Optimal football strategies: AC Milan versus FC Barcelona

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Abstract

In a recent UEFA Champions League game between AC Milan and FC Barcelona, played in Italy (final score 2-3), the collected match statistics, classified into four offensive and two defensive strategies, were in favour of FC Barcelona (by 13 versus 8 points). The aim of this paper is to examine to what extent the optimal game strategies derived from some deterministic, possibilistic, stochastic and fuzzy LP models would improve the payoff of AC Milan at the cost of FC Barcelona.

Keywords: football game, mixed strategies, fuzzy, stochastic, Nash equilibria

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1. Introduction

The main objective of the teams' managers is to find their optimal strategies to win the match. Thus, it should be appropriate to use game theory to analyze a football match. Moreover, as always with game applications, the access of accurate data to estimate the payoffs of the selected strategies is very difficult. According to Carlton & Perloff (2005), only a few mixed strategy models have been estimated in Industrial Economics. In addition to that, contrary to professional business managers who have a solid managerial, mathematic or economic education, team managers lack the necessary formal knowledge to use the game theoretic methods. Football managers, when they decide their most appropriate tactical move or strategy, rely more on their a-priori beliefs, intuition, attitude towards risk and experience.

In a football game if we exclude fortune and simple mistakes, by players and referees as well, goals scored or conceived are often the results of good offensive and/or bad defensive tactics and strategies. There are a varying number of strategies and tactics. As is well known, tactics are the means to achieve the objectives, while strategy is a set of decisions formulated before the game starts (or during the half-time brake), specifying the tactical moves the team will follow during the match, depending upon various circumstances. For instance, the basic elements of a team's tactics are: which players will play the game, which tasks they will perform, where they will be positioned, and how the team will be formed and reformed in the pitch. Similarly, a team's strategies might be to play a short passing game with a high ball possession, attacking with the ball moving quickly and pressing high up its competitors, while another team's strategy might be to defend with a zonal or a man-to-man system, and using the speed of its fullbacks to attack, or playing long passes and crosses as a counter attacking (see http://www.talkfootball.co.uk/guides/football_tactics.html).

Consequently, both managers need somehow to guess correctly how the opponents will play in order to be successful. All decisions made by humans are vulnerable to any cognitive biases and are not perfect when they try to make true predictions.

Not only the number of tactics and strategies in a football match is large, their measures are very hard indeed. How can one define and measure correctly "counter attacks", "high pressure", "attacking game", "long passes", "runs" etc? The existing data on match-statistics cover relatively "easy" variables, like: "ball possession", "shots on target", "fouls committed", "corners", "offside" and "yellow" or "red cards", (see for instance UEFA's official site http://www.uefa.com/uefachampionsleague/season=2012/statistics/index.html).

If one wants to measure the appropriate teams' strategies or tactics, one has to collect such measures, which is obviously an extremely time-consuming task, especially if a "statistically" large sample of matches, where the same teams are involved, is required. In this case-study, I have collected detailed statistics from just one match, a UEFA Champions League group match, between AC Milan (*ACM*) and FC Barcelona (*FCB*), held in Milan on November 23, 2011, where *FCB* defeated *ACM* by 3-2. Despite the fact that both teams were practically qualified before the game, the game had more a prestigious character and would determine to a large extent, which team would be the winner of the group. Given the fact that I have concentrated on six strategies per team, four offensives, and two defensive, and that *FCB* wins over *ACM* in more strategy pairs, the aim of this paper is indeed to examine to what extent the optimal game strategies derived from some deterministic, possibilistic and fuzzy LP models would improve the payoff of *ACM*.

Obviously, there are two shortages with the use of such match statistics. First, we can't blame the teams or their managers for not using their optimal pure or mixed strategies, if the payoffs from the selected strategies were not known in advance, but were observed when the game was being played. Second, it is unfair to blame the manager of *ACM* (the looser), if his players did not follow the correct strategies suggested by him. It is also unfair to give credits to the manager of *FCB* (the winner), if his players did not follow the (possibly) incorrect strategies suggested by him. Thus, we modify the purpose and try to find out the optimal strategies, assuming that the managers anticipated the payoffs and the players did what they have been asked to do.

On the other hand, the merits of this case study are to treat a football match not as a trivial zero-sum game, but as a non-constant sum game, or a bi-matrix game, with many strategies. It is not the goal scored itself that is analyzed, but merely under which mixed offensive and defensive strategies the teams (and especially *ACM*) could have done better and collected more payoffs. As is known, in such games, it is rather difficult to find a solution that is simultaneously optimal for both teams, unless one assumes that both teams will have Nash beliefs about each other. Given the uncertainty in measures of some or all selected strategies, possibilistic and fuzzy formulations are also presented.

The structure of the paper consists of five sections: In section 2 we discuss the selected strategies and how we measured them. In section 3, using the payoffs from section 2, we formulate the following models: (i) classical optimization; (ii) maximum of minimum payoffs; (iii) LP with complementary constraints; (iv) Nash; (v) Chance Constrained LP; (vi) Possibilistic LP; (vii) Fuzzy LP. In section 4 we present and comment on the results from all models and section 5 concludes the paper.

2. Selected strategies and Data

FCB and *ACM* are two worldwide teams who play a very attractive football. They use almost similar team formations, the 4-3-3 system (four defenders, three midfielders, and three attackers). All football fans know that *FCB*'s standard strategy is to play an excellent passing game, with highball possession, and quick movements when it attacks. According to official match statistics, *FCB* had 60% ball possession, even if a large part of the ball was kept away from *ACM*'s defensive area. All managers who face *FCB* expect that to happen, and knowing that *FCB* has the world's best player, Messi, they must decide in advance some defensive tactics to neutralize him.

Since the official match statistics are not appropriate for our selected strategies[1], I recorded the game and played it back several times in order to measure all interesting pairs of payoffs. Both teams are assumed to play the following six strategies: *(i) shots on goal, (ii) counter-attacks, (iii) attacking passes, (iv) dribbles, (v) tackling* and *(vi) zone marking.* The first four reflect offensive strategies and the last two defensive strategies. Most of these variables are hard to observe (and measure). It is assumed that the payoffs from all these strategies are equally worth. One can of course put different weights.

(i) shots on goal (SG)

Teams with many *SG*, are expected to score more goals. In a previous study (Papahristodoulou, 2008), based on 814 UEFA CL matches, it was estimated that teams need, on average, about 4 *SG* to score a goal.

In this paper all *SG* count, irrespectively if they saved by the goalkeeper or the defenders, as long as they are directed towards the target, and irrespectively of the distance, the power of the shot and the angle they were kicked[2]. *SG* from fouls, corners, and head-nicks are also included.

According to the official match statistics, *FCB* had 6 *SG* and 3 corners. According to my own definition, *FCB* had 14 *SG*. The defenders of *ACM* blocked 13 of them (including the 4 savings by the goal-keeper). Xavi turned one of the shots into goal. On the other hand, the other two goals scored do not count as *SG*, because the first was by penalty (Messi) and the other by own goal (van Bommel). Similarly, according to the official match statistics, *ACM* had 3 *SG* and 4 corners, while in my measures *ACM* had 13 *SG*. *FCB* blocked 11 of them (including a good saving by its goalkeeper), and two of them turned into goals (by Ibrahimovic and Boateng).

(ii) counter-attacks (CA)

The idea with *CA* is to benefit from the other team's desperation to score, despite its offensive game. The defendant team is withdrawn into its own half but keep a man or two further up the pitch. If many opponent players attack and loose the ball, they will be out of position and the defendant team has more space to deliver a

[1] Since a game theoretic terminology is applied, we use the term "strategy" in the entire paper, even if we refer to tactics.
[2] Pollard and Reep (1997) estimated that the scoring probability is 24% higher for every yard nearer goal and the scoring probability doubles when a player manages to be over 1 yard from an opponent when shooting the ball.

long-ball for the own strikers, or own players can run relatively free to the competitors' defensive area and probably score. This tactic is rather risky, but it will work if the defendant team has a reliant and solid defense, and excellent runners and/or ball kickers.

In this study *CA* have been defined as those which have started from the own defense area and continued all the way to the other team's penalty area. On the other hand, a slow pace with passes and/or the existence of more defenders than attackers in their correct position do not count.

