a MP, the choice of characterising MPs in terms of the solution requiring CR is valid.

Further, whether a task is a MP or an exercise, is often referred to as a relation between the solver and the task (e.g. Schoenfeld, 1985), but in this thesis it refers to a relationship between the task and what has been presented earlier in the textbook or earlier textbooks in the same series (assuming that the students follow the same book series). Supported by the methodology of the study of Palm et al. (2011), and based on an operational definition of a MP (Paper II) and the conceptual framework of Lithner (2006 & 2008), the operational definition of a MP in Paper II, and this thesis, reads:

\begin{quote}
A task is considered as a mathematical problem if no explanations, solved examples, rules, theorems, described facts or earlier tasks that provide a solution schema, are to be found in the textbook or earlier textbooks used by the solver.
\end{quote}

This operational definition demands definitions of the words textbook and task and also the expression solution schema. An argumentation for each definition is elaborated on in Paper II and the operational definitions reads: Textbook; A mathematics textbook is a book intended for teaching and learning mathematics according to a national curriculum, following the ‘explanation–example–exercises’ format; Task; a task is a question or exhortation in the textbook that the students are supposed to find an answer to; Solution schema; a solution schema is the solutions to the whole set of sub-tasks within the task.

Finally, the choice of calculus is a methodological choice and not a part of the purpose or aim of the thesis. Examining all tasks for all textbooks in all textbook series for the Swedish upper secondary school would be too extensive. From all conceivable ways of limiting the selection of tasks, examining tasks from one specific mathematical area was chosen. According to

\textsuperscript{16} See chapter 2.1, literature review; problem solving, for the definitions. For the reasoning according to the mutual interweaving between them I refer to Paper II.

\textsuperscript{17} I supplement the brackets as CR is in Lithner (2008) called CMR, meaning Creative Mathematically founded Reasoning.
the aim of the thesis, calculus was considered as an appropriate choice due to its connection to future MP solvers (Steen, 1988).

3.2.2 Rationale for the analytical questions
The first research question of this thesis is operationalized through four analytical questions: What proportion of the tasks could be classified as MPs? Of those which can be, where in the book are they found, at what level of difficulty are they and in what context are they produced? The rationale for the analytical questions is elaborated on here, as it is deficient in Paper I due to lack of space.

The purpose of examining the proportion of MPs in textbooks is twofold: 1) Students learn what they have the opportunity to learn (Hiebert, 2003) and Boesen et al. (2014) claim that students’ work in the mathematics classroom mainly consists of solving tasks in the textbook. At the same time MPS is accented in national curricula, internationally as well as in Sweden. Under these circumstances it is reasonable to claim that the proportion of MP-tasks in textbooks illustrates one important measure of the students’ opportunity to develop their ability in MPS, as is desirable in (the Swedish) national curricula. 2) When all MPs in a textbook have been located, it is possible to categorise them in order to illuminate certain aspects of how MPS is represented in the textbook. These aspects may differ depending on a wide range of causes such as: emphasis on MPS in the national curricula, specific research interests, teachers’ administration of the textbook etc. The suggested aspects in Paper I; placement, level of difficulty and context, are chosen to illuminate what opportunities the textbook provides to students’ in learning mathematics and MPS.

Placement: One might summarise different roles of MPS in teaching mathematics in terms of learning mathematics for, about or through MPS (Ryve, 2006; Wyndham et al., 2000). Learning for means first learning the tools, then applying them in a MPS situation; learning about means MPS is an explicit topic to learn and; learning through means seeing MPS as a vehicle for learning mathematics. The placement of the MPs indicates whether a textbook advocates learning mathematics for or through MPS. MPs at the beginning of a chapter can be interpreted as indicating learning through MPS, where an introducing MP becomes a vehicle for learning the mathematics that is needed to solve the MP. MPs at the end of a chapter, where MPs become a summary of what has been learned, or subjects to apply mathematical tools to, can be interpreted as learning mathematics for MPS. As such, the notion of placement is important to conclude whether the textbook emphasises MPS as learning mathematics for or through MPS.

