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APPROACHING CURRICULUM RESOURCES

EXAMINING THE POTENTIAL OF TEXTBOOKS AND TEACHER GUIDES TO SUPPORT MATHEMATICS LEARNING AND TEACHING

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Abstract

The driving forces of conducting the two studies presented in this thesis are: the strong position of curriculum resources in the mathematics classroom in general; teachers' faith in curriculum resources to convey the intentions and goals of mathematics education; and the expected benefits to mathematics education through expanding the knowledge base of research in the area of curriculum resources in general, and teacher guides in particular. This thesis builds upon two studies conducted in Sweden. The first is a study on how teachers use teacher guides. The results of this study could be seen as one piece in the building of a theoretical understanding of how teachers use their tools in planning and enacting teaching. The data for Study I were collected through semi-structured interviews with five early-years mathematics teachers. In advance, the teachers were asked to copy the pages from the teacher guide they had used the week before the interview. The analysis of the interviews shows that less experienced teachers desire a wider scope of content in their teacher guide. More experienced teachers desire support from teaching activities. Another interesting result was that all teachers in our study, regardless of prior experience, knowledge, beliefs, etc., want the teacher guide to offer connections between theory and practice. The second study investigates what content is represented in Swedish textbooks. The results of this study map the terrain over how the most commonly used textbooks in Sweden construct the topic of proportion and proportional reasoning. My choice of textbooks is based on a questionnaire among all compulsory school mathematics teachers in the sixth largest municipality in Sweden, revealing that two curriculum resources together covered 97% of the classrooms in Grades 7-9. The

study provides insights into the strengths and weaknesses in the presented structure regarding proportional reasoning in the most commonly used textbooks in Swedish lower secondary school. Teachers could use these insights when making decisions about how to use their textbooks, and to possibly reconsider how to organize the enactment of the national curriculum in order to develop deep understanding. The analyses could provide authors of teacher guides and textbooks with valuable information to consider regarding structure and design.

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List of Papers

This thesis is based on the following Papers, which are referred to in the text by their Roman numerals.

Paper I Ahl, L., Koljonen, T. & Hoelgaard, L. (in press). How are mathematics teacher guides used for support and inspiration in teaching? *Proceedings for The Seventh Nordic Conference on Mathematics Education NORMA14*. Turku, June 3-6, 2014.

Paper II Ahl, L. (2014). The potential of dominating mathematics textbooks in Sweden to support the learning and teaching of proportional reasoning. *Manuscript submitted for publication.*

Contents

Chapter 1 Introduction

> Understanding how teachers use curriculum materials[1] to craft instruction requires being explicit about the representations curriculum materials use to communicate concepts and actions, being attentive to the ways in which teachers perceive and interpret these representations, and understanding how these representations can constrain and afford teacher practice. (Brown, 2009, p. 18)

1.1 Background

The most important tool[2] in mathematics education is the curriculum resources used by teachers and students (Grouws, Smith & Sztajn, 2004; Brown, 2009). The area of research in how curriculum resources contribute to mathematics learning and teaching is growing but still inconclusive (cf. Pepin & Haggerty, 2003; Thomson & Fleming, 2004; Stein, Remillard & Smith, 2007; Lloyd, Remillard & Herbel-Eisenmann, 2009; Fan, Zhu & Miao, 2013; Pepin, Gueudet & Trouche, 2013). Especially teacher guides have received minimal attention (Remillard, 2005) but emerging research shows potential in textbooks to support students' learning, and in teacher guides to support teachers' teaching (cf. Ball & Cohen, 1996; Brown, 2009; Davis & Krajnic, 2005; Stein, Remillard, & Smith, 2007). Therefore, it is important to conduct further studies on how curriculum resources impact mathematics education.

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[1] Consequently, I will use *resources* instead of *materials,* as materials call to mind printed texts while today more and more curriculum resources are available in digital format and online. I refer to all resources used by teachers inside and outside the classroom for planning and enacting lessons (Remillard, 2005; Pepin, Gueudet & Trouch, 2013).

[2] Synonymous with artifacts with power to assist people in achieving goals they presumably could not accomplish on their own.

Research findings show that curriculum resources have the potential to function as one important bridge between the written curriculum and the enactment in classroom practice (Jaworski, 2009). The pillars of this bridge consist of how and what teachers use from their curriculum resources (Remillard, 2005; Brown 2009). The same curriculum resource can result in different teaching, due to teachers' different characteristics (Lloyd 1999; Collopy, 2003), such as experience, intentions and abilities (Brown, 2009). In addition, Brown (2009) highlights curriculum resources' "potential for representing ideas, conveying practices, reinforcing cultural norms, and influencing teachers" (p. 21). Hence, the features of the curriculum resource have an impact on what teachers use from curriculum resources when planning and enacting mathematics education.

Mathematics textbooks have a strong position in Swedish classrooms (Boesen, Lithner & Palm, 2010). The Trends in Mathematics and Science study, TIMSS (Skolverket, 2008) found that more than 90% of Swedish compulsory school students are taught according to the structure in the textbook, from Page 1 onward. Further, they spend most of their mathematics lessons working by themselves in their textbooks. In addition, Swedish teachers, as well as their colleagues around the world, have faith in curriculum resources to support them in interpreting the written curriculum (Budiansky 2001; Haggerty & Pepin 2002; Johansson, 2006).

Considering this altogether, my rationales for this thesis are a) the strong position of curriculum resources in the mathematics classroom in general, b) teachers' faith in curriculum resources to convey the intentions and goals of mathematics education, and c) the expected benefits for mathematics education of expanding the knowledge base of research in the area of curriculum resources in general, and teacher guides in particular.

This thesis builds upon two studies conducted in Sweden. The first is a case study on how teachers use teacher guides. The results of this study could be seen as one piece in the building of a theoretical understanding of how teachers use their tools in planning and enacting teaching. The second Study investigates what content is represented in Swedish textbooks. The results of this study map the terrain over how the most commonly used textbooks in Sweden construct the topic of proportion and proportional reasoning.

My first aim with this thesis is to contribute to the knowledge of how teachers use teacher guides, and what they use. My second aim is to investigate what kind of content is available for students and teachers to use when developing understanding for proportional reasoning. As mathematics education relies heavily on curriculum resources, it is highly interesting to broaden the knowledge base of the potential support for teachers' enactment of lessons. Authors of curriculum resources could use the results from the studies as one way to develop teachers' most important tools. Teachers and school managers could use the results to inform themselves of possible strengths and weaknesses in the support offered by curriculum resources.

1.2 Overview of the thesis

This thesis consists of a total of five chapters (including this introduction), references and two appendices. Chapter 2 reviews relevant research on teachers' use of curriculum resources. I will briefly describe four different perspectives (Remillard, 2005), to then throughout the rest of the literature review concentrate on research in the spirit of the adopted perspective, for this thesis. The next section deals with research on the design of curriculum resources. I will briefly describe the features of curriculum resources and specify the focus of this thesis. Finally, I review the large body of research on proportional reasoning. I focus my review on definitions and known obstacles to students' learning of proportional reasoning. This picturing is necessary to understand the framework for analysis, used for assessing the potential of Swedish textbooks to support the learning of proportion and proportional reasoning. Chapter 3 includes an initial methodological discussion of the two research studies. It describes collection of data, the Swedish context and analytical approaches, as well as a discussion of the studies in relation to criteria of quality in research on mathematics education. As the last discussion cannot be completed before we know what the actual findings are, parts of it are postponed until Chapter 5. In Chapter 4, summaries of both studies are presented, with an emphasis on the findings and conclusions. Chapter 5 closes this thesis with a discussion of the contributions from both studies individually. A second methodological discussion is then provided, again addressing the degree of trustworthiness, generality and importance of both studies. Finally, I comment on questions raised during the studies and gaze ahead to new challenges for this research area.