According to that definition, *FCB* had 15 *CA* and *ACM* 13.

(iii) attacking passes (AP)

The golden rule in football is to "pass and move quickly". There are not many teams which handle to apply it successfully though. *FCB* mainly, and *ACM* to a less extent, are two teams which are known to play an entertaining game with a very large number of successful passes. In a recent paper (Papahristodoulou, 2010) it was estimated that *ACM*, in an average match, could achieve about 500 successful passes and have a ball possession of more than 60%. (For all Italian teams see for instance, http://sport.virgilio.it/calcio/serie-a/statistiche/index.html). Similarly in a previous study (Papahristodoulou, 2008), *FCB* achieved even higher ball possession. Moreover, very often, the players choose the easiest possible pass, and many times one observes defenders passing the ball along the defensive line.

There is a simple logic behind this apparently attractive strategy. By keeping hold of the ball with passes, the opponents get frustrated, try to chase all over the pitch, be tired and disposed and consequently leave open spaces for the opponent quick attackers to score.

Given the fact that the number of passes is very large, compared to the other observations, the payoff game matrix will be extremely unbalanced and both teams would simply play their dominant *AP* strategy. To make the game less trivial, I have used a very restrictive definition of *AP*, assuming the following criteria are fulfilled:

Only successful passes and head-nicks, which start at most approximately 15 meters outside the defendant team's penalty area, count.

The passes and head-nicks should be directed forward to the targeted team player who must be running forward too (i.e. passes to static players are excluded).

Backward passes count as long as they take place within the penalty area only.

Neither long crosses, nor passes from free kicks and corners count.

Consequently, *FCB* had 17 successful *AP* and *ACM* had 13 ones. *ACM* managed to defend successfully 14 times while *FCB* defended successfully every third pass that *ACM* attempted.

(iv) dribbles (D)

Dribbling, i.e. the action to pass the ball around one or more defenders through short skillful taps or kicks, can take place anywhere in the pitch. Moreover, since *D* in this paper is treated as offensive strategy, only the offensive ones are of interest. The action will be measured if it starts no more than 15 meters outside the defendant team's penalty area and the player must move forward. Dribbling counts even if the player turns backward, as long as he remains within the penalty area. If the offensive player manages to dribble more than one player but with different actions subsequently, the number of *D* increases analogically.

According to that definition, each team had 14 *D*.

(v) tackling (T)

A standard defensive strategy is to tackle the opponents in order to stop them from gaining ground towards goal, or stop their *SG*, *AP* and their *D*. Tackling is defined when the defender uses either his left or right leg (but not both legs) to wrest possession from his opponent. Even sliding in on the grass to knock the ball away is treated as *T*. The tackle must always be at the ball, otherwise it may be illegal and often punished by the referee, especially if the player makes contact with his opponent before the ball, or makes unfair contact with the player after playing the ball.

Very often, teams, which use *T* frequently, play a man-to-man marking, i.e. when certain defenders who are responsible to guard a particular opponent are forced into that action, because they are dispossessed or are slower than the opponents are. Man-to-man marking is particularly effective when the team has a sweeper who has a free role and supports his teammates who are dispossessed or having problems with the opponents.

Only *T* at less than approximately 15 meters outside the defendant team's penalty area is counted. Tackling (and head-nicks as well) from free kicks and corners are also counted, because in these cases, the defenders play the man-to-man tactic. On the other hand, *SG*, *CA*, *AP* and *D* stopped by unjust *T* and punished by the referee, does not count.

According to these criteria, *FCB* defenders had 6 successful *T* against *SG*, 8 against *CA*, 6 against *AP* and 8 against *D*. Similarly, *ACM* had, 4, 9, 8 and 7 successful *T* respectively.

(vi) zone marking (ZM)

In *ZM* every defender and the defensive midfielders too, are responsible to cover a particular zone on the pitch to hinder the opponent players from *SG*, *AP*, *D* or *CA* into their area. In a perfect *ZM*, there are two lines of defenders, usually with four players in the first and at least three in the second line, covering roughly the one-half of the pitch. A successful *ZM* requires that every defender fulfills his duties, communicates with his teammates, covers all empty spaces, and synchronizes his movement. In that case, the defensive line can exploit the offside rules and prevent the success of long-balls, *CA*, *AP*, *D* and *SG*. Bad communication from the defenders though can be very decisive, especially if the opponents have very quick attackers who can dribble, pass, and shot equally well.

Since measuring *ZM* is very difficult, the following conditions are applied to simplify that tactic.

The two lines of defenders should be placed at about less than 10 and 20 meters respectively, outside the defendant team's penalty area, i.e. *ZM* near the middle of the pitch does not count. (Normally, *ZM* near the middle of the pitch is observed when the team controls the ball through passes or when it attacks).

To differentiate the *ZM* from the *T*, the own defender(s) should be at least 4-5 meters away from their offensive player(s) when he (they) intercepted the ball.

Despite the fact that offside positions are the result of a good *ZM*, do not count.

Precisely as in *T*, unjust actions by *ZM* do not count.

According to these conditions, *FCB* defenders had 5 successful *ZM* against *SG*, 6 against *CA*, 7 against *AP* and 10 against *D*. Similarly, *ACM* had, 9, 7, 6 and 10 successful *ZM* respectively.

The payoff of the game for all six strategies is depicted in the Table 1 below. Notice that some entries are empty because both teams can't play simultaneously offensive or defensive. When one team attacks (defends) the other team will defend (attack). The first entry refers to *FCB* and the second entry to *ACM*. Consequently, since the payoff from a team's offensive strategy is not equal to the negative payoff from the other team's defensive strategy, the game is a non-zero sum and the payoff matrix is bi-matrix.

There seem to be some doubtful pairs, where the defensive values are higher than the offensive ones, such as (a_4, b_6). How can 8 *D* be defended by 10 *ZM*? Simply, some *D* which counts was defended occasionally by a *ZM* which also counts; the ball is then lost to the offensive player who tried to dribble again, but failed. Consequently, the new *D* attempt does not count while the new *ZM* does.

Table 1: The payoff matrix

			B = AC Milan (ACM)						$\sum FCB$
			Offensive				Defensive		
			b_1	b_2	b_3	b_4	b_5	b_6	
A = FC Barcelona (FCB)	**Offensive**	a_1	0	0	0	0	5, 4	9, 9	14
		a_2	0	0	0	0	8, 9	7, 7	15
		a_3	0	0	0	0	11, 8	6, 6	17
		a_4	0	0	0	0	6, 7	8, 10	14
	Defensive	a_5	6, 6	8, 7	6, 8	8, 5	0	0	28
		a_6	5, 7	6, 6	7, 5	10, 9	0	0	28
$\sum ACM$			13	13	13	14	28	32	

a_1 = *SG*; a_2 = *CA*; a_3 = *AP*; a_4 = *D*; a_5 = *T*; a_6 = *ZM*; b_1 = *SG*; b_2 = *CA*; b_3 = *AP*; b_4 = *D*; b_5 = *T*; b_6 = *ZM*

Notice also that there are no pure dominant strategies. However, despite the fact that there are no pure dominant strategies, *FCB* gets more points than *ACM* from the match. For instance, *FCB* had 17 *AP*, (a_3), in comparison with *ACM*, which had only 13, (b_3). As a whole, *FCB* beats *ACM* in six offensive-defensive pairs by a total of 11 points, is beaten by *ACM* in five pairs, by 8 points, while in five pairs there is a tie. The highest differences in favor of *FCB* are in (a_3, b_5), i.e. when *FCB* plays its *AP* and *ACM* does not succeed with its defensive *T*, and in (a_5, b_4), when *ACM* tries with its *D* but *FCB* defends successfully with its *T*.

3. Models

In this section, I will present four deterministic models, one chance constrained, one possibilistic and one fuzzy LP. Five of them are formulated separately for each team and two simultaneously for both teams.

3.1 Classical Optimization

Let *A* and *B* represent *FCB* and *ACM* respectively, their respective six strategies a_i and b_j, with (0, 1) bounds. Each team maximizes separately the sum of its payoffs times the product of a_i and b_j of the relevant strategy pairs. Consequently, the objective functions given below, are non-linear.