Level of difficulty: Even if Boesen et al. (2014) claim that students’ work in the classroom mostly consists of solving tasks in the textbook, Lithner (2004) claims that most students do not have time to solve all recommended
tasks in the textbook. Thus, the actual number of tasks the students work with is a subset of the tasks in the textbook. If the MPs found are at the most difficult level, it is reasonable to assume that most students do not have time to try to solve them and that only the strongest students will work on them. Thus, most of the students are left without practice in MPS. As Hiebert (2003) claims that students may learn what they are given opportunity to learn, the consequence might be that many students will be left without the opportunity to learn MPS from the textbook. As such, the notion of level of difficulty is important to conclude whether the textbook emphasises MPS for all students or only for the strongest.

Context: According to Davies–Dorsey, Ross and Morrison (1991) the context in which a MP is produced, affects the mental representation of the problem for the solver and the motivation to solve the problem. Also a motivation to solve the task is mentioned as a part of a definition for MPS (Hagland et. al. 2005; Jonassen, 2000), and through the context in which a task in a textbook is produced, it is important to provide a task that interests the solver. In defining mathematical literacy in PISA 2012 it is claimed; “Mathematical literacy is an individual’s capacity to formulate, employ, and interpret mathematics in a variety of contexts.” (OECD, 2013, p. 25), and four contexts in which the tasks are produced, are used: Personal which relates to the individual’s and family’s everyday life; Societal which relates to the individual as a part of a society, globally as well as locally, nationally as well as internationally; Occupational which relates to workplaces; and Scientific which relates to the use of mathematics in science and technology. In Paper I a fifth category called Pure Mathematical was used, in which all MPs that are not embedded in a specific context, but only a mathematical one, was introduced. As such, the notion of context is important to conclude whether the textbook represents MPS in a way that captures the students’ interest or presents the use of mathematics in different situations.

3.3 Remarks

3.3.1 Limitations of the studies
I argue that limitations of a study can be explained and understood at different levels or viewpoints. A study may be limited by, for instance, the context in which the research is conducted, the data collected, the research questions, and the analysis of the data or the literature review. Further, limitations of the literature review may affect the interpretation of the analysis and, by extension, the result and what conclusions can be drawn. Also, the researchers own experiences and beliefs may be a limiting aspect of a study.

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Limiting factors for the research in Paper I are mainly the context in which it is conducted, the data source, and the analysing process. The context and data source is limited to three Swedish textbook series for upper secondary school and the mathematical area calculus. The choice of calculus leads to focus on the three courses 3c, 4 and 5, as calculus is introduced in course 3. A piece of research in another country, at another stage, for other textbooks or for some other mathematical area, may lead to other results and conclusions than this research. Also, calculus is a new concept introduced to the students at course 3, which may affect the result in comparison to the case where some other, previously introduced and well-known area, was chosen. It is plausible to assume that a newly introduced area leads to more detailed explanations, more solved examples, more exercising tasks and considerably less MP tasks, than a well-known and established area.

Further, in the analysing process, tasks are compared with earlier tasks, solved examples and explanations in the textbooks and no account is taken of teacher guides, student solutions, students’ interests, students’ background, teachers’ explanations etc. A task that is classified as a MP in this study may be “deproblemised” if, for instance, the teacher (or someone else) communicates (parts of) a solution scheme for the task to the student. Research shows that it is rather common that teachers support students in such ways where mathematical problems are turned into routine tasks not requiring any CR (Smith & Stein, 2011). The interpretation of this fact is that the number of MPs found in this study is the highest number of MPs possible to get from the examined textbooks.

Limitations in Paper II mainly concern my limits as a researcher and the literature study. As the analytical tool presented in Paper II is a theoretical piece of work, it is dependent on the researchers experience in the field and, in extension, the amount of literature studies that is put into the theoretical argumentation. It is possible, or even plausible, that more literature studies and more experience in the research field of mathematics education would lead to changes in the suggested tool. Also, the tool would benefit from empirical studies to confirm its validity. Thus the tool is not claimed to be complete, but rather (as emphasised) a suggestion for further research. However, the current focus is, nevertheless, productive.

3.3.2 Ethical considerations and trustworthiness

Ethical considerations

Since the object of study is tasks in textbooks and no surveys, interviews and contact with people is used, the ethical considerations for this thesis and the two papers are limited to plagiarism and self-plagiarism. The issue of plagiarism applies primarily to the description of the conceptual framework used (Lithner, 2008), which is most elaborated on in Paper II, and a figure
connected to it, used in both the papers. The description of the framework is necessarily close to the original text as it defines words and how these words are used. To make the text easier to read, quotation marks are not used in these sections in the papers. Concerning the figure, the author asked for permission to use it as it is presented in the papers. According to self-plagiarism many pieces of text in this thesis are similar to the two papers. I have not been explicit about every place this occurs by using quotation marks or explicitly mentioning that a piece of text is equivalent to the text in one of the papers.