Chapter 2 Related research

2.1 Research on teachers' use of curriculum resources

Teachers' use of curriculum resources is a growing but still underdeveloped research field (Lloyd, Remillard & Herbel-Eiselmann, 2009). The concept of *use* holds a variety of activities that teachers engage with when they interact with their curriculum resources: "It includes how teachers engage or interact with these resources as well as how and the extent to which they rely on them in planning and enacting instruction, and the role resources play in teachers' practice" (Lloyd et al., 2009, p. 7).

In her article from 2005, Remillard examines key concepts in research on teachers' use of curriculum resources from the past 25 years. She learned that that this area of research has produced a number of contradictory results, and she also found a possible explanation for this. The results of different studies are to be seen in the light of the researchers' interpretation of the relationship between teachers and curriculum resources. She found that different views on curriculum use could explain some diverse research findings from this field. Remillard found four main (partly overlapping) conceptualizations of curriculum use. First, she found that several studies are conducted under the presumption that teachers either *follow* or *subvert* the text. This view is predominant when curriculum resources are seen as a vehicle for instructional reform. A presumption is that a close fidelity between the written and enacted curriculum is possible and desirable. The curriculum is seen as a fixed representation of the enacted curriculum and the teacher is seen as an enactor of the planned curriculum. However, studies have shown that there will be a discrepancy between the written and the enacted curriculum, even when teachers intend to follow the curriculum literally (cf. Collopy, 2003; Sherin & Drake, 2009). This insight leads to that other studies are conducted under the conceptualization that teachers *draw on* curriculum resources (Remillard's second). Fidelity between the written and the enacted curriculum is possible, but not desirable, and the teacher has agency over the curriculum. Curriculum resources are seen as one resource among others, and the teacher is seen as an active agent in the process of enacting the curriculum. While fidelity between written curriculum and enacted curriculum is possible in the *drawing on* perspective, it is impossible in the *interpretation* perspective (Remillard's third). Research conducted within this perspective focus on how teachers interpret curriculum resources and how those interpretations relates to teachers' beliefs and prior experiences. The sole focus of the teacher is a distinction from Remillards' fourth perspective, *participating with the texts,* in which both curriculum resources and teachers are seen as participatory influencing the enacted curriculum. In participating with the text curriculum resources are seen as tools in the meaning of artifacts, sprung from the sociocultural evolution with power to shape human activity (Brown, 2009). The view that teachers use their resources in a participatory relationship proposes that the features of the teacher guides matter as much as the teachers' characteristics (Pepin et al., 2013). This perspective is adopted in this thesis, and taking this stance has several consequences for what teaching is, and for what teachers and the features of curriculum resources can bring to the collaborative relationship between teachers and curriculum resources.

With the fourth conceptualization in mind, we move on to the work of Matthew Brown (2009) and Brown & Edelson, (2003), who has elaborated further on the teacher-tool relationship under the presumption that teachers participate with their curriculum resources. Brown identifies three key components for understanding the interaction between teachers and tools: "a) curriculum resources play an important role in affording and constraining teachers' actions; b) teachers notice and use such artifacts differently given their experience, intentions, and abilities and c) 'teaching by design' is not so much a conscious inevitable reality" (p. 19). All three of these are interesting aspects of teachers' interaction with curriculum resources.

Key component (c), teaching by design frames the interaction between teachers and curriculum resources as a process where teachers appropriate and mobilize curriculum resources in order to craft instruction. By *design*, Brown refers to a process of *selecting, interpreting, reconciling-, accommodating,* and finally *modifying* curriculum resources. Key component (b) refers to the teacher's contribution to the participatory relationship is his or her personal resources, i.e. subject matter knowledge (Shulman, 1986; Ball, 1991; Stodolsky & Grossman, 1995), pedagogical content knowledge (Shulman, 1996), and goals and beliefs (Ball & Cohen, 1999)[3]. Due to different personal resources, teachers read, interpret and use curriculum resources differently (Remillard, 2000; Sherin & Drake, 2009). As mentioned above, key component (a) establish that the structure, features and quality of curriculum resources matter as much as teachers' personal resources for the instructional outcome (Pepin et al., 2013). The consequence of these three aspects is that the authors' intentions with the curriculum resources will be interpreted, adapted and brought to life in unique ways by each teacher who chooses to use them.

Brown and Edelsson (2003) found three basic patterns of how teachers participate with curriculum resources: *offloading, adapting* and *improvising*. In this thesis, these constructs are useful for discuss how the teachers talk about ways to use curriculum resources both to discuss how the teachers' talks about their use in the first study, and as potential ways to use the textbooks in study two. Offloading is when teachers relinquish their agency to the curriculum resources. An example of this is when a teacher offloads how to calculate a procedure to scripted instructions in the curriculum resources, deliberately giving all agency to the resource. In another situation, the same teacher used a lab setup based on her own experience and intentions as well as the instructions from curriculum resources. This shared agency between teacher and curriculum resources is an example of what Brown denotes adapting curriculum resources. When improvising the teacher had total agency, for example when improvising and orchestrating a whole class discussion relying entirely on her own strategies

[3] The contributions from the features of curriculum resources will be described in the next section.

(around issues raised during work with a lab task from the curriculum resources). So, in a sequence towards a certain learning goal, a teacher's use of curriculum resources can vary between offloading, adapting and improvising. An aspect of these three patterns is that teachers with different personal resources (knowledge, skills, commitment) can engage in equal processes of adaptation, offloading or improvisation and achieve different instructional outcomes. The more agency a teacher has, the more important her personal resources are for the outcome of the instruction, e.g. imagine an improvised discussion led by an engaged, experienced mathematically proficient teacher in contrast to a less experienced, skilled and engaged teacher. Let us now move on to consider the other part of this participatory relationship, the curriculum resources.

Design of curriculum resources is a growing area of research (Stein et al., 2007). An increasing interest for educative curriculum resources has generated several theoretical and some empirically driven studies over effective design of these resources (cf. Ball & Cohen, 1998; Brown, 2009; Davis, Palincar, Arias, Bismarck, Marulis & Iwashyna, 2014; Davis & Krajcik, 2005; Remillard, 2012).

In this thesis the focus is on 1) what teachers use from their teacher guides, and 2) the structure in textbooks. For the analysis of teachers' use, a framework for educative features of curriculum resources is used as a lens for drawing attention to what the teachers talk about in the interviews. This framework is developed by Hemmi, Koljonen, Hoelgaard, Ahl & Ryve (2012), building on the work of Ball and Cohen (1996) and Davis and Krajcik (2005). A closer description of the framework is presented in section 3.1.3. Further, the work of Shield and Dole (2002; 2008; 2013) are used for the analysis of the textbooks structure (physical objects and representation of physical objects, representation of tasks and procedures and domain representation/concepts) is in focus. This framework is presented in section 3.2.3.