Two models have been formulated: (a) unrestricted, i.e. the sum of all six strategies is equal to unit; (b) restricted, i.e. both offensive and defensive strategies must be played. Consequently, in model (b) the two conditions $\sum_{i=1}^{6} a_i = 1,\ \sum_{j=1}^{6} b_j = 1,$ are modified into the four:

$$a_1 + a_2 + a_3 + a_4 = 1,\ \ a_5 + a_6 = 1,\ \ b_1 + b_2 + b_3 + b_4 = 1,\ \ b_5 + b_6 = 1$$

<u>Model (a)</u>

$$\begin{aligned}
\max\ A = {} & (5a_1 + 8a_2 + 11a_3 + 6a_4)b_5 + \\
& (9a_1 + 7a_2 + 6a_3 + 8a_4)b_6 + \\
& (6a_5 + 5a_6)b_1 + (8a_5 + 6a_6)b_2 + \\
& (6a_5 + 7a_6)b_3 + (8a_5 + 10a_6)b_4 \\
s.t.\ & \\
& \sum_{i=1}^{6} a_i = 1,\ \sum_{j=1}^{6} b_j = 1, \\
& 0 \le a_i \le 1,\ i = 1,..,6,\ 0 \le b_j \le 1,\ \ j = 1,...,6
\end{aligned}$$

$$\begin{aligned}
\max\ B = {} & (6b_1 + 7b_2 + 8b_3 + 5b_4)a_5 + \\
& (7b_1 + 6b_2 + 5b_3 + 9b_4)a_6 + \\
& (4b_5 + 9b_6)a_1 + (9b_5 + 7b_6)a_2 + \\
& (8b_5 + 6b_6)a_3 + (7b_5 + 10b_6)a_4 \\
s.t.\ & \\
& \sum_{i=1}^{6} a_i = 1,\ \sum_{j=1}^{6} b_j = 1, \\
& 0 \le a_i \le 1,\ \ i = 1,..,6,\ \ 0 \le b_j \le 1,\ \ j = 1,...,6
\end{aligned}$$

Model (b) [3]:

$$a_1 + a_2 + a_3 + a_4 = 1 \qquad b_1 + b_2 + b_3 + b_4 = 1$$
$$a_5 + a_6 = 1 \qquad b_5 + b_6 = 1$$

This formulation ensures that team *A* for instance, will receive its respective payoffs from its offensive strategy a_3, if team *B* will play its b_5 and/or its b_6. In fact, when team *A* or *B* maximize, both strategies b_j and a_i are decided simultaneously. Obviously, without the strategies of the other team, the objective function would be trivial or even erroneous since the highest payoff strategy would not be ensured.

3.2 Max-min

Let v_1 be the minimal value from all four offensive strategies and v_2 is the minimal value from both defensive strategies for *FCB*. Similarly, let z_1 and z_2, be the respective minimal values for *ACM*. Each team maximizes separately the sum of these minimal respective values. Again, the model is non-linear because each one of the offensive (defensive) strategies of one team is multiplied by the defensive (offensive) strategies of the other team.

Model (a)

$$\begin{aligned}
&\max A = v_1 + v_2\\
&s.t.\ (5a_1 + 8a_2 + 11a_3 + 6a_4)b_5 \ge v_1\\
&\quad (9a_1 + 7a_2 + 6a_3 + 8a_4)b_6 \ge v_1\\
&\quad (6a_5 + 5a_6)b_1 \ge v_2\\
&\quad (8a_5 + 6a_6)b_2 \ge v_2\\
&\quad (6a_5 + 7a_6)b_3 \ge v_2\\
&\quad (8a_5 + 10a_6)b_4 \ge v_2\\
&\sum_{i=1}^{6} a_i = 1,\ \sum_{j=1}^{6} b_j = 1,\\
&0 \le a_i \le 1,\ i = 1,..,6,\ 0 \le b_j \le 1,\ j = 1,...,6
\end{aligned}$$

$$\begin{aligned}
&\max B = z_1 + z_2\\
&s.t.\ (6b_1 + 7b_2 + 8b_3 + 5b_4)a_5 \ge z_1\\
&\quad (7b_1 + 6b_2 + 5b_3 + 9b_4)a_6 \ge z_1\\
&\quad (4b_5 + 9b_6)a_1 \ge z_2\\
&\quad (9b_5 + 7b_6)a_2 \ge z_2\\
&\quad (8b_5 + 6b_6)a_3 \ge z_2\\
&\quad (7b_5 + 10b_6)a_4 \ge z_2\\
&\sum_{i=1}^{6} a_i = 1,\ \sum_{j=1}^{6} b_j = 1,\\
&0 \le a_i \le 1,\ i = 1,..,6,\ 0 \le b_j \le 1,\ j = 1,...,6
\end{aligned}$$

3.3 LP formulation with complementary conditions

While the first two models assume that teams optimize separately, we turn now to a simultaneously optimal decisions. Normally, for a bimatrix game with many strategies, it is rather difficult to find a solution that is simultaneously optimal for both teams. We can define an equilibrium stable set of strategies though, i.e. the well-known Nash equilibrium. In the following two sections, I will formulate two models to find the Nash equilibrium.

As is known, the max-min strategy is defined as:

$$\left(a_1^*,..,a_6^*\right) = \arg \max_{(a_1,...,a_6)} \min_{(b_1,...,b_6)} payoff_A\{(a_1,...,a_6),(b_1,...,b_6)\}$$

$$\left(b_1^*,..,b_6^*\right) = \arg \max_{(b_1,...,b_6)} \min_{(a_1,...,a_6)} payoff_B\{(b_1,...,b_6),(a_1,...,a_6)\}$$

A standard model to find a max-min to both teams is to use a simultaneous LP, with complementary conditions. The complementary conditions are to set the product of each one of the six respective slack, times the six

[3] In order to save space, model (b) will be excluded in all subsequent formulations.

respective strategies, equal to zero. According to this formulation, both teams behave symmetrically, since they maximize their own minimal payoffs obtained from their own selected strategies. Compared to the previous models, each team selects now only its own strategies.

Notice also the two extra constraints, which ensure that both teams can't play entirely offensively or defensively[4]. For instance, the upper bound for all offensive strategies is set arbitrarily equal to 1.2 and the lower bound for the defensive strategies is set arbitrarily equal to 0.8.

Model (a)

$$\max A+B = v_1+v_2+z_1+z_2$$
$$
\begin{aligned}
s.t.\ & 5a_1+8a_2+11a_3+6a_4-sla_1=v_1; && 6b_1+7b_2+8b_3+5b_4-slb_1=z_1;\\
& 9a_1+7a_2+6a_3+8a_4-sla_2=v_1; && 7b_1+6b_2+5b_3+9b_4-slb_2=z_1;\\
& 6a_5+5a_6-sla_3=v_2; && 4b_5+9b_6-slb_3=z_2;\\
& 8a_5+6a_6-sla_4=v_2; && 9b_5+7b_6-slb_4=z_2;\\
& 6a_5+7a_6-sla_5=v_2; && 8b_5+6b_6-slb_5=z_2;\\
& 8a_5+10a_6-sla_6=v_2; && 7b_5+10b_6-slb_6=z_2;\\
& a_1\times sla_1=0; && b_1\times slb_1=0;\\
& a_2\times sla_2=0; && b_2\times slb_2=0;\\
& a_3\times sla_3=0; && b_3\times slb_3=0;\\
& a_4\times sla_4=0; && b_4\times slb_4=0;\\
& a_5\times sla_5=0; && b_5\times slb_5=0;\\
& a_6\times sla_6=0; && b_6\times slb_6=0;\\
& 0\le sla_i,\ i=1,..,6, && 0\le slb_j,\ j=1,..,6;
\end{aligned}
$$
$$\sum_{i=1}^{6}a_i=1;\ \sum_{j=1}^{6}b_j=1;\ \sum_{i=1}^{4}a_i+\sum_{j=1}^{4}b_j\le 1.2;\ \sum_{i=5}^{6}a_i+\sum_{j=5}^{6}b_j\ge 0.8;$$
$$0\le a_i\le 1,\ i=1,..,6,\ 0\le b_j\le 1,\ j=1,...,6$$