In addition to plagiarism, responsibility towards publishers and writers of the examined textbooks is taken by only examining, and in no way evaluating or judging the textbooks.

**Trustworthiness**

Inspired by the reading of Hemmi (2006) the trustworthiness of the papers and this thesis is reflected upon and argued for through the words originality, generality, reliability and objectivity.

**Originality**

The four-step analytical schema used in Paper I and elaborated on in Paper II, may be seen as a custom application of the framework for CR and IR (Lithner 2006 & 2008). Thus the framework is adapted into a new context. Also the four analytical questions to examine how MPS is represented in textbooks in Paper I, is a product of my own. That makes me feel comfortable to argue that the concept of originality is fulfilled in the papers and this thesis.

**Generality**

I consider generality as an observation or principle having general application. In Paper I only textbooks for Swedish upper secondary school, from three textbook series, three textbooks from each series and only in the mathematical area calculus, are examined and I cannot claim any further generalizations of the results.

Thus, the analytical tool suggested and elaborated on in Paper II may be considered as a general method to analyse any textbook with respect to MPS. Other researchers may apply and replicate the method to any textbook or mathematical area and obtain equivalent or conflicting results, which will make the result of Paper I more and more generalizable the more the method is used. This kind of fuzzy generalization is advocated in pedagogical research as it promotes another researcher to start from where the first ended (Bassey, 1999).
As such, the tool suggested in Paper II might be applicable in a various range of situations\(^{18}\). The wide applicability in itself is an important aspect of generality, but more important is the general base it is built on. The four-step analytical schema emerges from operational definitions, which, in turn, emerges from accepted definitions of a MP. Also it may be seen as a custom application of the conceptual framework of different types of mathematical reasoning. The definitions and the framework are both generally accepted and applied in several situations and the suggested tool is built on generally applicable rules. Thus, the suggested tool may be considered to fulfil the aspects of generality.

**Reliability**

One factor in the term *reliability* is *stability* (Bryman, 2001), and concerns repeatability for a piece of research, given the same circumstances. For Paper I the textbooks and the analytical schema it is possible for anyone to use and *stability* (and reliability) is therefore high. Though, in the classification procedure, there is a critical moment when deciding whether a task is a MP or an exercise. The interpretation of the analytical schema may be different if someone else does it, even if it is used for the same textbooks. To minimise differences in the interpretation efforts are made to be as explicit as possible about the analytical procedure in the given examples in the papers, enabling other scholars to replicate the study. Further, to verify the interpretations in the process of categorisation, a seminar, presented in Paper I, was conducted. The seminar revealed a strong consensus among the researchers as to whether a task was categorised as a MP or an exercise. No divergent interpretations were made within the seminar and therefore no statistic calculations were made.

**Objectivity**

Objectivity may be seen as a sub-set of reliability as being objective in the observations affecting the reliability of the results obtained. For this thesis the objectivity is high because within the method and the analysing process there are few aspects that might be influenced by emotions or personal prejudices and all tasks are observable to anyone. An opposition to the objectiveness is the fact that even if the same schema for categorisation is used, other researchers might interpret tasks differently. As all interpretations are influenced not only by explanations of methods and definitions of words, but also by the interpreter’s personal perceptions and beliefs\(^{19}\), there is always a matter of bias in interpretative situations.

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\(^{18}\) This is further elaborated on in Paper II and is not taken any further here.

\(^{19}\) The concept of beliefs is not further developed here as it is not a key concept for this thesis.
Validity

Are the research facts and findings what they claim to be? Are the claims well grounded? These questions lead back to the question; are the methods valid for its purpose? The results in Paper I are claimed to be valid, because the method is well-grounded in established definitions and frameworks, is generalizable and produces desired results according to the research questions.

Transparency is one subset of validity as “… research transparency allows a researcher’s audience to evaluate claims and form an evidentiary and logical basis for treating the claims as valid” (Lupia, A., & Elman, C., 2014, p. 22). To obtain transparency in the research, all data from the categorisation process is saved in order to pass on to any reviewer to test the validity of the result.
CHAPTER 4

Summary of papers

This chapter is a brief and summary overview of each of the two papers. In the previous section the methodology of the paper was described. This overview focuses on the result of each paper. Then, in chapter five, conclusions from the papers are discussed.