2.2 Proportional reasoning in mathematics education

When scanning the literature on proportional reasoning I learned that there is a large body of research, which seems to unanimously 1) agree that proportional reasoning is a cornerstone in mathematics and a prerequisite for successful further studies (Behr, Harel, Post, & Lesh, 1992; Carpenter, Fennema, & Romberg, 1993; Harel & Confrey, 1994; Karplus, Pulos & Stage, 1983; Lamon, 2007; Lesh, Post & Behr, 1988; Sowder, Armstrong, Lamon, Simon, Sowder, & Thompson, 1998); and 2) demonstrate that proportional reasoning is hard to define, and appears to be used as an umbrella term for anything referring to ratio, rate and proportion (Lamon, 2007). Thus, definitions in the conceptual field of proportional reasoning are not crystal clear. However, in the following section, my intention is to specify how the concepts present in my analysis are interpreted and defined in this article.

Karplus et al. (1983) defined proportional reasoning as "a term that denotes reasoning in a system of two variables between which there exists a linear functional relationship - leads to conclusions about a situation or phenomenon that can be characterized by a constant ratio" (p. 219). Thus, it is clear that many mathematical topics will be involved in proportional reasoning, since whenever there is a linear functional relationship present, proportional reasoning is applicable. Rate, gradient of a linear function, similarity, percentage, geometry and trigonometry, for example, are topics that require proportional reasoning. From this it is obvious that no proportional reasoning will take place without the presence of a proportion. Proportion is an underlying mathematical concept, fundamental for the understanding of a range of topics (Shield & Dole, 2008). Proportions4 can be represented with words (i.e. the constant relationship between two variables), as linear functions that go through origin $y = kx$, with graphs or written as ratios $a: b$, or in fraction form $\frac{a}{b}$. I want to clarify

[4] Definitions vary between countries. Proportion is also defined as equivalence between ratios, e.g. a:b = c:d (Miyakawa & Winslow, 2009). This definition is not used in this paper, however, as the Swedish national curriculum defines proportion as the constant relationship between two variables.

that I use the concept of ratio as a representation form of proportion5. The concept of rate is used to describe magnitudes related to time, distance, cost and so on, e.g. speed (km/h; m/s), resistance (U/I), price per kilogram (SEK/kg) etc. (Miyakawa & Winslow, 2009).

One critical part of proportional reasoning is the multiplicative relationship represented in the situation (cf. Dole & Shield, 2008; Fernández, & Llinares, 2009; Lamon, 2007; Sowder et al., 1998; Van Dooren, De Bock, Vleugels, & Verschaffel, 2010). Through research, we know students have difficulty recognizing multiplicative situations and distinguishing them from additive situations (Van Dooren, De Bock, Hessels, Janssens & Verschaffel, 2005). The difference between additive and multiplicative relationships is that the former requires reasoning about quantities. To exemplify, consider this problem:

> Team A played a basketball game against Opponent A. Team B played a basketball game against Opponent B. The captains of Team A and Team B argued about which team beat its opponent by more. The captain of Team B won the argument by 8 points. Team A scored 79 points. Opponent A scored 48 points. Team B scored 73 points. How many points did Opponent B score? (Sowder et al., 1998 p. 133)

This problem involves enough complexity to require that the students make sense of the quantities involved (Sowder et al., 1998). It is not enough to simply apply a procedure to calculate the right answer; this problem requires reasoning about quantities in the additive relationships. In their early years, children rely on additive reasoning. Making the transition to be able to use both additive and multiplicative reasoning

[5] This may seem very basic, and is presumably entirely clear to anyone engaged in the field of mathematics education. However, having received vague answers when I asked several senior researchers, teacher educators and teachers, I decided to include this clarification.

requires the ability to distinguish between the two kinds of reasoning. Therefore, it is important for students to meet situations in which both additive and multiplicative comparisons are involved (Van Dooren et al., 2010; Sowder et al., 1998), as in the following problem. Both additive and multiplicative comparisons are necessary to solve this problem. This gives the student the opportunity to reflect over the different features of additive and multiplicative reasoning.

> Dieter A: "I lost 1/8 of my weight. I lost 19 pounds."
>
> Dieter B: "I lost 1/6 of my weight, and now you weigh 2 pounds less than I do."
>
> How much weight did Dieter B lose? (Sowder et al., 1998, p. 137)

The difference between ratios and fractions is another source of confusion for students. For example, if a class of students consists of 15 girls and 14 boys, the part/whole fractions 15/29 and 14/29 arise. The part/part ratio between girls and boys is 15:14. When a ratio connects two parts of the same whole, students may not adequately recognize the difference between the part/part and part/whole relationships (Clark, Berenson, & Cavey, 2003). It is not easy for students to discover the difference that ratios consist of a part/part comparison while the comparison in a fraction refers to part/whole, since ratios can be written in fraction form and obey the same mathematical laws as fractions (Shield & Dole, 2002).

According to Karplus et al. (1983), two types of problems convey the nature of proportional reasoning: missing value problems and comparison problems.

> *1. Missing Value Problem*: Yesterday I bought 28 candies with 12 quarters. Today, if I go to the same store with 15 quarters, how many candies can I buy? (Lo & Watanabe, 1997)

2. Comparison Problem: Car A is driven 180 km for 3 hours. Car B is driven 400 km for 7 hours. Which car is driven faster? (Karplus et al., 1983, p. 220)

More recent research suggests that the construction of missing value problems and comparison problems also embodies proportional reasoning problems (Tjoe & de la Torre, 2012). The missing value problem (1) above is strictly multiplicative in its nature, and can be solved through several methods. It can be approached with proportional reasoning within or between ratios, cross-multiplication, or other strategies (not relying on ready-made procedures). Approaching the missing value problem (1) above will give us the following multiplicative comparison, when reasoning within ratios (same measure space):

Measure space candies	Measure space quarters
28	12
↓	↓
?	15

Fig. 1 Reasoning within measure space

Within the measure space *quarters* we can reason that 12 times 15/12 is 15. Since we have a multiplicative comparison, the same relation must be true for the measure space *candies*. Consequently, the number of candies you can buy for 15 quarters is 28 times 15/12. If we instead reason between measure spaces, we get the relation 12 times 28/12 is 28. Then 15 times 7/3 gives us the answer to how many candies you can buy with 15 quarters.

Measure space candies		Measure space quarters
28	←	12
?	←	15

Fig. 2 Reasoning between measure spaces

Another way to approach the problem is to apply cross-multiplication. Students are often taught to solve missing value problems with the method of cross-multiplication, when there are three known values and one unknown (Karplus et al., 1983).

Measure space candies		Measure space quarters
28		12
	↖↗	
?		15

Fig. 3 Cross-multiplication

With cross-multiplication we get the equation $12x = 15 \cdot 28$. While this is an easy way for students to approach missing value problems, research has shown that it turns a proportion problem into an algebraic equation and prevents the understanding of proportional reasoning (Lamon, 2007; Shield & Dole, 2008). Teaching how to solve missing-value problems is not synonymous with teaching proportional reasoning (Tjoe & de la Torre, 2014). Students tend to apply this procedure without considering the nature of the problem. For example, without considering the applicability in the problem *Farmer Gus needs 8 hours to fertilize a square pasture with sides of 200 meters. Approximately how much time will he need to fertilize a square pasture with sides of 600 meters?*, applying proportionality will give the wrong (and unrealistic) answer of 24 hours (Van Dooren et al., 2010). To promote the understanding of the concept of proportion and avoid over-use of the cross-multiplication procedure, it should be delayed until students have attained an understanding of proportional reasoning. Though procedures can be very helpful in solving problems involving proportional reasoning, if you attained a profound understanding of the multiplicative relationship represented in proportional situations (Lamon, 2007).