3.4 Nash strategies

As is known in the Nash equilibrium, each team selects its probability mixture of strategies (or pure strategy) to maximize its payoff, conditional on the other team's selected probability mixture (or pure). The probability mixture of a team is the best response to the other team's probability mixture. Consequently, the $\left[\left(a_1^*,..,a_6^*\right),\left(b_1^*,..,b_6^*\right)\right]$ is a Nash equilibrium if and only if it satisfies the following conditions:

$$payoff_A\left(a_1^*,..,a_6^*\right),\left(b_1^*,..,b_6^*\right)\ge payoff_A\left\{(a_1,...,a_6),(b_1^*,..,b_6^*)\right\}\ \forall(a_1,...,a_6)\in\begin{pmatrix}\sum_{i=1}^{6}a_i=1,\\ 0\le a_i\le 1,\ i=1,..,6\end{pmatrix}$$

$$payoff_B\left(a_1^*,..,a_6^*\right),\left(b_1^*,..,b_6^*\right)\ge payoff_B\left\{(a_1^*,...,a_6^*),(b_1,..,b_6)\right\}\ \forall(b_1,...,b_6)\in\begin{pmatrix}\sum_{j=1}^{6}b_j=1,\\ 0\le b_j\le 1,\ j=1,..,6\end{pmatrix}$$

It is also known that, if min-max and Nash equilibria coincide, the game has a saddle point. Such saddle points are rather frequent in zero-sum games but not in bi-matrix non-zero sum games.

[4] Without these additional constraints, both teams played offensively; *FCB* plays 55.55% *SG* and 44.45% *AP*, while *ACM* plays 57.14% *AP* and 42.86% *D*.

I applied the package by Dickhaut & Kaplan (1993) programmed in Mathematica, to find the Nash equilibria. In model (a) the entire payoff matrix was used. In model (b) I used two sub-matrices; when *FCB* (*ACM*) was playing offensively and *ACM* (*FCB*) defensively.

3.5 Chance-Constrained Programming (CCP)

When teams are uncertain about competitors' actions or about the payoff matrix, games become very complex. According to Carlton & Perloff (2005) much of the current research in game theory is undertaken on games with uncertainty. I move now to some more plausible models and modify the deterministic parameters and constraints.

In CCP the parameters of the constraints are random variables and the constraints are valid with some (minimum) probability.

Let us assume that the deterministic parameters are expected values, independent and normally distributed random variables with the means as previously, and variances[5] given in Table 2. The first entry depicts the variance for *FCB* and the second for *ACM.*

Table 2: The variance of the payoff matrix

			B = AC Milan (ACM)					
			Offensive				Defensive	
			$\sigma^2 b_1$	$\sigma^2 b_2$	$\sigma^2 b_3$	$\sigma^2 b_4$	$\sigma^2 b_5$	$\sigma^2 b_6$
A = FC Barcelona (FCB)	Offensive	$\sigma^2 a_1$	*0*	*0*	*0*	*0*	*9, 10*	*17, 12*
		$\sigma^2 a_2$	*0*	*0*	*0*	*0*	*16, 15*	*15, 13*
		$\sigma^2 a_3$	*0*	*0*	*0*	*0*	*17, 14*	*10, 11*
		$\sigma^2 a_4$	*0*	*0*	*0*	*0*	*13, 13*	*15, 14*
	Defensive	$\sigma^2 a_5$	*10, 9*	*12, 11*	*15, 15*	*11, 12*	*0*	*0*
		$\sigma^2 a_6$	*10, 12*	*11, 10*	*14, 13*	*16, 16*	*0*	*0*

Moreover, in CCP, when we maximize for one team, we assume that the other team's values are deterministic and disregard their variance. We also assume that, Josep Guardiola, the manager of *FCB*, might expect that the probability of the expected value of his team's defensive strategies a_5 and a_6 is at least *90%*, while the probability of all four expected values of offensive strategies, a_1, a_2, a_3 and a_4 is at least *95%*.

The first stochastic constraint is now formulated as:

$$P\{v_1 \le (5a_1 + 8a_2 + 11a_3 + 6a_4)b_5\} \ge 1 - \alpha = F\left(\frac{v_1 - (5a_1 + 8a_2 + 11a_3 + 6a_4)b_5}{\sqrt{9a_1^2 + 16a_2^2 + 17a_3^2 + 13a_4^2}}\right),$$

where, *F* is the cumulative density function of the standard normal distribution. If *F* $(K_{\square})$ is the standard normal value such that $F\ (K_\alpha) = 1 - \alpha$, then the above constraint reduces to: $\left\{\frac{v_1 - (5a_1 + 8a_2 + 11a_3 + 6a_4)b_5}{\sqrt{9a_1^2 + 16a_2^2 + 17a_3^2 + 13a_4^2}}\right\} \ge (K_\alpha)$

Given $\alpha = 0.10$, the constraint is simplified to:

[5] The variances of the payoffs are obviously very subjective and are given just to show the formulation of the model. Moreover, based on my numerous playing back of the match, the variances reflect rather well the uncertainty of the respective payoffs.

$$(5a_1+8a_2+11a_3+6a_4)b_5+1.282\sqrt{9a_1^2+16a_2^2+17a_3^2+13a_4^2}\geq v_1$$

Similarly, given $\alpha = 0.05$, the first defensive constraint is modified to:

$$(6a_5+5a_6)b_1+1.645\sqrt{10a_5^2+10a_6^2}\geq v_2$$

So, the CCP model (a) for *FCB* is:

$$\max A = v_1+v_2$$
$$s.t.\ (5a_1+8a_2+11a_3+6a_4)b_5+1.282\sqrt{9a_1^2+16a_2^2+17a_3^2+13a_4^2}\geq v_1$$
$$(9a_1+7a_2+6a_3+8a_4)b_6+1.282\sqrt{17a_1^2+15a_2^2+10a_3^2+15a_4^2}\geq v_1$$
$$(6a_5+5a_6)b_1+1.645\sqrt{10a_5^2+10a_6^2}\geq v_2$$
$$(8a_5+6a_6)b_2+1.645\sqrt{12a_5^2+11a_6^2}\geq v_2$$
$$(6a_5+7a_6)b_3+1.645\sqrt{15a_5^2+14a_6^2}\geq v_2$$
$$(8a_5+10a_6)b_4+1.645\sqrt{11a_5^2+16a_6^2}\geq v_2$$
$$\sum_{i=1}^{6}a_i=1,\ \sum_{j=1}^{6}b_j=1,$$
$$0\leq a_i\leq 1,\ i=1,..,6,\ 0\leq b_j\leq 1,\ j=1,...,6$$

A similar formulation applies for *ACM*, assuming that its manager Massimiliano Allegri expects that the probability of the expected value of his team's defensive strategies b_5 and b_6 is also at least *90%*, while the probability of all four offensive strategies, b_1, b_2, b_3 and b_4 is at least *95%*. Allegri also treats Barcelona's values as deterministic and therefore the problem is formulated similarly.

3.6 A Possibilistic LP (PLP) model

No matter how well one has defined and measured the six variables, the observed payoffs are still rather ambiguous.

The ambiguity of measured values can be restricted by a symmetric triangular fuzzy number, determined by a center a_i^c, and a spread w_{a_i}, respectively b_j^c, and w_{b_j} which is represented as: $A_i = \left\langle \alpha_i^c, w_{a_i} \right\rangle$, respectively $B_j = \left\langle b_j^c, w_{b_j} \right\rangle$. For instance, the estimate of *CA* for *FCB*, when teams play (a_2, b_5), can be restricted by a fuzzy number $A_{2,5}$ with the following membership function: $\mu_{A_{2,5}}(x) = \max\left(0,\ 1-\frac{|x-8|}{3}\right)$. Thus, the center is 8 (i.e. the initial value), its upper value is 11 and its lower value is 5. Consequently, that fuzzy *CA* variable is expressed as: $A_{2,5} = \langle 8,\ 3\rangle$.

In addition to that, we can use possibility measures in order to measure to what extent it is possible that the possibilistic values, restricted by the possibility distribution $\mu_{A_{i,j}}$, are at least equal to some certain values.