4.1 Paper I
Title: Problem solving in Swedish mathematics textbooks for upper secondary school.

The aim of this study is to analyse how MPS is represented in mathematics textbooks for Swedish upper secondary school. The analysis comprises dominant Swedish textbook series and relates to uncovering a) the quantity of tasks that actually are MPs b) the location of tasks in the chapter, c) the difficulty level and d) the context of the actual problems. The first part in the analysing process is to separate tasks into MPs or exercises. An analytical tool for that purpose was created and is elaborated on in Paper II. The second part consists of the aspects placement, level of difficulty and context of the MPs found. Together these aspects are claimed to illuminate how MPS is represented in the textbooks according to what opportunities students are given to learn MPS, by using the textbooks.

Based on an analysis of 5,722 tasks from the area of calculus the result shows that the textbooks themselves contain very few tasks that may be defined as MPs, the ones that are MPs are found at the end of a chapter, at the most difficult level and are produced in pure mathematical context. In detail, 312 of the tasks (5.45%) were MPs. Of these 312 MPs, not one was found at level 1 (the easiest level), four MPs were found at level 2 (1.28% of the MPs) and 308 at level 3 (98.72% of the MPs). The placement of the MPs showed that 264 of the 312 (84.62%) was found at the end of a chapter and of the five defined content areas, pure mathematical was the most common with 195 of the 312 (62.50%) MPs, followed by professional (50 of 312 tasks, 16.03%) and personal (33 of 312 tasks, 10.58%). The patterns were
similar when looking at each book series separately for each category. Implications are discussed in the paper and are further elaborated on in section five in this thesis.

4.2 Paper II

Title: An analytical tool for separating mathematical problems from exercises in mathematics textbooks.

The result of Paper II is an analytical tool for separating tasks in a mathematics textbook into MPs and exercises, and the aim of the paper is to argue for the validity of it. The tool, which is a four-step schema, functions as a custom application of the conceptual framework of Lithner (2006 & 2008). Each step of the schema emerges from different types of imitative reasoning described in the framework. Inspired by the methodology of the study of Palm et al. (2011) the schema is designed to examine what type of reasoning it is possible to use in relation to solved tasks and examples, explanations, rules, theorems, described facts etc. presented earlier in the book or in earlier courses. A task in the textbook is categorised as a MP, if there are, in the textbook or earlier textbooks used by the solver:

i) no examples given (explicit or implicit) regarding how to solve the task. This is connected to MR, where the answer is to be found in a solved example, AR where a procedure is to be found in a solved example, or text-guided AR, where one simply has to mimic a solved example.

ii) no earlier tasks that have the same or similar surface similarities. This is mainly connected to delimiting AR, if the task has surface similarities to an earlier task that limit the possible algorithms to use. It might also be connected to text-guided AR, where it is possible to mimic an algorithm from an earlier task or familiar MR/AR, where it is possible to recall a memorized answer or used algorithm.

iii) no solution scheme possible to recall from an earlier part of the textbook or textbooks from earlier courses. This is connected to familiar MR/AR, where it is possible to recall a memorized answer or used algorithm.

iv) no guidance within the task in how to solve it. This is connected to guided AR, where however the guidance is within the task instead of from an external source. See Example 2.
If no such *imitative reasoning* is possible to use it is plausible to believe that *creative reasoning* is needed to solve the task. Further it is argued that a task that needs creative reasoning to be solved is a MP, even if Lithner (2008) distinguishes between creative reasoning and MPS. In Paper II the schema is followed by explanatory and dilating comments and is illuminated by four examples.

The building of the tool consists of two steps. First, three recurrent definitions of a MP are discussed and a ranking between them is argued for. This ranking is then, together with operational definitions of words and expressions such as *task, solution schema* and *mathematics textbook*, made into an *operational definition* for what a MP is in the context of a textbook. The second step is the conceptual framework of different types of mathematical reasoning (Lithner, 2006 & 2008) where *creative reasoning* and different types of *imitative reasoning* are explained and it is shown how they are to be understood within the framework and the suggested tool. The two steps, the operational definition of a MP and the conceptual framework, then lead to the analytical schema.
CHAPTER 5

Conclusion and discussion

In this section the research questions are answered and conclusions from the results presented in chapter 4 are discussed according to the literature studies. Also, contributions to the research field of mathematics education and ‘the practice’ are discussed and issues for further research are raised.

