



School of Education, Culture and Communication
Division of Applied Mathematics

BACHELOR THESIS IN MATHEMATICS / APPLIED MATHEMATICS

The Omega Function
A Comparison Between Optimized Portfolios

by

Ali Salih

Kandidatarbete i matematik / tillämpad matematik

DIVISION OF APPLIED MATHEMATICS

MÄLARDALEN UNIVERSITY
SE-721 23 VÄSTERÅS, SWEDEN

Bachelor thesis in mathematics / applied mathematics

Date:

2010-11-24

Project name:

The Omega function: a comparison between optimized portfolios

Author:

Ali Salih

Supervisor(s):

Oskar Schyberg and Robin Lundgren

Examiner:

Anatoliy Malyarenko

Comprising:

15 ECTS credits

Acknowledgements

First of all I would like to thank God for the strength and power that made me complete this work. Very special thanks to my supervisors Robin Lundgren and Oskar Schyberg for their expert knowledge, feedback and their time. Mike Bartholomew-Biggs for the external help and constructive suggestions. Finally I would like to thank my friends and family especially my wife for the moral support.

Abstract

The traditional way to analyze stocks and portfolios within the area of finance have been restricted to Sharpe and Markovitz. The Omega function and its properties enlighten the field of finance and differs from the traditional ways when it comes to the volatility of the stocks.

The Omega function, the Sharpe performance criteria and mean-variance model by Markovitz will be used. All calculations are done in Matlab and the data sheets are excel tables. The aim of this thesis is to investigate the nordic small cap market by using the Omega function, Sharpe performance criteria and the mean variance model by Markovitz. In order to to see how the purposed methods differs.

Contents

1	Introduction	1
2	Ranking assets	2
2.1	The Omega function	2
2.2	Sharpe ratio	6
2.3	Differences within the measures	6
3	Optimizing portfolios	8
3.1	Optimizing Omega	8
3.2	Markovitz model	9
4	Methodology	10
4.1	Framework	11
	4.1.1 Choosing assets	11
	4.1.2 Selecting portfolios	12
4.2	Methods of calculations	12
4.3	Description of the constructed programs	13
	4.3.1 Sharpe/Markovitz	13
	4.3.2 Omega	14
	4.3.3 Transaction Costs	15
5	Analysis	15
6	Result and Conclusion	18
7	References	20

1 Introduction

"Money alone sets all the world in motion"

(100 BC, Publius Syrus, Maxims)

Ever since ancient times mankind have always had the urge to get wealthier. Investments may be a good way of earning money, but there is always a risk involved and as it is impossible to predict the future price of a stock nobody can guarantee how much the payoff will be. Different methods and strategies has been studied in order to find out which of the stocks that are the most optimal according to various criteria's proposed by respected scholars within the field of finance. In this thesis the focus will be on studying the most efficient assets according to Sharpe performance criteria and the omega value, statistically based on the daily increment so that we later on can create two optimized portfolios according to existing models such as Markovitz model and Omega. The Omega portfolio is constructed according to its loss and profit variables and it will be compared to the Markovitz portfolio which is constructed according to its mean and variance which is the well known mean-variance model of Markovitz. The introduced models will be explained in through this thesis.

2 Ranking assets

"Finance is the art of passing money from hand to hand until it finally disappears."

(Robert W. Sarnoff 1918-1997)

There are many ways to rank assets statistically and many methods are calculated with respect to expected value and variance. The Omega function approaches the area of finance with a new modern technique that rank assets according to their volatility, skewness and kurtosis[1]. This differs from traditional methods where the interesting parameters have been variance and expected value.

In this thesis we will concentrate on comparing two portfolios, where one is optimized according to the Omega function and the second portfolio is based on the mean-variance model.

2.1 The Omega function

The Omega function is a performance measurement tool that will measure assets and rank them according to their omega value. When calculating the omega function we start from the cumulative distribution function of the asset returns which we call $F(x)$.