> *2. Comparison Problem*: Car A is driven 180 km for 3 hours. Car B is driven 400 km for 7 hours. Which car is driven faster? (Karplus et al., 1983, p. 220)

Let us return to the comparison problem (2) above. This can be solved with different approaches as well. For this problem the students need to compare ratio 180/3 and 400/7 to determine which car is faster. The students can avoid the proportional reasoning part of the problem by using the method of manipulating the speed formula. Manipulating $s = v * t$ will give the students speed per hour without considering a comparison of ratios (Karplus et al., 1983). Hence, if our learning goal is to promote students' understanding of proportional reasoning, we are in great danger of missing this goal if we teach them to manipulate formulas before they have had extensive opportunities to practice proportional reasoning.

To sum up: The critical part of proportional reasoning is the multiplicative relationship represented in the situation. Identifying multiplicative relationships requires the ability to distinguish between additive and multiplicative situations. Reasoning proportionally includes reasoning within and between measure spaces, and being able to compare ratios. Students are often confused about the difference between ratios and fractions. The introduction of procedures should be delayed until students have attained an understanding of the above-mentioned hurdles associated with proportional reasoning, known from research.

Chapter 3 Methodology

3.1 Research design: Study I

Below I outline the research design for the first study of this thesis as a way to elaborate the methodological approach used to answer the research question: What content do teachers use from their teacher guide when planning lessons?

3.1.1 Data

The data for Study I were collected through semi-structured interviews of five early-years mathematics teachers. Our selection of teachers was based on the criteria that the group represents both teachers with long teaching experience and those with only some years in the profession; and that they use one of the most commonly used teaching materials for Grades 1 to 3 in Sweden (*Matte Eldorado* and *Matte Direkt Safari*). Our rationale for choosing teachers with different experience is that research has shown that they desire different support from their teacher guides (Brown, 2009; Stein et al. 2007). All participating teachers teach children in Grades 1 to 3. They volunteered for the interview, knowing that the study was about how they use teacher guides. In advance, the teachers were asked to copy the pages from the teacher guide that they had used the week before the interview. The same researcher conducted all interviews in November 2013 (14th-18th), filming them with her laptop computer. Four of five interviews lasted approximately 30 minutes, while the interview with Doris lasted 60 minutes. All teachers except one (Doris) brought copied pages from the teacher guide, showing the content they had used the week before the interview. However, Doris had not used any particular content from the teacher guides the week before the interview had instead occupied her students with practical exercises of measuring.

This study is part of a larger comparative study on how teachers use their teacher guides. Swedish, Icelandic and Finnish researchers drafted the interview questions together for this collaborative project. Altogether, we arrived at eight questions for semi-structured interviews (Bryman, 2012). The questions aim to capture how the teachers start their planning; how they use their teacher guides; which parts of the guides they usually use; what they value in the guides; and what they would like to see in them (see Appendix A). In short, all questions concerned *what* they use and *how* they use it; as well as the important issue of *what* support they feel is lacking and wish to see in their teacher guides.

3.1.2 Context

In this section I give a brief description of the participating teachers and the teacher guides as a foundation for our analysis of the data. The teachers in our study differ concerning their education and experience.
The participating teachers are:

- Annika, with 30 ECTS in mathematics. She has been teaching for less than two years.
- Beret, with 90 ECTS in mathematics. She has been teaching for more than ten years.
- Camilla, with 30 ECTS in mathematics. She has been teaching for more than ten years.
- Doris, with 60 ECTS in mathematics. She has been teaching for more than ten years.
- Erika, with30 ECTS in mathematics. She has been teaching for less than two years.

The teacher guides in our study are the most commonly used on the market in Sweden. We know this from our own experience as teachers, and from a quantitative

analysis of a survey carried out among all teachers teaching mathematics in the sixth largest municipality in Sweden (Neuman, Hemmi, Ryve & Wiberg, 2014).
The teaching resources the teachers use are:

1. *Matte Direkt Safari* (Falk et al., 2011), a Swedish textbook and teacher guide that has been on the market for several years (used by teachers Beret, Doris[6] and Erika); and
2. *Matte Eldorado* (Olson & Forsbäck, 2011), a Swedish textbook and teacher guide that has been on the market only a few years (used by teachers Annika and Camilla).

Matte Direkt Safari (S): At the beginning of the teacher guide there are four pages with information for the teacher about the structure of the students' textbook and the teacher guide. Each chapter in the teacher guide starts with the goals for the chapter. The students' textbook has a picture as an introduction to each chapter. The teacher guide offers the teacher suggestions for questions to ask the children. The picture is meant to engage the children and make them curious about the content in the upcoming chapter. There are also practical exercises, worksheets and a page of activities that can be carried out either indoors or outdoors. Some chapters, but not all, have one page of descriptions and explanations of the methods presented in the students' textbook.

Matte Eldorado (E): The information to the teacher is extensive. Around 40 introduction pages at the beginning of each teacher guide deal with different aspects of mathematics, such as algorithms, teaching methods and assessments. The authors of this teacher guide have emphasized visualizing clear connections between the content in the textbook and the national curriculum. After this introduction, the guide follows the structure in the textbook. Each chapter starts with two pages of additional information to the teacher about what to pay extra attention to in teaching to achieve the

[6] Doris uses a range of teacher guides, different ones for different grades.

chapter's goals. Goals for working with the chapter, worksheets and concrete practical materials are represented in every chapter. There are also suggestions for how to individualize teaching for those who need more of a challenge and those who need easier tasks. See Hoelgaard, Hemmi and Ryve (2014) for more information about the teacher guides.

3.1.3 Analytical approach

The framework we used for our analysis of data in Study I is developed from the work of Ball and Cohen (1996) and Davis and Krajcik (2005). In their article from 2005, Davis and Krajcik present guidelines for how to design curriculum materials, with the intention to promote teacher as well as student learning. They use the work of Ball and Cohen (1996) to develop nine design heuristics for *educative curriculum materials* in science. These design heuristics rely on five high-level guidelines for how to design curriculum materials. We used these guidelines to develop an analytical tool to analyze the content of mathematics teacher guides (Hemmi, Koljonen, Hoelgaard, Ahl & Ryve. 2013). It contains five categories for educative curriculum materials (Table 1). In using the tool to analyze the interviews, we want to test whether the tool works to reveal what content the teachers considered useful. We want to be clear that we use the tool to draw our attention to what kind of content the teachers talk about in the interviews; we do not aim to use it as a scheme to code every sentence.

Table 1: Five categories for data analysis

1a) General knowledge of students' ideas and strategies	Describes why students might hold particular ideas about mathematical concepts and exemplifies common strategies among students.
1b) Suggestions for how to encounter students' ideas and strategies	Gives suggestions for how to deal with/encounter various ideas and strategies of students and how to enhance their learning and prevent future difficulties.

2) Concepts and facts	Describes concepts and facts within mathematics such as history, field of application, derivations, methods, proofs, and correct terminology.
3) Progression and connections	Shows the mathematics progression throughout the school years as well as connections between mathematical topics; for example, explains the future development of methods and concepts.
4) Connecting theory and practice	Supports the teacher's actions in practice beyond the curriculum materials by connecting theory and practice. Exposes the central ideas in national curriculum and research results for promoting teachers' autonomy.
5) Design of Teaching[7]	Supports the teacher's ability to act in practice through suggestions with respect to the design and enactment of lessons, tasks, formative assessment, individualization of teaching, homework, etc.