5.1 Conclusions

The aims of the thesis, examining how MPS is represented in the new mathematics textbooks for Swedish upper secondary school and presenting an analytical tool to separate tasks in a mathematics textbook into MPs and exercises, is operationalized through two research questions. Conclusions and discussions are made adjacent to the answers of these research questions.

What characterises the distribution of MP tasks in three selected Swedish textbook series?

The distribution of MP tasks in the textbooks examined is characterised by a few MPs being placed at the end of a chapter, at the most difficult level and given in a pure mathematical context.

The results in Paper I show that it is possible to solve the vast majority of the tasks by following examples or solution schemas from earlier tasks. As the textbook is the major resource for teacher planning and student practice (e.g. Boesen et al., 2014; Jablonka & Johansson, 2010) and operationalizes the steering documents (Boesen et al., 2014) it is concluded that the low number and rate of MPs indicates a mismatch between the emphasis on and intention of MPS in the steering documents and the possibility for the students to develop their MPS-ability by using the textbooks examined. The conclusion causes an extended responsibility for teachers not to rely so heavily on the textbooks and to create possibilities for the students to develop the ability in MPS from other sources than the textbooks.

Also, in a system with steering documents emphasising MPS, where textbook writers interpret and ‘translate’ the steering documents into textbooks,
and teachers’ interpret/implement these textbooks, there is an imminent risk of misinterpretations in several steps. If the authors of the steering documents, the textbook writers and the teachers do not increase the consensus of what a MP is, a conclusion is that the intention in the steering documents is not fulfilled according to what the students actually meet (in the textbooks and via the teacher) when they are supposed to develop their ability in MPS.

Further, the results in Paper I show that the MPs are found at the end of a chapter and at the most difficult level. Thus, I conclude that in the textbooks, MPS seems to be treated as something difficult (the most difficult level) and summon up what has been learned (at the end of the chapter) and thus, promote the view ‘learning mathematics for MPS’. As raised by Lithner (2004) the placement and level of difficulty of the MPs also makes it plausible to conclude that the large majority of the students will never be able, or have enough time, to work with the existing MPs according to their placement and level of difficulty. Thus, as students learn what they have opportunity to learn (Hiebert, 2003) and their work in the classroom is dominated by solving tasks in the textbook (Boesen et al., 2014) I conclude there is an imminent risk that mathematics for these students will never be more than practicing standard solutions by following examples and learning some mathematical tools they afterwards do not know how to use. To obtain more MPS training for the students, I also conclude that teachers have to avoid using the textbooks by following the content and order as presented in the book (e.g. Pepin & Haggerty, 2002 & 2003; Skolverket, 2008), and relying on them to fulfil the written curriculum and ensure effective teaching (Thomson & Fleming, 2004; Vincent & Stacey, 2008), and start to use the textbook in a more conscious way. Further, according to level of difficulty, although there is an emphasis on MPS in the description of the examined textbooks (see chapter 3.1) for all levels of difficulty, there were no MP-tasks found at the easiest level and only four tasks at level two. This might suggest that there is a discrepancy and unconsciousness in how to interpret what a MP and the concept of MPS is. An unconsciousness that is illuminated by Berqvist and Bergqvist (2012), who found that more than 60% of Swedish upper secondary school teachers interpret MPS in a way that is not based on any definition of MPS.

Also, most of the problems found in the textbooks were stated without superficial context (62.5% pure mathematical), which makes me conclude that in the examined textbooks, MPS is treated as an internal mathematical activity only seldom connected to non-mathematical contexts. As the context in which a MP is produced affects the mental representation of the problem for the solver and the motivation to solve the problem (Davies–Dorsey, Ross & Morrison, 1991), the lack of MPs stated in a superficial context is concluded to be a disadvantage to creating motivation to solve the tasks.
Which aspects are important to consider in designing an analytical tool to separate tasks into MPs and exercises in a mathematics textbook?

As Schoenfeld (1992) emphasises the need for an operational definition of what the author means, I conclude that the most important aspect in designing a tool to separate tasks into MPs and exercises is a clear and well-founded operational definition of what a MP is. In contrast, I also conclude it is important to be clear about what characterises an exercise.