I will follow Inuiguchi & Ramik, (2000) who used possibility and/or necessity measures to de-fuzzify a fuzzy LP.

Given two fuzzy sets, *F* and *Z*, and a possibility distribution μ_F of a possibilistic variable κ, the possibility measure is defined as:

$$\Pi_F(Z) = \sup_r \min(\mu_F(x), \mu_Z(x))$$

If $Z = (-\infty,\ g]$, i.e. Z is a deterministic (non-fuzzy) set of real numbers not larger than g, the possibility index is defined as: $Pos(\kappa \le g) = \Pi_F((-\infty,\ g]) = \sup\{\mu_F(x) | x \le g\}$

If $Z = [g,\ +\infty)$, the respective possibility index is defined as:

$$Pos(\kappa \ge g) = \Pi_F([g,\ +\infty)) = \sup\{\mu_F(x) | r \ge g\}$$

The necessity measures measure to what extent it is certain that the possibilistic values, restricted by the possibility distribution μ_F, are at least or at most some certain values.

The necessity measures and the necessity index are similarly defined as:

$$N_F(Z) = \inf_r \max(1 - \mu_F(x), \mu_Z(x))$$

$$Nes(\kappa \le g) = N_F((-\infty,\ g]) = 1 - \sup\{\mu_F(x) | x \succ g\}$$

$$Nes(\kappa \ge g) = N_F([g,\ +\infty)) = 1 - \sup\{\mu_F(x) | x \prec g\}$$

In my estimates, I assume a spread equal to 3 for the most “fuzzy” measures, *CA*, *D* and *ZM*, equal to 2 for *AP* and equal to 1, for the less “fuzzy” value, *SG*. Thus, I use the following fuzzy sets:

$$A_{1,5} = \langle 5,\ 1\rangle, A_{1,6} = \langle 9,\ 1\rangle, A_{2,5} = \langle 8,\ 3\rangle, A_{2,6} = \langle 7,\ 3\rangle, A_{3,5} = \langle 11,\ 2\rangle, A_{3,6} = \langle 6,\ 2\rangle, A_{4,5} = \langle 6,\ 3\rangle, A_{4,6} = \langle 8,\ 3\rangle$$
$$A_{5,1} = \langle 6,\ 2\rangle, A_{5,2} = \langle 8,\ 2\rangle, A_{5,3} = \langle 6,\ 2\rangle, A_{5,4} = \langle 8,\ 2\rangle, A_{6,1} = \langle 5,\ 3\rangle, A_{6,2} = \langle 6,\ 3\rangle, A_{6,3} = \langle 7,\ 3\rangle, A_{6,4} = \langle 10,\ 3\rangle$$
$$B_{1,5} = \langle 6,\ 1\rangle, B_{1,6} = \langle 7,\ 1\rangle, B_{2,5} = \langle 7,\ 3\rangle, B_{2,6} = \langle 6,\ 3\rangle, B_{3,5} = \langle 8,\ 2\rangle, B_{3,6} = \langle 5,\ 2\rangle, B_{4,5} = \langle 5,\ 3\rangle, B_{4,6} = \langle 9,\ 3\rangle$$
$$B_{5,1} = \langle 4,\ 2\rangle, B_{5,2} = \langle 9,\ 2\rangle, B_{5,3} = \langle 8,\ 2\rangle, B_{5,4} = \langle 7,\ 2\rangle, B_{6,1} = \langle 9,\ 3\rangle, B_{6,2} = \langle 7,\ 3\rangle, B_{6,3} = \langle 6,\ 3\rangle, B_{6,4} = \langle 10,\ 3\rangle$$

I will also make the right-hand side parameters ambiguous and use only possible measures. I assume that the *certainty* degrees of both defensive strategies being at least equal to 0.5, is not less than 60%. Similarly, I assume that the *certainty* degrees of all four offensive strategies being at least equal to 2, is not less than 90%. These bounds apply to both teams and are very moderate compared to the deterministic estimates from the previous models.

Given the symmetric triangular fuzzy values, and the assumptions above, the PLP <u>model (a)</u> for *FCB* is:

$$\begin{aligned}
&\max \quad A = v_1 + v_2 \\
&s.t. \quad (5a_1 + 8a_2 + 11a_3 + 6a_4)b_5 - v_1 + 0.9(a_1 + 3a_2 + 2a_3 + 3a_4)b_5 \ge 2 \\
&\qquad (9a_1 + 7a_2 + 6a_3 + 8a_4)b_6 - v_1 + 0.9(a_1 + 3a_2 + 2a_3 + 3a_4)b_6 \ge 2 \\
&\qquad (6a_5 + 5a_6)b_1 - v_2 + 0.6(2a_5 + 3a_6)b_1 \ge 0.5 \\
&\qquad (8a_5 + 6a_6)b_2 - v_2 + 0.6(2a_5 + 3a_6)b_2 \ge 0.5 \\
&\qquad (6a_5 + 7a_6)b_3 - v_2 + 0.6(2a_5 + 3a_6)b_3 \ge 0.5 \\
&\qquad (8a_5 + 10a_6)b_4 - v_2 + 0.6(2a_5 + 3a_6)b_4 \ge 0.5 \\
&\sum_{i=1}^{6} a_i = 1,\ \sum_{j=1}^{6} b_j = 1,\ 0 \le a_i \le 1,\ i = 1,..,6,\ 0 \le b_j \le 1,\ j = 1,...,6
\end{aligned}$$

A similar formulation applies for *ACM*.

3.7 Van Hop's Fuzzy LP model

Let us finally make both left-and right-hand side parameters fuzzy. Van Hop (2007) formulated a Fuzzy LP model, using superiority and inferiority measures.

Given two fuzzy numbers, $\tilde{F} = (u,a,b),\ \tilde{Z} = (v,c,d)$ where, (u,v) = central values and $(a,b,c,d \in R)$, i.e. the left and right spreads respectively, and if $\tilde{F} \le \tilde{Z}$,

the superiority of $\tilde{Z}$ over $\tilde{F}$ is defined as: $Sup(\tilde{Z},\tilde{F}) = v - u + \dfrac{d-b}{2}$,

and the inferiority of $\tilde{F}$ to $\tilde{Z}$ be defined as: $Inf(\tilde{F},\tilde{Z}) = v - u + \dfrac{a-c}{2}$.

Similarly, given two triangular fuzzy random variables $\tilde{\tilde{A}} \le \tilde{\tilde{B}}$, the superiority of $\tilde{\tilde{B}}$ over $\tilde{\tilde{A}}$ is defined as: $Sup(\tilde{\tilde{B}},\tilde{\tilde{A}}) = v(w) - u(w) + \dfrac{d(w)-b(w)}{2}$, and the inferiority of $\tilde{\tilde{A}}$ to $\tilde{\tilde{B}}$ be defined as: $Inf(\tilde{\tilde{A}},\tilde{\tilde{B}}) = v(w) - u(w) + \dfrac{a(w)-c(w)}{2}$.

Let us now assume the following symmetric triangular type, fuzzy random parameters.

The four offensive fuzzy parameters (for *FCB*) are:

$$\left(\tilde{\tilde{A}}_1,\tilde{\tilde{v}}_1\right) = \begin{cases} (\tilde{A}_{1,w1},\tilde{v}_{1,w1}) = \left[\{\tilde{5},\ \tilde{9}\},\{\tilde{8},\ \tilde{7}\},\{\tilde{11},\ \tilde{6}\},\{\tilde{6},\ \tilde{8}\},\{\tilde{2}\}\right] \\ (\tilde{A}_{1,w2},\tilde{v}_{1,w2}) = \left[\{\tilde{6},\ \tilde{10}\},\{\tilde{10},\ \tilde{9}\},\{\tilde{12},\ \tilde{8}\},\{\tilde{7},\ \tilde{11}\},\{\tilde{2}.2\}\right] \end{cases}, \text{ with } p(w_1) = 0.75,\ p(w_2) = 0.25$$

Notice that the first row is identical to the respective deterministic values (first entries of Table 1) and has a probability of 75%. In order to be consistent with the PLP model previously, we assume that the fuzzy $\{\tilde{2}\}$, is the expected value above the minimum value v_1. The second row consists of the respective "fuzzy" variables and has a lower probability.