Categories 1 to 4 consist of written support, which aims to increase the teachers' resources in form of subject matter knowledge and pedagogical content knowledge. Category 1 (a and b) could help teachers anticipate and act on misconceptions, by describing why students hold particular ideas. Category 2 concerns deepening teachers' subject matter knowledge (Shulman, 1986; Ball, 1991; Stodolsky and Grossman, 1995) by closely presenting concepts and facts within the subject area. Category 3 helps the teacher consider her lessons within a larger context, by showing expected progression over time and connections between topics. Category 4 visualizes the authors' rationales for their pedagogical judgments. This can promote teachers' autonomy, e.g., by giving them opportunities to consider the applicability to their students in their context. Thus, Categories 1 to 4 help teachers "find productive ways of adapting curriculum materials" (Davis & Krajcik, 2005), and support Category 5 – the teachers' design capacity. Category 5 consists of support for the Design of Teaching, which refers to the support for the process of selecting, interpreting, reconciling, accommodating, and finally modifying curriculum materials (Brown, 2009).

[7] Brown's definition of Design of Teaching, which refers to a process of *selecting, interpreting, reconciling, accommodating,* and finally *modifying* curriculum materials.

3.2 Research design: Study II

Below I outline the research design for the second study of this thesis as a way to elaborate the methodological approach used to answer the research question: What is the potential of Swedish mathematics textbooks to support the learning and teaching of proportion and proportional reasoning?

3.2.1 Data

My choice of textbooks is based on a questionnaire among all compulsory school mathematics teachers in the sixth largest municipality in Sweden. The questionnaire was distributed in August 2012. The results showed that two curriculum resources together covered 97% of the classrooms in Grades 7-9 (Neuman et al., in press). Swedish publishers' sales numbers are not official, but based on the dominance of these textbooks in our questionnaire and my own experience from teaching, it is likely that they are the most commonly used curriculum resources in Swedish classrooms. In this analysis they are called Textbook A and Textbook B. I briefly introduce these textbooks, as a foundation for the analysis that focuses on how proportion and proportional reasoning is represented in the books.

In Textbook A, every chapter starts with a description of the content and the goals for working with the chapter. At the top of each page or spread there is a square containing key concepts relating to the exercises. There are few worked examples in the textbook, but when they are present they are in this square. The intention of the authors is that all students should work with a basic course. After this, they perform a diagnosis. Depending on the results, the students then continue with either a blue course (slightly easier than the basic course) or a red course, which is more demanding than the basic course. The chapter closes with a summary and a spread with more demanding problems.

In Textbook B, every chapter starts with a list of goals for that chapter and a summary of key concepts the students will encounter. Then follows a number of sections containing theory, worked examples and tasks on four levels, where 1 is the most basic and 4 the most demanding. The intention of the authors is that the students should work with two levels. The chapter closes with a summary and a set of mixed tasks on different levels, aiming to cover the learning goals of the chapter. Each chapter also includes a diagnosis, extra tasks for those who need more practice, a theme and problem-solving.

3.2.2 Context

Teachers in Sweden are free to choose which curriculum resources to use. Since the beginning of 1991, there is no state control over curriculum resources (Jablonka & Johansson, 2010) and the textbooks and teacher guides available on the market are produced commercially. This gives the authors of curriculum resources an important role, as research findings show that curriculum materials have potential to function as one important bridge between the written national curriculum and classroom practice (Jaworski, 2009). The authors interpret the national curriculum and provide teachers with a structure for how to enact the national curriculum. Hence this is especially important in the Swedish context since mathematics textbooks have a strong position in Swedish classrooms (Skolverket, 2008).

Sweden has a new national curriculum from 2011, *Curriculum for compulsory school, preschool class and leisure-time center 2011*. As a frame for learning and teaching, this curriculum emphasizes the explicit demand that in all education, it is important that overall, well-balanced historical, environmental, international, and ethical perspectives be established (pp. 11-12). Further "Teaching in mathematics should essentially give pupils the opportunities to develop their ability to:

- formulate and solve problems using mathematics and also assess selected strategies and methods,
- use and analyze mathematical concepts and their interrelationships,
- choose and use appropriate mathematical methods to perform calculations and solve routine tasks,

• apply and follow mathematical reasoning, and
• use mathematical forms of expression to discuss, reason and give an account of questions, calculations and conclusions." (Skolverket, 2011a, pp. 59)

The importance of the application of mathematics in real-world situations is also explicit, as is the demand for creating learning situations in contexts familiar to the learners. The national curriculum provides brief descriptions of learning goals for Grades 3, 6 and 9. These learning goals are then further elaborated on in a separate commentary text (Skolverket, 2011b). Skolverket (2011b) defines proportionality as the constant relationship between two variables. Proportionality can serve as a model in different contexts. If students understand proportionality they can transfer this model thinking to calculations of, for example, percent or scale at reductions or enlargements. Skolverket (2011b) stresses that it is important to master the concept proportionality in many situations in everyday life and society, such as repayment of loans or calculating and comparing the costs of mobile phone subscriptions. It is important that the learning in this field of knowledge is based on student-close situations. This gives them the opportunity to gradually develop a more abstract and general understanding of how to use mathematical expressions to describe changes and rate of change. (Skolverket, 2011b)

In sum, Swedish mathematics lower secondary schools are highly dependent on textbooks to plan and enact lessons. These textbooks are produced commercially on an open market, which gives the authors agency to interpret the national curriculum. Teaching students to master proportionality in many different situations is part of the core content for Grades 7-9. The learning and teaching should take place in a context that is familiar to students in a good balance of historical, environmental, international, and ethical perspectives.

3.2.3 Analytical approach

This study assesses the potential of the above-described textbooks to support the learning and teaching of proportional reasoning. Shield and Dole (2013, pp. 8-9) developed the tool I used for assessing the potential in Swedish textbooks to support the learning of proportion and proportional reasoning, during studies of Australian mathematics textbooks. My rationale for choosing this framework is that the framework is deeply rooted in research. Further, Australia is one of the OECD countries and therefore shares the same rationale for compulsory mathematics education, namely: "Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens" (OECD, 2013 p. 25). Shield and Dole's (2013) framework for assessing the potential of mathematics textbooks to promote deep learning consists of five learning goals: *1) Additive and multiplicative comparison contrasted through use of authentic life-related situations; 2) Identification of multiplicative structure and proportional thinking; 3) Meaningful symbolic representation; 4) Related fraction ideas explicitly connected;* and *5) Effective use of a range of representations*.

I conducted the analysis in three steps. Initially, I analyzed the textbooks for Grades 7, 8 and 9 and documented which parts of the textbooks dealt with proportion and proportional reasoning (Appendix B). In step two, I solved all exercises in relation to ratio, proportion and proportionality, and analyzed the characteristics of the problem (missing value, ratio-proportion construction, ratio comparison and other[8]). In step three, I read the instructions to the students and searched the textbook for worked examples, presented procedures and written support for develop the students' understanding of proportional reasoning in relation to the five learning goals in the framework I used for analysis:

[8] For example interpret proportional graphs

Learning Goal 1. Additive and multiplicative comparison contrasted through use of authentic life-related situations.

Indicators:

(a) Opportunities to differentiate between additive and multiplicative comparisons are provided.

(b) The multiplicative relationship in proportional situations is made explicit (cf. the additive nature of non-proportional comparisons).

(c) Examples and exercises use authentic comparisons.

Learning Goal 2. Identification of multiplicative structure and proportional thinking

Indicators:

(a) Multiplicative comparative relationship of ratio situations is clearly defined.

(b) Use of the operations of multiplication and division is highlighted (inverse).

(c) Use of both within and between thinking is evident.