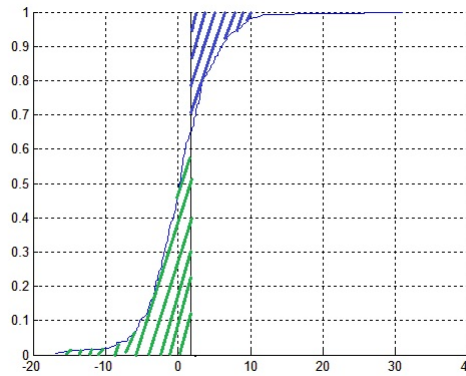


Fig. 1: cumulative distribution function of an asset

Figure 1 shows the cumulative distribution function for the returns of an asset given a threshold r , which is where the function is split into two areas in Figure 1. The threshold an important part of the omega function since it decides the loss and gain proportion of the distribution function. We can see the loss proportion on the green area where the values are less than r and the gain proportion on the blue area where values are greater than r . Loss is defined as any return less than r .

In order to calculate the omega value we simply divide the blue striped area with the green striped area, thus the following formula.

$$\Omega(r) = \frac{\int_r^b (1 - F(x)) dx}{\int_a^r F(x) dx}.$$

The formula given above integrates the distribution function of the asset returns where the given integrals are based on either loss or gain proportion. The loss and gain proportion are decided with respect to the threshold value i.e r . Every value less than r is defined as loss and every value greater than r is defined as gain. The distribution function of the asset returns are limited from a to b where the theoretical value of a is $-\infty$ and ∞ for b . We can also describe a and b pragmatically and define them as the lowest and the highest return of an asset. That will limit r to be a value between a and b .

When the Omega value is calculated we can solve for the omega function, which is calculated by making a function from the obtained omega values when r is moving from $-\infty$ to ∞ [2].

Hence, the following lemma.

Lemma 1. *Let $F(x)$ be the cumulative distribution of an asset, then if we increase r in the omega function the value of omega will decrease monotonically.*

Proof. In order to prove that the Omega function is a decreasing function we will have to study the derivative of Omega with respect to the threshold r .

When the derivative is negative we know that the Omega function is decreasing and in this proof the study will be based on the definition of derivatives, which is shown below.

$$\lim_{h \rightarrow 0} \frac{1}{h} \frac{\int_{r+h}^b (1-F(x))dx}{\int_a^{r+h} F(x)dx} - \frac{\int_r^b (1-F(x))dx}{\int_a^r F(x)dx} =$$

$$\lim_{h \rightarrow 0} \frac{1}{h} \frac{\int_r^b (1-F(x))dx - \int_r^{r+h} (1-F(x))dx}{\int_a^r F(x)dx + \int_r^{r+h} F(x)dx} - \frac{\int_r^b (1-F(x))dx}{\int_a^r F(x)dx}.$$

In order to clarify the calculations we will extend the second fraction with $(1 + \frac{\int_r^{r+h} F(x)dx}{\int_a^r F(x)dx})$.

$$\lim_{h \rightarrow 0} \frac{1}{h} \frac{\int_r^b 1-F(x)dx - \int_r^{r+h} 1-F(x)dx}{\int_a^r F(x)dx + \int_r^{r+h} F(x)dx}$$

$$= \frac{\int_r^b 1-F(x)dx (1 + \frac{\int_r^{r+h} F(x)dx}{\int_a^r F(x)dx})}{\int_a^r F(x)dx (1 + \frac{\int_r^{r+h} F(x)dx}{\int_a^r F(x)dx})}.$$

After some algebraic manipulation we get the same denominator.

$$\lim_{h \rightarrow 0} \frac{1}{h} \frac{\int_r^b (1-F(x))dx - \int_r^{r+h} (1-F(x))dx}{\int_a^{r+h} F(x)dx}$$

$$= \frac{\int_r^b (1-F(x))dx - \frac{\int_r^b (1-F(x))dx * \int_r^{r+h} F(x)dx}{\int_a^r F(x)dx}}{\int_a^{r+h} F(x)dx}$$

$$= \lim_{h \rightarrow 0} \left(\frac{-\frac{1}{h} \int_r^{r+h} (1-f(x)) dx - \frac{\frac{1}{h} \int_r^b (1-f(x)) dx \int_r^{r+h} (f(x)) dx}{\int_a^r (f(x)) dx}}{\int_a^{r+h} (f(x)) dx} \right)$$

We know from the fundamental theorem of calculus that

$\lim_{h \rightarrow 0} \frac{1}{h} \int_r^{r+h} F(x) dx = F(r)$ when h tends to 0. Thus we can simplify calculations.

$$\frac{-(r - F(r)) - \frac{F(r) \int_r^b (1-F(x)) dx}{\int_a^r F(x) dx}}{\int_a^r F(x) dx}$$

where we easily can see that $\frac{\int_r^b (1-F(x)) dx}{\int_a^r F(x) dx}$ is equal to $\Omega(r)$ which is positive, since it is a ratio between to areas.