Similarly, the two defensive fuzzy parameters (again for *FCB* only) are:

$$\left(\tilde{\tilde{A}}_2,\tilde{\tilde{v}}_2\right) = \begin{cases} (\tilde{A}_{2,w1},\tilde{v}_{2,w1}) = \left[\{\tilde{6},\ \tilde{8},\ \tilde{6},\ \tilde{8}\},\{\tilde{0}.5\}\right] \\ (\tilde{A}_{2,w2},\tilde{v}_{2,w2}) = \left[\{\tilde{7},\ \tilde{8},\ \tilde{8},\ \tilde{9}\},\{\tilde{0}.7\}\right] \\ (\tilde{A}_{2,w1},\tilde{v}_{2,w1}) = \left[\{\tilde{5},\ \tilde{6},\ \tilde{7},\ \tilde{10}\},\{\tilde{0}.5\}\right] \\ (\tilde{A}_{2,w2},\tilde{v}_{2,w2}) = \left[\{\tilde{7},\ \tilde{9},\ \tilde{9},\ \tilde{14}\},\{\tilde{0}.7\}\right] \end{cases}, \text{ with } p(w_1) = 0.75,\ p(w_2) = 0.25$$

Notice that in this matrix, the first and third rows are the respective deterministic values from Table 1, while the second and fourth rows are the true "fuzzy" ones.

In order to be consistent with the symmetric triangular fuzzy values in the PLP model previously, we keep the same spreads. Thus, we have the following fuzzy numbers:

$\mu_{A_1 1,5} = \mu_{A_1 1,6} = \mu_{A_2 1,5} = \mu_{A_2 1,6} = 1,\ \mu_{A_1 3,5} = \mu_{A_1 3,6} = \mu_{A_1 5,1} = \mu_{A_1 5,2} = \mu_{A_1 5,3} = \mu_{A_1 5,4} = \mu_{A_2 3,5} = \mu_{A_2 3,6} = \mu_{A_2 5,1} =$
$\mu_{A_2 5,2} = \mu_{A_2 5,3} = \mu_{A_2 5,4} = 2,\ \mu_{A_1 2,5} = \mu_{A_1 2,6} = \mu_{A_1 4,5} = \mu_{A_1 4,6} = \mu_{A_1 6,1} = \mu_{A_1 6,2} = \mu_{A_1 6,3} = \mu_{A_1 6,4} =$
$\mu_{A_2 2,5} = \mu_{A_2 2,6} = \mu_{A_2 4,5} = \mu_{A_2 4,6} = \mu_{A_2 6,1} = \mu_{A_2 6,2} = \mu_{A_2 6,3} = \mu_{A_2 6,4} = 3$

Finally, based on the fuzzy numbers above, we construct an average fuzzy number for the respective offensive and defensive constraints, such as: *$(a_1, a_2, a_3, a_4) = (b_1, b_2, b_3, b_4) = (1+3+2+3)/4 = 2.25$* and *$(a_5, a_6) = (b_5, b_6) = (2+3)/2 = 2.5$*

Following Van Hop, the corresponding LP model (a) for *FCB* is:

$$\max \quad A = v_1 + v_2 - 0.75\left(\sum_{k=5}^{6} \lambda_{1k}^{\sup} + \sum_{m=1}^{4} \lambda_{2m}^{\sup}\right) - 0.25\left(\sum_{k=5}^{6} \lambda_{1k}^{\inf} + \sum_{m=1}^{4} \lambda_{2m}^{\inf}\right)$$

$$s.t.\ (5a_1 + 8a_2 + 11a_3 + 6a_4)b_5 - v_1 + 2 + \frac{(a_1 + 3a_2 + 2a_3 + 3a_4)b_5 - 2.25}{2} = \lambda_{15}^{\sup}$$

$$(9a_1 + 7a_2 + 6a_3 + 8a_4)b_6 - v_1 + 2 + \frac{(a_1 + 3a_2 + 2a_3 + 3a_4)b_6 - 2.25}{2} = \lambda_{16}^{\sup}$$

$$(6a_1 + 10a_2 + 12a_3 + 7a_4)b_5 - v_1 + 2.2 + \frac{2.25 - (a_1 + 3a_2 + 2a_3 + 3a_4)b_5}{2} = \lambda_{15}^{\inf}$$

$$(10a_1 + 9a_2 + 8a_3 + 11a_4)b_6 - v_1 + 2.2 + \frac{2.25 - (a_1 + 3a_2 + 2a_3 + 3a_4)b_6}{2} = \lambda_{16}^{\inf}$$

$$(6a_5 + 5a_6)b_1 - v_2 + 0.5 + \frac{(2a_5 + 3a_6)b_1 - 2.5}{2} = \lambda_{21}^{\sup}$$

$$(8a_5 + 6a_6)b_2 - v_2 + 0.5 + \frac{(2a_5 + 3a_6)b_2 - 2.5}{2} = \lambda_{22}^{\sup}$$

$$(6a_5 + 7a_6)b_3 - v_2 + 0.5 + \frac{(2a_5 + 3a_6)b_3 - 2.5}{2} = \lambda_{23}^{\sup}$$

$$(8a_5 + 10a_6)b_4 - v_2 + 0.5 + \frac{(2a_5 + 3a_6)b_4 - 2.5}{2} = \lambda_{24}^{\sup}$$

$$(7a_5 + 7a_6)b_1 - v_2 + 0.7 + \frac{2.5 - (2a_5 + 3a_6)b_1}{2} = \lambda_{21}^{\inf}$$

$$(8a_5 + 9a_6)b_2 - v_2 + 0.7 + \frac{2.5 - (2a_5 + 3a_6)b_2}{2} = \lambda_{22}^{\inf}$$

$$(8a_5 + 9a_6)b_3 - v_2 + 0.7 + \frac{2.5 - (2a_5 + 3a_6)b_3}{2} = \lambda_{23}^{\inf}$$

$$(9a_5 + 14a_6)b_4 - v_2 + 0.7 + \frac{2.5 - (2a_5 + 3a_6)b_4}{2} = \lambda_{24}^{\inf}$$

$$\sum_{i=1}^{6} a_i = 1,\quad \sum_{j=1}^{6} b_j = 1,\ 0 \le a_i \le 1,\ i = 1,..,6,\ 0 \le b_j \le 1,\ j = 1,..,6$$

$$\lambda_{1k}^{\sup} = \lambda_{2m}^{\sup} = \lambda_{1k}^{\inf} = \lambda_{2m}^{\inf} \ge 0,\ k = 5, 6,\ m = 1,..,4$$

A similar formulation applies for *ACM*.

4. Results

The unrestricted offensive and defensive strategies, model (a), are presented in Table 3 and the restricted ones, model (b), are presented in Table 4. The maximizing team is in bald and the other team in italics. In LP with complementary constraints and in the Nash, both teams maximize and are in bald.

In classical optimization model (a), both teams play pure strategies, *FCB* receives 10 points and *ACM* 9. It is rather surprising because *FCB* plays defensively and *ACM* plays offensively, no matter which team maximizes. In both cases, *FCB* plays *ZM* and *ACM* plays *D*, (*$a_6 = b_4 = 1$*).

In model (b), the strategies change. When *FCB* maximizes, it plays the pure strategies *AP* and *ZM* ($a_3 = a_6 = 1$) and receives 21, under the condition that *ACM* plays also its pure strategies *T* and *D*, ($b_5 = b_4 = 1$). When *ACM* maximizes, it receives 17, by playing *AP* and *T*, ($b_3 = b_5 = 1$), given that *FCB* plays *T* and *CA*, ($a_5 = a_2 = 1$)[6].

In the maximization of the minimum payoffs, model (a), both teams use mixed offensive strategies when they maximize separately. When *FCB* maximizes, it gets 3.95 points, if it plays offensively (75.45% *AP* and 24.55% *SG*) and *ACM* plays defensively (58.58% *ZM* and 41.42% *T*). Similarly, when *ACM* maximizes, it gets 3.37 points when it also plays offensively (42.71% *AP* and 57.29% *D*) and *FCB* plays defensively (46.28% *ZM* and 53.72% *T*).