Learning Goal 3. Meaningful symbolic representation

Indicators:

(a) Representation supports identification of within and between relationships in the proportion situation.

(b) Links between symbolic representation across problem types are made explicit (i.e. solution procedures are based on consistent symbolic representation for problems that share the same structure).

(c) Introduction of the formal "proportion equation" is delayed until extensive experience with other representations has been attained.

Learning Goal 4. Related fraction ideas explicitly connected

Indicators:

(a) Clear links are made with ideas of fractions and equivalence.

(b) Part/whole fraction and part/part/whole ratio relationships are explicitly distinguished.

(c) Fraction notation meaning in use (e.g. part/whole, ratio, and quotient) is clearly signaled.

Learning Goal 5. Effective use of a range of representations

Indicators:

(a) Tables are used to highlight multiplicative relationships.

(b) Graphs of proportional situations are straight lines that go through the origin.

(c) Graphs are used to extrapolate and interpolate solutions and/or make predictions.

I used the ratings from Shield and Dole (2013) to analyze textbooks A and B. I have read and interpreted Shield and Dole's work (see also 2008). However I cannot know if my interpretation slightly diverges from the original as qualitative studies contains some subjective human impact from the researcher. The rating of the indicators is on a scale from *No evidence* to *High support* (Table 1). The table contains examples of each rating.

Table 2: Explanation of rating

Rating	Explanation
No evidence	Used when there is no evidence of the indicator. The indicator is not mentioned at all in the curriculum resource.
Low support	Used when the indicator is present in the curriculum resource with no further, or only shallow, explanation.
Medium support	Used when the indicator is present in the curriculum resource without full correspondence with the statement in the indicator.
High support	Used when the statement in the indicator corresponds with the curriculum resource.

Example of No evidence: Neither Textbook A nor B provides any exercises or problems requiring the students to differentiate between additive and multiplicative situations. Consequently, for Learning Goal 1, Indicator (a) *Opportunities to differentiate between additive and multiplicative comparisons are provided*, both textbooks received the rating *No evidence*.

Example of Low support: Textbook A rated low on Learning Goal 2, Indicator (a) *Multiplicative comparative relationship of ratio situations is clearly defined.* In Textbook A, Grade 9, basic course, similarity is described as objects and figures with the same shape (e.g., a triangle) that are enlargements or diminishments of each other, without explicitly pointing out that the lengths of the sides in the enlargement are x times the length in the smaller triangle. In the more demanding red course, triangles are again used to illustrate similarity. This time, the proportional sides are represented as ratios. Again, this is done without explicitly pointing out that the same multiplicative factor will enlarge the sides of the smaller triangle to the lengths of those of the larger one. In the summary of the *Geometry* chapter, similar triangles are defined as enlargements or diminishments of each other, with the same angles between corresponding sides and the same ratio between corresponding sides. Again, there is no explicit statement that multiplying (or dividing if going from larger to smaller) by the same factor (the proportionality constant) will give us the lengths of the other triangle. In the more demanding red course, when working with comparing proportionalities, the rule of three (first calculating the unit rate SEK/kg, liter/mile etc. and then multiplying by number of kilograms, miles etc.) is introduced in a worked example, without considering comparisons of within or between ratios, which could have been used to illustrate the multiplicative relationship in ratio situations. To sum up, since there is no explicit definition of the multiplicative comparative relationship of ratio situations, I rated the support for textbook A low.

Example of Medium support: For Learning Goal 1, Indicator (c) *Examples and exercises use authentic comparisons,* both Textbooks A and B rated *Medium support*. Both textbooks have a clear emphasis on presenting the problems in a context that is familiar to the students. Purchasing candy, renting a personal trainer and sports gear, and buying gym memberships are situations familiar to most students. However, the chosen contexts are harmless and do not engage the students in a historical-, environmental-, international- or ethical perspective. To sum up, since the contexts lacks of authentic comparisons in historical-, environmental-, international- or ethical perspective's, which is an explicit demand in the national Swedish national curriculum (2011a), I rated the support for both textbook A and B medium.

Example of High support: For Learning Goal 5, Indicator (b) *Graphs of proportion situations are straight lines that go through the origin*, both Textbooks A and B rated *High support*. In both textbooks proportionality is defined as a linear function that goes through the origin, and there is a range of graphs of proportionalities in both textbooks. Proportionalities are mixed with linear function that is non-proportional. Graphs clearly illustrate the difference. To sum up, since both textbook A and B explicitly shows that graphs of proportion situations are straight lines that goes through the origin, I rated the support for both textbooks high.

3.3 Ethical Considerations

These studies brought up ethical considerations regarding the participating teachers, as well as the authors and publishers of the teacher guides used by the teachers in Study I, and the textbooks investigated in Study II. Regarding the teachers in Study I, I considered the basic ethical principles of harm to participants, lack of informed consent, invasion of privacy, and deception (Bryman, 2008).

The teachers in Study I volunteered to participate in the study, and were informed of its aim and methods. Their anonymity is protected by pseudonyms and the lack of information on the city and schools where the interviews were conducted. The data are used only for the declared purpose of the study, and are stored so that unauthorized individuals cannot access it. Regarding the teacher guides, I was indecisive as to whether to refer to them by title or pseudonym. I chose to refer to those in Study I by their actual titles, for two reasons. First, this study aims to capture the teachers' perceived support and inspiration provided by the guides. This is a positive approach that highlights their positive aspects; hence, I doubt that their authors and publishers will resent the results. The teachers also answered questions about what they would like to see in their teacher guides. Authors and publishers could perceive this as criticism, but I believe the gain from transparency outweighs the risk of resentment.

In Study II I chose to refer to the textbooks as Textbook A and Textbook B, instead of using their titles. The reason for this approach is that I investigated the textbooks with a prescriptive framework. Though my aim is not to criticize them, it is inevitable that the weaknesses found can be perceived as such. My sole intention is to make visible the structures of how proportion and proportional reasoning are presented in the dominating Swedish textbooks. I do not aim to assess which is better, but rather to simply describe how these commonly used textbooks present proportion and proportional reasoning.

3.4 Trustworthiness, generality and importance

Some words about the trustworthiness, generality and importance (Schoenfeld, 2007) of the studies are appropriate at this point of the description of method and design. Starting with the last, importance is closely connected with relevance; i.e., is there interest from the community of mathematics education in the research questions asked? As mentioned earlier, we have a situation where the knowledge about the impact of, and teachers' use of, resources for mathematics education needs more research to be understood; thus the answer to the question is yes.

Addressing the dimension of generality, these studies do not claim to provide solid evidence of any phenomena (Schoenfeld, 2007). However, there are some general characteristics of the circumstances under which the studies are carried out. The teachers in the first study were not specially chosen due to any extraordinary contributions to mathematics education. They are teachers in general, those you would find in ordinary lower secondary schools all over Sweden. Perhaps, to some extent, you could argue that they might be a bit more engaged than teachers in general, considering that they volunteered for the study. The curriculum resources in both studies are those most commonly used in Swedish classrooms, easily found for replication studies (a dimension of trustworthiness). The discussion of how efficient my research design is postponed to chapter 5, when the results have been presented. Considering the scope of Study I, existence proofs in the form of demonstration of a phenomenon worth investigating further are a realistic and satisfying generality of results (Schoenfeld, 2007). In Study II, I extrapolate from a sample to a population to map the terrain of Sweden. Extrapolation is considered to be a strong argument for generalization (Firestone, 1993). I will return to the dimensions of importance, generality and (the not yet addressed) dimensions of trustworthiness in Chapter 5.