Finally we can see the proof where the following calculations are less than 0.

$$\frac{F(r) - r - F(r)\Omega(r)}{\int_a^r F(x) dx} < 0$$

Since $F(r)$ is a probability where $a < r < b$, it will always be greater than 0 and less than 1. The Omega value is a ratio between two areas, where the value of the function tends to infinity as r gets closer to a . When r gets closer to b the Omega value will tend to 0 so $\Omega(r) > 0$. Thus we can see a proof of the Omega function being a decreasing function, since the derivative is negative. \square

The Omega function is a decreasing function. An interesting property of the Omega function is that the expected value of the asset return is shown when $\Omega(r) = 1$ [2]. The figure below shows that graphically, thus the second lemma on the next page.

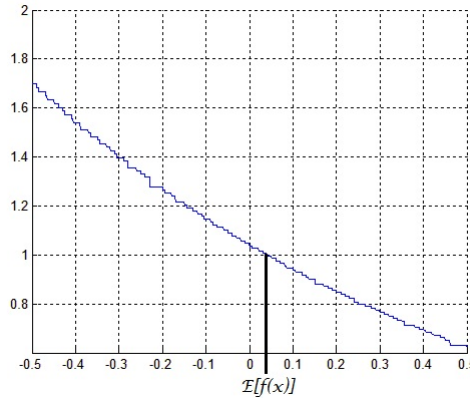


Fig. 2: The omega function

Figure 2 shows the omega function where one can see that when $r = E[f(x)] \Rightarrow \Omega(r) = 1$

Lemma 2. *The Omega value equals 1 iff r is equal to the Expected value*

Proof. Let $s(r) = \int_{-\infty}^r F(x)dx - \int_r^{\infty} 1 - F(x)dx$. In order to prove the lemma we need to show that $s(\mu) = 0$, where μ is the expected value of the asset returns.

Since $F(x)$ is the cumulative distribution function we get that $1 - F(x) = \int_x^{\infty} f(y)dy$, which we can use as a substitute s.t.

$$\begin{aligned} s(r) &= \int_{-\infty}^r \int_{-\infty}^x f(y)dydx - \int_r^{\infty} \int_x^{\infty} f(y)dydx \\ &= \int_{-\infty}^r \int_y^r f(y)dx dy - \int_r^{\infty} \int_r^y f(y)dx dy \\ &= \int_{-\infty}^r \int_y^r f(y)dx dy + \int_r^{\infty} \int_y^r f(y)dx dy \end{aligned}$$

Now we solve one of the integrals in order to simplify and make the double integral single.

$$\begin{aligned} s(r) &= \int_{-\infty}^r (r-y)f(y)dy + \int_r^{\infty} (r-y)f(y)dy \\ &= \int_{-\infty}^{\infty} (r-y)f(y)dy \\ &= \int_{-\infty}^{\infty} rf(y)dy - \int_{-\infty}^{\infty} yf(y)dy \end{aligned}$$

Where we now from basic probability that $\int_{-\infty}^{\infty} yf(y)dy$ is the definition of the mean value, and therefore we can write it as $E[y] = \mu$ and since $\int_{-\infty}^{\infty} f(y)dy = 1$ it will be a lot easier to calculate the last step.

$$s(r) = r - \mu$$

Thus by choosing $r = E[y]$ we will get that $s(\mu) = 0$ and consequently

$$\Omega(\mu) = 1$$

□

We can now state more characteristics of omega since we know that omega of the expected value is equal to one and that the function is a ratio between the gain proportion and the loss proportion. If we get a omega value greater than 1, we can for sure say there is higher probability to gain more than the given threshold. If we get a omega value less than 1 we will know that the probability is higher to have a lower value than the threshold.