In model (b), when *FCB* maximizes, it continues with the same offensive game but it plays 100% *T* as well. Given the fact that *ACM* continues with the same defensive game and almost equally balanced with all the offensive strategies, *FCB* gets 3.95 + 1.71 = 5.66 points. When *ACM* maximizes, it continues with almost the same weights in *AP* and *D*, and also plays almost 97% *ZM* and 3% *T*. Since *FCB* continues with the same mixture in defense, and also with all four offensive ones, with *AP* just above 30%, *ACM* gets 3.37 + 1.92 = 5.29 points.

In the LP with complementary constraints model (a), *FCB* plays mainly defensively (almost 80% *T*) and *ACM* almost offensively (51.87% *SG* and 45.8% *CA*), with two positive slacks ($sla_4 = 1.59$, $sla_6 = 1.59$), giving *ACM* more points than *FCB*!

In model (b), *FCB* mixes two offensive strategies (55.55% *SG* and 44.45% *AP*) and plays also 100% *T*. *ACM* shifts strategies by playing 50% *SG* and 50% *CA*, and also mixing its defensive strategies, with more weights in *ZM*. *FCB* gets more points from its offensive strategies ($v_1 = 7.67$, $v_2 = 6$), while *ACM* gets slightly more points from its defensive strategies ($z_1 = 6.5$, $z_2 = 6.68$). Notice though that in this case there are five positive slacks, $sla_4 = 2$, $sla_6 = 2$, $slb_4 = 1.25$, $slb_5 = 0.25$, $slb_6 = 1.93$.

In Nash, model (a), Mathematica found seven equilibria, with three of them being pure strategies and four mixed ones. The three pure strategies and one of the four mixed ones are identical in model (b) as well. Notice also that the pure strategies Nash equilibrium ($a_6 = b_4 = 1$) is identical with the solution from the classical optimization when *FCB* maximizes and is the only one where *ACM* plays offensively. Apart from the Nash payoff (*9*, *9*), in all other equilibria *FCB* gets more points than *ACM*, with the largest difference (*11*, *8*) when *FCB* plays 100% *AP* and *ACM* defends with 100% *T*. In another Nash equilibrium, (*4.74*, *4.5*), the difference is approximately 5% in favor of *FCB*. That equilibrium is found if *FCB* plays 50% its a_1 and 50% its a_6, while *ACM* plays 52.63% its b_6 and 47.37% its b_4. In that case the product for *FCB* is: $(0.5*0.5263*9) + (0.5*0.4737*10) = 4.74$. Similarly the product for *ACM* is: $(0.5*0.5263*9) + (0.5*0.4737*9) = 4.50$.

In CCP model (a), when *FCB* maximizes, it plays 100% *CA* if *ACM* defends by 45.58% with *T* and 54.42% by *ZM*, giving *FCB* 8.77 points. On the other hand, when *ACM* maximizes, it mixes four strategies, with *D* dominating by 90.38%, given that *FCB* defends by about 2/3 *T* and 1/3 *ZM*, and giving *ACM* 8.53 points, i.e. a rather balanced game.

In model (b), both teams shift strategies. When *FCB* maximizes, it plays 100% *AP* and 100% *ZM*, while *ACM* plays all six strategies with changes in defense weights. When *ACM* maximizes, it shifts to two pure strategies, 100% *CA* and 100% *T*, while *FCB* plays all six strategies too and changes its defense weights. In this model, the offensive strategies give 8.37 points to *FCB* and 7.38 points to *ACM*. On the other hand, both teams get almost the same points (7.36 versus 7.35) from their defensive strategies.

[6] Notice that ACM would receive 17 points too if it accepted the solution in which *FCB* maximizes, i.e. ($a_3 = a_6 = 1$) and ($b_5 = b_4 = 1$).

Table 3: Unrestricted offensive and defensive strategies

Model (a)	Team	v_1 ; v_2	z_1 ; z_2	Offensive strategies				Defensive	
				SG	*CA*	*AP*	*D*	*T*	*ZM*
Classic Opt.	***FCB*** *ACM*	**10**	-				 *1*		**1**
	ACM *FCB*	-	**9**				**1**		 *1*
Max-min of payoffs	***FCB***	**3.95; 0**	- -	**0.2455**		**0.7545**			
	ACM	-	-					*0.4142*	*0.5858*
	ACM	- -	**3.37; 0**			**0.4271**	**0.5729**		
	FCB	-	-					*0.5372*	*0.4628*
LP & Compl. Constr.	***FCB***	**1.56; 4.78**	- -	**0.1128**		**0.0902**		**0.7970**	
	ACM	- -	**6.48; 0.02**	**0.5187**	**0.4580**	**0.0203**		**0.0015**	**0.0015**
Nash	***FCB*** ***ACM***	**10**	 **9**				 **1**		**1**
	FCB ***ACM***	**5.24**	 **4.24**			**0.5294**	 **0.5238**	 **0.4762**	**0.4706**
	FCB ***ACM***	**11**	 **8**			**1**		 **1**	
	FCB ***ACM***	**4.34**	 **3.89**	**0.1622**		**0.4054**	 **0.4340**	 **0.1887**	**0.4324** **0.3773**
	FCB ***ACM***	**7.67**	 **6.86**	**0.2857**		**0.7143**		 **0.3333**	 **0.6666**
	FCB ***ACM***	**4.74**	 **4.5**	**0.5**			 **0.4736**		**0.5** **0.5263**
	FCB ***ACM***	**9**	 **9**	**1**					 **1**
CCP	***FCB***	**8.77; 0**	- -		**1**				
	ACM	-	-					*0.4558*	*0.5442*
	ACM	- -	**7.49; 0.04**		**0.0456**	**0.0427**	**0.9038**	**0.0078**	
	FCB	-	-					*0.6695*	*0.3305*
PLP	***FCB***	**3.05; -0.5**	- -		**0.6287**	**0.2594**	**0.1119**		
	ACM	-	-					*0.4582*	*0.5418*
	ACM	- -	**2.47; -0.63**	**0.0022**	**0.9278**	**0.0640**	**0.0031**		**0.0029**
	FCB	-	-					*0.4620*	*0.5380*
Fuzzy	***FCB***	**0.88; 1.22**	- -					**0.9891**	**0.0109**
	ACM	-	-	*0.2817*	*0.2193*	*0.2808*	*0.2182*		
	ACM	- -	**0.88; 1.56**					**1**	
	FCB	-	-	*0.2199*	*0.2716*	*0.3078*	*0.2007*		

In PLP model (a), the results are rather similar as in CCP. Both teams, when they maximize, mix their offensive strategies, with most weights in *CA*. Both teams mix also their defensive strategies (with almost identical weights) when the other team maximizes. *FCB* gets 3.05 and *ACM* 2.47 points. Notice though the two negative values in the defensive strategies, v_2 = *- 0.5* and z_2 = *- 0.63*, indicating that the certainty degree of defensive strategies being at least equal to 0.5, should not be less than 60%, is violated. For *ACM*, the additional *- 0.13* is explained by the fact that b_6 = *0.0029*. On the other hand, the *certainty* degrees of all four offensive strategies being at least equal to 2, should not be less than 90%, is valid.