Chapter 4 Summary of papers

4.1 Paper I

Title: How are mathematics teacher guides used for support and inspiration in teaching?

It is well known that the design of mathematics resources[9], in and for teaching, has a strong impact on how teaching, learning and classroom practice will be orchestrated. Notwithstanding this strong position, this area is short of research. Especially teacher guides are short of research (Remillard, 2012). Therefore, it is of particular interest to broaden the knowledge of how teachers interact with their teacher guides. This study focuses on how teachers talk about their use of mathematics teacher guides, for support and inspiration in teaching. We aim to broaden the knowledge of how teachers use the support from mathematics teacher guides in and for teaching. Our research question is: What content do teachers use from their teacher guide when planning lessons?

Data are collected through a study with semi-structured interviews of five early-years mathematics teachers. The teachers brought copies of the pages from the teacher guide they had used the week before the interviews. The interviewer filmed the interviews with her laptop computer. Our selection of teachers was based on the criteria that they represent both teachers with long teaching experience and teachers with only some years in the profession; and that they use one of the most commonly used teaching resources in Sweden. All participating teachers teach children in Grades 1 to

[9] All the resources developed and used by teachers in their interaction with mathematics for learning and teaching, both inside and outside classroom (Pepin, Gueudnet & Trouche, 2013, p. 929).

3. They volunteered for the interview, knowing that the study was about how they use their teacher guides. We, the authors, made a joint analysis of the films and the copied pages from the teacher guides to ensure that we had reached mutual understanding of the data. In the analysis of data we use the analytical tool we had developed from Davis & Krajcik (2005) in a study by Hemmi et al. (2013). It contains five categories of content in educative curriculum resources (Table 1). We use the tool to label the support and lack of support the teachers talk about in the interviews.

The results show that less experienced teachers desire a wider scope of content in their teacher guide. More experienced teachers desire support for teaching activities in the Design of Teaching in classroom practice. These results are in line with Brown's (2009) finding that teachers use their tools[10] differently depending on experience, abilities and intentions. So, parts of our study confirm previous research findings, but the results also make a new contribution to the understanding of how teachers use their teacher guides. All teachers in our study (regardless of their differences in experience, knowledge, beliefs, etc.) want the teacher guide to offer connections between theory and practice. This finding shows that outer driving forces can gather teachers, with very different personal resources, to demand the same kind of support. In this case, the new national curriculum was the other driving force.

A question raised during the work is what teachers consider to be the characteristics of a teacher guide and what impact these interpretations have on how they are used. The view that teachers use their resources in a participatory relationship proposes that the features of the teacher guides matter as much as the teachers' characteristics (Pepin et al., 2013). Further studies on the impact of the design of mathematics teaching resources are required to reach a better understanding of the participatory relationship between teachers and tools.

[10] Curriculum resources are seen as artifacts that assist teachers in achieving goals they presumably could not or would not accomplish on their own.

4.2 Paper II

Title: The potential of dominating mathematics textbooks in Sweden to support the learning and teaching of proportional reasoning

My second study focused on how dominating Swedish textbooks support the learning and teaching of a topic that is considered to be the most cognitively challenging in compulsory school (Lamon, 2007); namely, proportion and proportional reasoning. Several studies have revealed a discrepancy between the represented ideas in text-books and educational goals. A study by Vincent and Stacey (2008) found that some of the best-selling books in Australia emphasized memorization and procedures with-out connections. Again in Australia, Shield and Dole (2002) analyzed textbooks for lower secondary school and found limitations in definitions for the topics of ratio, rate and proportion. Fan and Zhu (2007) studied textbooks from China, Singapore and the US, and found a notable gap between each stated curriculum goal and the mathematical content in the textbooks. Lundberg (2011) found that Swedish text-books for Grade 10 are short of problems requiring proportional reasoning and deep understanding of concepts. Most exercises in her analyzed textbooks required no more than imitating the procedure presented in worked examples. However, there are no earlier studies on textbooks' representation of the topic of proportion and propor-tional reasoning for Grades 7 to 9 in Sweden.

Hence, the aim of this Study is to map the terrain of the potential of Swedish sec-ondary school textbooks to support the learning and teaching of proportional reason-ing. This Study is likely to reflect the general situation in Sweden. Based on an in-quiry of all mathematics teachers in the fifth largest municipally in Sweden, we found that these textbooks covered 97% of the classrooms. It is plausible that there are local variations in which other textbooks are dominant, but the sample indicates that the analyzed textbooks have a dominant market share. Lower secondary school is highly interesting, due to textbooks' strong influence in Swedish classrooms[11]. My rationale for the chosen topic, proportion and proportional reasoning, is that it is a cornerstone in mathematics and a prerequisite for all further studies of mathematics (Behr, Post & Lesh, 1988). This topic has attracted great interest from researchers for decades, which makes it possible to use research results and investigate how these insights are put to work in textbooks. Proportion and proportional reasoning is also considered to be one of the most cognitively challenging (Lamon, 2007) to learn. Consequently, it is also difficult to teach. My research question is: How do mathematics textbooks support the learning of proportion and proportional reasoning?

For the analysis of the textbooks I use a framework developed by Shield and Dole (2013) for assessing the potential of textbooks to support learning and teaching, to develop the students' understanding of proportional reasoning in relation to the five learning goals: *1) Additive and multiplicative comparison contrasted through use of authentic life-related situations; 2) Identification of multiplicative structure and proportional thinking; 3) Meaningful symbolic representation; 4) Related fraction ideas explicitly connected;* and *5) Effective use of a range of representations.*

The study shows that the analyzed textbooks have limited potential to support the learning and teaching of proportion and proportional reasoning. Strength in both textbooks is the representation of tables that can be plotted into graphs. There are also several illustrations in both textbooks of proportional graphs as straight lines that go through the origin. A weakness is that the identification of multiplicative structure and proportional thinking is hindered by the absence of representation of within and between thinking in both textbooks. Both textbooks' lack of links between symbolic representations across problem types is made explicit (i.e., solution procedures are based on consistent symbolic representation for problems that share the same structure). Textbook A has few worked examples in the basic course. This gives the teacher agency, but no support, to choose what solution methods to emphasize. In red

[11] Skolverket (2008) reported that more than 90% of Swedish compulsory school students are taught with the textbook.

course, procedures are often introduced together with the introduction of new mathematical content. In Textbook B, the worked examples give students different solution methods for speed, scale, ratio and proportion problems without being explicitly introduced to the underlying multiplicative structure, which risks their getting the impression that the problems are essentially different. My analysis reveals that both textbooks offer low support for the learning and teaching of the topic proportion and proportional reasoning.

Chapter 5 Discussion

This final chapter closes this thesis, with a discussion of the contributions of both studies. A second methodological discussion is then provided, again addressing the degree of trustworthiness, generality and importance of both studies. Finally, I gaze forward to new challenges for this research area.

5.1 Contributions

In Study I, we found that teachers use teacher guides differently and search for different kinds of support for their planning and enactment of lessons. The results align with Remillard (1992; 2000), who found that teachers' reading of textbooks is selective and interpretative. Brown (2009) stresses that these differences are due to teachers' different experience, abilities, and goals and beliefs. We also found that the new Swedish national curriculum was an outer driving force that united the teachers' need for support from the teacher guides. All teachers wanted support in interpreting the learning goals and progression in the national curriculum, regardless of their different personal resources. Authors could use these results to consider the structure and design of teacher guides. The study indicates that there is content suitable and desirable for all teachers. At the same time, there are different needs for support that could be structured with multiple entry points of access (cf. Brown, 2009), to allow teachers to enter the guide at a suitable level of support for their needs. Researchers could use the results to further elaborate on how the teacher guides can be designed to suite a range of different teachers.