The resulting analytical schema in Paper II is based on an operational definition of what a MP is, intertwined with a conceptual framework of different types of mathematical reasoning (Lithner 2006 & 2008). The operational definition of a MP is, in turn, based on existing definitions of a MP (e.g. Blum & Niss, 1991; Lesh & Zawojewski, 2007; Schoenfeld, 1985), which the conceptual framework also is based on. The basic idea of the analytical schema is that a MP is connected to the creative reasoning aspect novelty in Lithner’s framework, which in turn emerges from the MP-definition no ready solution schema for how to solve the task (Schoenfeld, 1985). Further, different types of imitative reasoning in Lithner’s framework define aspects of a task to be considered as an exercise. Thus I conclude that the resulting analytical schema is a custom application of the conceptual framework, applicable to any textbook and any mathematical area and I would like to put it forward as a conjecture that needs to be substantiated by further empirical studies.

5.2 Contributions

The results from the papers contribute to both the research community of mathematics education and the practice. Paper I contributes to the research community of mathematics education by providing a mapping of a current state of how MPS is presented in frequently used textbooks, covering a large part of the Swedish market for upper secondary school. Thus, the result is not limited to a statistical selection of a specific number of classrooms or teachers, but rather covers a substantial part of the Swedish context. Consequently the result may function as e.g. one piece of knowledge in mapping characteristics of mathematical textbooks (e.g., Bruin Muurling, 2010; Li, Chen & An, 2009; Van Stiphout, 2011; Vincent & Stacey, 2008) and as such contribute to future international comparative textbook studies. The contribution to the practice of Paper I is mainly to teachers and textbook writers. For teachers the result from Paper I is informative according to how to view and use the textbooks in the classroom e.g. when planning how to create MPS lessons. For textbook writers the result is informative according to how they can view and present MPs in future textbooks.
Paper II contributes to the research community of mathematics education as the analytical schema is designed to examine any textbook in any context worldwide. It may also serve as a frame for research on MP-tasks in tests. According to the practice, Lithner (2008) discusses there is a lack of terminology and frameworks to communicate researched insights between researchers as well as to ‘the educational line’\textsuperscript{20}; politicians, syllabus constructors, administrators, textbook writers, teachers and students. The clearing up of defining a MP within the analytical tool in Paper II has the potential to make it more understandable how to define a MP and to increase the consensus of what a MP is among this ‘educational line’. An optional application of the analytical tool is the possibility to use it as a frame, or guideline for thinking, when creating MPs in different situations, e.g., tasks in tests that are supposed to examine MPS ability. Thus, further contributions of Paper II to the practice are mainly to teachers and textbook writers. For textbook writers the resulting analytical schema may serve as a guideline for the development of future textbooks with respect to MPS and how to create MPs according to solved examples, explanations etc. (Selden, Mason and Selden, 1989). For teachers, and other constructors of assessment tasks, the result in Paper II contributes to the development of MP-tasks in tests, as the analytical schema may function as a guideline for creating such tasks.

5.3 Further research

According to Lithner’s (2008) discussion concerning the lack of terminology and frameworks to communicate researched insights, I argue that all parts of an ‘educational line’ would benefit from an increased consensus of what a MP is. Thus, I call for research on how to obtain such consensus to what MPS is, and better and more effectively communicate it through and within the whole ‘educational line’. Such research may include the suggested analytical tool in empirical situations to examine if and how the tool may support such consensus and communication.

Further, the analytical schema, the arguing for it and the intended implications of it, is a theoretical piece of work. Shortcomings depending on my lack of experience as a researcher and, in extension, the amount of literature studies that is put into the theoretical argumentation, makes it possible, or even plausible, that more literature studies and more experience in the research field of mathematics education would lead to changes in the proposed tool. Thus, the analytical schema is a subject for further research and empirical studies to consolidate its accuracy.