Table 4: Restricted offensive and defensive strategies

Model (b)	**Team**	v_1 ; v_2	z_1 ; z_2	**Offensive strategies**				**Defensive strategies**	
				SG	*CA*	*AP*	*D*	*T*	*ZM*
Classic Opt.	***FCB***	**21**	**-**			**1**			**1**
	ACM						*1*	*1*	
	ACM	**-**	**17**			**1**		**1**	
	FCB				*1*			*1*	
Max-min of payoffs	***FCB***	**3.95; 1.71**	**- -**	**0.2455**		**0.7545**		**1**	
	ACM	-	-	*0.2857*	*0.2143*	*0.2857*	*0.2143*	*0.4142*	*0.5858*
	ACM	**- -**	**3.37; 1.92**			**0.4323**	**0.5677**	**0.0314**	**0.9686**
	FCB	-	-	*0.2172*	*0.2720*	*0.3168*	*0.1939*	*0.5372*	*0.4628*
LP & Compl. Constr.	***FCB***	**7.67; 6.00**	**- -**	**0.5555**		**0.4445**		**1**	
	ACM	**- -**	**6.50; 6.68**	**0.50**	**0.50**			**0.4642**	**0.5358**
NASH *FCB*: offensive *ACM*: defensive	***FCB***	**11**				**1**			
	ACM		**8**					**1**	
	FCB	**7.67**		**0.2857**		**0.7142**			
	ACM		**6.86**					**0.3333**	**0.6666**
	FCB	**9**		**1**					
	ACM		**9**						**1**
***ACM*: off BFC: def**	***FCB***	**10**							**1**
	ACM		**9**				**1**		
CCP	***FCB***	**8.37; 7.36**	**- -**			**1**			**1**
	ACM	-	-	*0.4319*	*0.3176*	*0.1723*	*0.0781*	*0.2805*	*0.7195*
	ACM	**- -**	**7.38; 7.35**		**1**			**1**	
	FCB	-	-	*0.5379*	*0.1091*	*0.1498*	*0.2032*	*0.4463*	*0.5537*
PLP	***FCB***	**3.09; 1.61**	**- -**		**1**				**1**
	ACM	-	-	*0.3105*	*0.2707*	*0.2399*	*0.1789*	*0.4755*	*0.5255*
	ACM	**- -**	**2.70; 1.89**		**0.4165**	**0.5835**			**1**
	FCB	-	-	*0.2209*	*0.2711*	*0.3058*	*0.2022*	*0.5505*	*0.4495*
Fuzzy	***FCB***	**5.36; 1.22**	**- -**	**0.9841**	**0.0159**			**0.9891**	**0.0109**
	ACM	-	-	*0.2817*	*0.2193*	*0.2808*	*0.2182*	*0.4705*	*0.5295*
	ACM	**- -**	**4.96; 1.56**		**0.3535**		**0.6465**	**1**	
	FCB	-	-	*0.2199*	*0.2716*	*0.3078*	*0.2007*	*0.5671*	*0.4329*

In PLP, model (b), both certainty degrees are satisfied. Both teams, when they maximize, play 100% *ZM*, *FCB* plays also 100% *CA*, while *ACM* mixes its *CA* with *AP*. When one team maximizes, the other team mixes all six strategies, with roughly similar weights. *FCB* gets 3.09 + 1.61 = 4.7 points, while *ACM* gets 2.70 + 1.89 = 4.59 points, again a rather balanced game.

Finally, in Fuzzy model (a), both teams, play similar strategies when they maximize, 100% *T* for *ACM* and almost 99% for *FCB*, and mix all four offensive strategies, with almost similar weights, when the other team maximizes. They get the same points from their offensive strategies, but *ACM* gets more points than *BFC* from its pure defensive strategy *T*.

In Fuzzy, model (b), while the strategies from model (a) remain unchanged, *FCB* plays 98.4% *SG*, 1.6% *CA* and *ACM* mixes about 1/3 *CA* and 2/3 *D*. Both teams mix also their defensive strategies when the other team maximizes. Again, *ACM* gets 0.34 more points from its *T*, while *FCB* gets 0.40 more points from its offensive strategies, leading to almost balanced game.

Notice that in all fuzzy models all inferior lambdas are positive, varying between 1.52 and 2.9.

In general, the "average" strategies from all models (a) are: *FCB* plays about 57% offensively, i.e. about 23% *AP*, 19% *SG* and 14% *CA.* Its highest weight though is in defense, *ZM* with about 28%. *ACM* plays more offensively, 61.8%, mainly through 41% *D*, and by about 12% *CA. ACM* balances its defensive strategies, by about 21% *ZM* and 17% *T.* The closest to these averages is the Nash equilibrium that gives *FCB* 4.34 points and *ACM* 3.89 points. Similarly, the "average" strategies from all models (b) are: *FCB* plays about 54.6% *AP*, 34.1% *SG* and 11.3% *CA* offensively and 42.7% *T* and 57.3% *ZM* defensively; *ACM* plays 32.4% *CA*, 31.6% *D*, 28.8% *AP* and 7.2% *SG* offensively and 53.7% *T* and 46.3% *ZM* defensively.

Conclusions

A number of deterministic and stochastic models where used to find out the optimal offensive and defensive strategies that *FCB* and *ACM* could have applied during their UEFA CL match, based on the selected match statistics. Since *FCB* won the match, the question posed was if *ACM* could have done better by following better strategies.

Despite the fact that the optimal strategies vary with the selected model, there are indeed four different strategies that *ACM* could have selected according to the following models: (*i*) in fuzzy model (a) *ACM* gets 16% more points than *FCB*, if it plays purely defensive, i.e. 100% *T*; (*ii*) in LP with complementary constraints model (a) *ACM* gets 2.5 % more points, if it plays offensively, i.e. 52% *SG* and 46% *CA*; (*iii*) in one Nash equilibrium where both teams get 9 points, if it plays 100% *ZM* in order to defend the 100% *SG* played by *FCB*; (*iv*) in fuzzy model (b) where *ACM* gets 1% less points than *FCB*, if it plays 100% *T* in defense and also 35.4% *CA* and 64.6% *D* in offense.

On the other hand, *ACM* does it badly with the following strategies: (*i*) in PLP model (a), if it relies mainly to *CA* (by 93%) with almost no defense; (*ii*) in one Nash equilibrium when *FCB* plays 100% *AP* and *ACM* 100% *T*; (*iii*) in another Nash equilibrium when *FCB* plays 53% *AP* and 47% *ZM* while *ACM* plays 52.4% *D* and 47.6% *T*; (*iv*) in classical maximization model (b), if both teams play 100% *AP*, and also *FCB* plays 100% *ZM* while *ACM* plays 100% *T*.

Therefore, the final suggestion to *ACM* is: When both teams maximize simultaneously, play offensively, start with counter-attacks with finish with shots on goal. *FCB* would try with tackling, but not with high success. On the other hand, excessive *CA* if SG does not follow it should be avoided since *FCB* can mix its two defensive strategies successfully. If *FCB* plays mainly its amazing passing game or its SG, defend with ZM instead of T.

I leave it open to the readers, the funs and the managers to conclude if *ACM*'s defeat can be explained because it did not followed the suggested strategies above.

References

Carlton, D.W. & J.M. Perloff: *Modern Industrial Organization*, 4th ed., Pearson Addison Wesley, (2005).

Dickhaut J. & T. Kaplan: A Program for Finding Nash Equilibria, in H.R. Varian, (ed.) *Economic and Financial Modeling with Mathematica*, Telos, Springer-Verlag (1993).

Inuiguchi, M. and J. Ramík: Possibilistic linear programming: a brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problem, *Fuzzy Sets and Systems*, 111 (2000), 3-28.

Luhandjula, M.K.: Optimization under hybrid uncertainty, *Fuzzy Sets and Systems* 146 (2004), 187-203.

Luhandjula, M.K.: Fuzziness and randomness in an optimization framework, *Fuzzy Sets and Systems* 77 (1996), 291-297.

Papahristodoulou, C.: The optimal layout of football players: A case study for AC Milan, Working Paper, Munich Personal RePec Archive, (2010), downloadable at: http://mpra.ub.uni-muenchen.de/20102/1/MPRA_paper_20102.pdf

Papahristodoulou, C.: An Analysis of UEFA Champions League Match Statistics, *International Journal of Applied Sports Sciences* 20 (2008), 67-93.

Pollard, R. and C. Reep: Measuring the effectiveness of playing strategies at soccer, *The Statistician*, 46, (1997), 541-50.

Taha, H.A.: *Operations research: An Introduction*, 8th ed., Pearson Prentice Hall, Upper Saddle River, NJ, (2007).

Van Hop, N.: Solving fuzzy (stochastic) linear programming problems using superiority and inferiority measures, *Information Sciences*, 177 (2007), 1977-91.

Wolfram Research, Mathematica: *Fuzzy Logic* (2003).

http://www.talkfootball.co.uk/guides/football_tactics.html

http://sport.virgilio.it/calcio/serie-a/statistiche/index.html

http://www.uefa.com/uefachampionsleague/season=2012/statistics/index.html