Study II maps the terrain of how dominating Swedish textbooks for Grades 7 to 9 support the learning and teaching of proportion and proportional reasoning. The study provides insights into the strengths and weaknesses in the presented structure regarding proportional reasoning in the most commonly used textbooks in Swedish lower secondary school. Teachers could use these insights when making decisions about how to use their textbooks. Thus, as rationales for the learning goals are gathered from research on the topic, teachers could also use the results to reconsider how to organize the enactment of the national curriculum in order to develop deep understanding. The analysis could provide authors with valuable information to consider for textbooks' structure and sequencing. The results broaden Lundberg's findings (2011) that Swedish textbooks for Grade 10 are short of problems requiring proportional reasoning and deep understanding of concepts. Most exercises in her analyzed textbooks required no more than imitating the procedure presented in worked examples. Similar results are reached for textbooks in Australia (Shield & Dole, 2013). These researchers found that the commonly used textbooks have low potential to promote deep learning of proportion and proportional reasoning. This study broadens the knowledge of how structures of concepts, tasks and representations are represented in Swedish textbooks for Grades 7 to 9.

A possible hypothesis for the author choices of structure is the phrasings in the Swedish national curriculum (Skolverket, 2011a) and its commentary text (Skolverket, 2011b). The use of graphs in student-close situations is explicit. Both textbooks have a range of graphs over both proportional and non-proportional situations. However there is no explicit phrasing to highlight the common multiplicative relationship represented in problems that share the same structure; the importance of being able to identify multiplicative relationships and the ability to distinguish between additive and multiplicative situations; reasoning within and between measure spaces, or being able to compare ratios. To find the answer for this hypothesis further studies including interviews with textbook authors are necessary.

5.2 Trustworthiness, Generality and Importance revisited

As mentioned in Section 3.4 I now return to the trustworthiness, generality and importance of this thesis. Starting with the last aspect, the importance of the thesis, see Section 5.1, where its contributions and relevance are described. Addressing the aspects of generality, as mentioned above, Study I provides an existence proof, worthy of further investigation (Shoenfeld, 2007), that there are outer driving forces that unite teachers' desire for support, regardless of the teachers' different personal resources. This finding is not yet general, but with further studies has the potential to be. Study II is likely to map the terrain of how the support of proportional reasoning is structured in Swedish textbooks for Grades 7 to 9. The claim for this generality is based on an extrapolation of a sample from a quantitative study showing that 97% of the teachers in Sweden's sixth largest municipality use these textbooks. Extrapolation from a sample is generally thought to be the strongest argument for generalization in qualitative research (Firestone, 1993).

The discussion of trustworthiness (validity and reliability) is centered on three themes: 1) the gathered data; 2) the performed analyses; and 3) the findings. As presented in Chapter 3, the data gathered for Study I consist of five filmed (with a laptop) interviews lasting 30 to 60 minutes and copied pages containing the content used the week before the interview. Our small sample of five teachers with different experience and education serves as a snapshot of how teachers use teacher guides for support and inspiration in teaching. The amount of video material (approximately 3 hours) and the copied pages were manageable to overview. We believe that this amount of data captured the essence of how the participating teachers use their teacher guides. An important aspect is that there was no time limit for the interviews; they continued until the teachers had said everything they wanted to say about their use of the teacher guides.

The analyses performed for Study I were supported by the framework for educative curriculum materials developed by Hemmi et al. (2013), based on the work of Ball and Cohen (1996) and Davis and Krajcik (2005). We used the framework as a lens to draw our attention to what kind of support the teachers talk about. While Categories 1 to 4 were easily detected, Category 5 was harder to capture. In our analysis, we include ready-made teaching activities for direct enactment in the classroom as support for; e.g. games, laboratory setups, pictures to discuss, extra worksheets, etc. However, revisiting my methodology, I found that support for Design of Teaching (Category 5) is complex and difficult to interpret. Ready-made teaching activities do not in themselves constitute support for Design of Teaching (Ball & Cohen, 1996; Brown 2002; 2009; Davis & Krajcik, 2005); however, an additional discussion of the strengths and weaknesses of a certain activity does. So, ready-made teaching activities that are not accomplished through such discussion can be separated into a new category. For the future use of this framework, some adjustments to and clarifications of Category 5 are necessary in order to be able to replicate the studies. Despite the uncertainty regarding the essence of Category 5, the findings hold true. Teachers with different personal resources still desire different support, and the common demand for connections to the new Swedish national curriculum also holds true.

Data for Study II consist of the most commonly used textbooks and teacher guides in Swedish lower secondary school, Grades 7 to 9. Gathering data in three steps, I scrutinized all content referring to the five learning goals for proportion and proportional reasoning in the framework developed by Shield and Dole (2002; 2008; 2013). Based on the rigor definitions of concepts and ratings used, the study should be possible to replicate. Due to clear definitions and the indicators of each learning goal, I believe that such a replicate study, using the same framework, would reveal the same results.

5.3 Further Research

From Study I, we learned that there are outer driving forces that unite teachers in their need for support, regardless of their differences in experience, abilities, and goals and beliefs. In this case, this driving force was the reform 2011, accompanied by a new Swedish national curriculum. Due to teachers' different needs, the design of teacher guides is a delicate mission for authors. Insights into catch-all desires for support can provide valuable information to consider in the design process of guides that appeal to a plethora of different teachers. A larger study with the aim of identifying the conditions under which teachers share their need for support could be important in developing the structure of teacher guides.

In Study II, the textbooks are investigated as tools for teaching. This research can be regarded as the first level of analysis, to provide a picture of how proportional reasoning is addressed in Swedish textbooks for Grades 7 to 9. The next level will be to investigate how the teacher guides, produced together with the textbooks, support the teaching of proportional reasoning. Teacher guides have received minimal attention in research (Remillard, 2005), but the research that has been conducted has shown us that there is a promising potential in teacher guides to support teachers' teaching (cf. Ball & Cohen, 1996; Davis & Krajcik, 2005; Stein, Remillard, & Smith, 2007; Brown, 2009). Therefore, it is important to conduct further studies on how teacher guides impact mathematics education.

Ongoing research on the teacher guides published together with Textbooks A and B in Study II indicates that both teacher guides serve as guides for how to use the textbooks. In Teacher Guide A the writing is in a narrative form, with the authors describing what is going on in the students' textbook without directly addressing the teacher. Also in Teacher Guide B, the comments describe the authors' intentions with the sections of the chapters. The writing lacks personal pronouns, and provides scripted instructions for how to introduce the procedures presented in the students' textbooks.

My analysis indicates that the teacher guides speak *through* the teacher, in contrast to speaking *to* the teacher. The differences between speaking *through* and *to* are explained by Remillard (2012): "Despite the invisibility of the authors, curriculum resources have a voice that is manifested through the way they communicate with the teacher. Most curriculum resources place primary emphasis on what the teacher should do. I think of this as talking *through* teachers. That is, the authors communicate their intent through the actions they suggest the teacher takes. Few resources speak *to* the teacher by communicating with teachers about the central ideas in the curriculum" (Remillard, 2012). Thus far, my analysis has revealed very little additional information to the teachers (cf. the framework in Study I) compared with the information that is already in the students' textbooks. The results of this study are expected to be complete in the fall.

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