\textsuperscript{20} The expression ‘the educational line’ is mine and is not derived from Lithner’s discussion.
Finally, putting together that 1) the textbook is the most used artefact in teachers planning and teaching (e.g., Jablonka & Johansson, 2010; Stein & Kim, 2009; Stein et. al., 2007), 2) textbooks focus on solution procedures and operations (e.g. Bruin Muurling, 2010, Li, Chen & An, 2009, Van Stiphout, 2011), 3) students’ work in the classroom is dominated by solving tasks in the textbook (Boesen et al., 2014), 4) teacher-made tests contain a small proportion of tasks that demand creative reasoning (Palm et al., 2011) and 5) the result of Paper I, paints a picture of an ‘educational line’ that strongly reduces and overrides the importance of MPS for the students. An importance that is emphasised in the steering documents and which teachers and educators agree on but do not seem to be able to live up to by using the textbooks examined, as they are designed and used today. As the teachers are the ones who are supposed to clarify to the student what to learn and guide the way to get there, they are also responsible for creating adequate MPS lesson events that form a logical whole in order to promote learning of MPS. To better support teachers in creating such lesson events, aimed at developing students’ ability in MPS, it is essential to understand teachers’ use of CMs (textbooks). And to do so it is essential to relate to this use as affordances and constraints of CMs (Remillard, 2005). The textbook analysis in Paper I might be seen as a first step into research on affordances and constraints of the textbook, which advantageously may be followed up by empirical studies on teachers’ use of the textbooks examined and other CMs. This would function as an extension of this thesis, highly relevant for future research.

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21 ‘Lesson events’ is a part of a framework developed by Clarke et al. (2008).
Finally, putting together that 1) the textbook is the most used artefact in teachers planning and teaching (e.g., Jablonka & Johansson, 2010; Stein & Kim, 2009; Stein et. al., 2007), 2) textbooks focus on solution procedures and operations (e.g. Bruin Muurling, 2010, Li, Chen & An, 2009, Van Stiphout, 2011), 3) students' work in the classroom is dominated by solving tasks in the textbook (Boesen et al., 2014), 4) teacher-made tests contain a small proportion of tasks that demand creative reasoning (Palm et al., 2011) and 5) the result of Paper I, paints a picture of an 'educational line' that strongly reduces and overrides the importance of MPS for the students. An importance that is emphasised in the steering documents and which teachers and educators agree on but do not seem to be able to live up to by using the textbooks examined, as they are designed and used today. As the teachers are the ones who are supposed to clarify to the student what to learn and guide the way to get there, they are also responsible for creating adequate MPS lesson events that form a logical whole in order to promote learning of MPS. To better support teachers in creating such lesson events, aimed at developing students' ability in MPS, it is essential to understand teachers' use of CMs (textbooks). And to do so it is essential to relate to this use as affordances and constraints of CMs (Remillard, 2005). The textbook analysis in Paper I might be seen as a first step into research on affordances and constraints of the textbook, which advantageously may be followed up by empirical studies on teachers' use of the textbooks examined and other CMs. This would function as an extension of this thesis, highly relevant for future research.

Summary in Swedish

Tidigare forskning visar att läroböcker är det studieminmaterial som i huvudsak används i matematikundervisning, både internationellt och i Sverige. Matematisk problemlösning är ett återkommande begrepp i tidigare och nuvarande läroplaner, ämnesbeskrivning för matematik och kursplaner i matematik, både internationell och i Sverige. Problemlösning är i den senast kursplanen för den svenska gymnasieskolan även en av sju kompetenser att bedöma vid betygssättning, för alla betygsnivåer. En naturlig fråga att ställa är således huruvida läroböckerna tillhandahåller möjligheter för elever att utveckla sin problemlösningsförmåga och om/hur de utgör ett verktyg för lärawar att skapa ett problemlösningsinriktat arbetssätt.

I den första artikeln undersöktas tre av de dominerande läromedelserierna för svenskt gymnasium i syfte att kartlägga andelen uppgifter som kan klassas som matematiska problem, samt var i boken/kapitlet dessa är placerade, vilken svårighetsgrad de är i och i vilken kontext de är skrivna. Resul tatet visade att utav 5722 analyserade uppgifter kunde 312 stycken (5,45%) klassas som matematiska problem. De som kan klassad es som problem finns i slutet av ett kapitel (84,62%), i den högsta svårighetsgraden (98,72%) och är skriven i en inom-matematisk kontext (62,50%).

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Bibliography

References


https://matematiklyftet.skolverket.se/matematik/content/conn/ContentServer/uuid/dDocName:LI64RH5PRO019853?rendition=web (150228)


MEE Notes Number 8. Mathematical Association of America, 1529 18th Street, NW, Washington, DC 20007.

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