After further analysis on properties of the Omega function we can investigate the extreme values. We can see that if we choose an extremely low threshold, then we will get a large omega value which will tend to go up to infinity as we decrease the threshold. If we study omega when we choose an extremely high threshold we will find that the omega value tends to go to zero as the threshold increases. Thus we can state that $\Omega(r)$ will always be greater than 0 and it can tend up to ∞ .

2.2 Sharpe ratio

Sharpe performance criteria ratio is another way to rank assets and probably the most traditional way. By dividing the expected value $E[f(x)]$ of the asset returns minus the risk free interest rate R_f with the standard deviation σ we will obtain the Sharpe value. The higher the Sharpe value the more return per standard deviation, which will indicate a more optimal asset. An interesting part of Sharpe ratio is that it uses the expected value and the standard deviation in order to rank assets which differs from the Omega function. The formula below gives a clear description of the Sharpe criteria ratio[3].

$$Sharpe = \frac{E[f(x)] - R_f}{\sigma}$$

The risk free rate can be equivalent to the interest rate that is offered when saving in banks or it can also be considered as government t-bills where the variance of the returns are zero.

2.3 Differences within the measures

There are many ways to analyze an asset, many analyzing methods uses standard deviation in order to get a risk value. The difference between the Omega function and the Sharpe ratio is that the Omega function does not depend on the standard deviation but considers the skewness and kurtosis instead[4]. Since the variance only show how much the asset will variate and will not consider the profit and the loss return we will get a better picture of an asset when using the omega function. Sharpe criteria ratio also has a weakness when it comes to measuring assets which are not normally distributed, since it does not consider higher moments of a distribution.

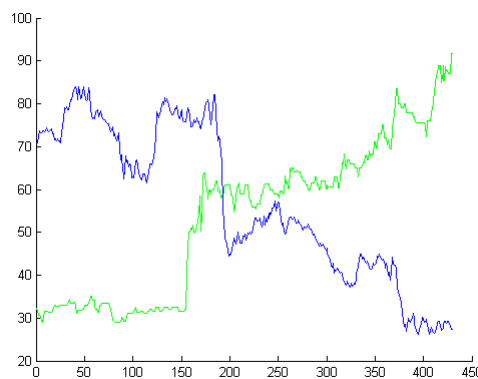


Fig. 3: comparison of two assets

Figure 3 shows how the price of two assets changes, the vertical axis defines the price which is given in SEK and the x axis defines the length of the series. Even though the blue marked

asset has a lower standard deviation on its returns, we can easily see the green lined asset have a higher expected return. Of course it is easy to prove that the standard deviation of the returns itself cannot explain the performance of an asset thereby Figure 4.

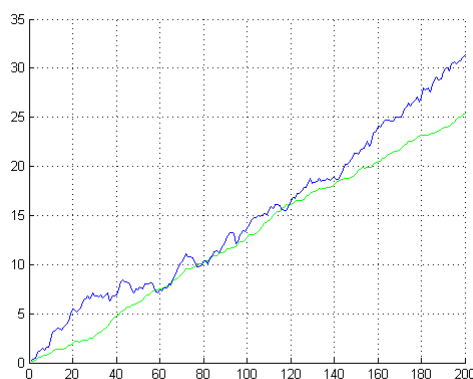


Fig. 4: comparison of two assets

By adding more information we will get closer to the reality. As it was written before, the variance itself will not show the whole picture but when we add the expected value we will have the Sharpe ratio. The graph above shows the price of two different assets. When analyzing the returns from the graph presented above, we will find that the expected value are almost equal for both assets (the blue-lined asset have a slightly higher expected value). The distinguish between the assets is the standard deviation where the blue-lined asset have higher standard deviation while the green have a lower standard deviation. It is easy to notice that the blue-lined asset have higher expected return, but according to Sharpe the green-lined asset is ranked higher that is because of a higher standard deviation which have a greater impact than the expected value on the Sharpe ratio.

When analyzing the same assets with the Omega function we can easily see a difference when Omega chooses the blue-lined asset as presented in the graph below.

Figure 5 shows the same assets presented in Figure 4 but implemented in the Omega function. We can see that the blue-lined asset it chosen when the threshold is set to be positive, that is since it has a greater profit proportion on the returns than the green lined asset. In this case we find the omega function more apprehensive since it is better to consider the profit and loss distribution. Which the omega function does, thus it will give the area of finance a more modern and wider approach.

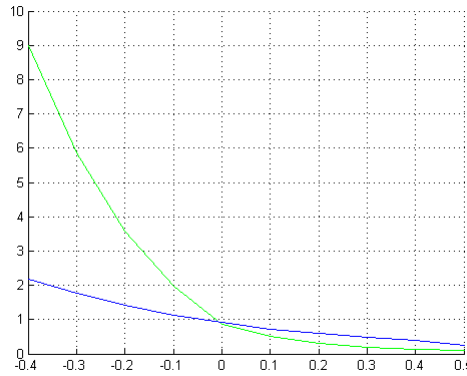


Fig. 5: The Omega function of the same assets

3 Optimizing portfolios

In order to decrease the risk and increase the expected return we will have to optimize portfolios and a good way is to diversify the portfolio by adding assets from different industries. The constructed programs when optimizing portfolios is a modification of S.J Kane and M.C. Bartholomew Biggs work in Optimizing Omega, since the original program were dealing with generated data. The most interesting part of Biggs & Kanes program is that it uses the built-in Matlab function `fminsearch` (which is the Nelder-Mead simplex method) in order to find optimal weights. The Nelder-Mead simplex method which is a direct search method is constructed to solve multidimensional unconstrained non linear minimization problems, and it gives outputs as local minimization points in the local function. The problem of using `fminsearch` in Matlab is that it will only find the local optimums. Biggs & Kane discussed the problem of finding a global optimum of the function in their paper "Optimizing Omega" and chose the MCS multilevel algorithm of Huyer and Neumaler. The MCS approach is a global minimum searcher and it is more useful to apply when using more data series[5]. Following sections will describe two different methods when optimizing portfolios. In order to create a portfolio we need to decide the weights of each asset in the portfolio, the weights for asset n is defined as w_n and the sum of all weights will be calculated as $w_1 + w_2 + \dots + w_{n-1} + w_n = 1$.

3.1 Optimizing Omega

When optimizing a portfolio according to the Omega function we will decide the weights according to the threshold value of the assets. In Mr Biggs and Kanes article there exists a formulas of portfolio optimizing with the omega measurement. The optimization formula is a general formula with unrestricted number of assets.

We will first start with finding the portfolio returns considering each weight for each asset.

$$R_p = \sum_{i=1}^{n-1} (r_i w_i) + r_n (1 - \sum_{k=1}^{n-1} r_i w_k).$$

Where r_i are the daily returns of asset i , R_p are the daily returns of the portfolio and w_i are the weight of asset i . Since the portfolio returns are defined we can now study the formula below.

$$\frac{1}{\Omega(r_t)} + \rho \sum_{i=1}^{n-1} (\min\{0, w_i\}) + \rho \min\{0, 1 - \sum_{k=1}^{n-1} (w_k)\}$$

In order to get proper weights we will maximize Omega for a certain threshold r_t by adjusting the weights. In order to avoid the problem of short selling a barrier parameter is created called ρ . Whenever a weight becomes negative the penalty function starts and it punishes the right hand side.

3.2 Markovitz model

The American economist Harry M. Markowitz received the noble award of Nobel's Prize in finance, Sweden 1990[6]. Markowitz is well known for his contribution in the area of finance. This thesis will focus on the mean-variance approach which is considered to be a fundamental area within modern portfolio theory where Markowitz contribution has been distinct.

Within the mean-variance model the holder is expected to be a utility maximizer and we will also assume that the holder is risk averse. The model is based on exactly as it sounds the mean value and the variance, which are the main factors when selecting a portfolio according to Markowitz. The formula given below shows how the variance of a portfolio is defined according to Markowitz.

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho_{1,2}$$

The formula above shows that the variance of the portfolio is dependent on the variance, standard deviation, weight and covariance of the assets. The goal is now to minimize portfolio variance[7]. In order to optimize the portfolio by the mean-variance model, we will have to create an efficient frontier where we can find all the optimal weights in the portfolio, see Figure 6. Weights will now be chosen where the maximum return is in the same point as we have the minimum variance. One question that immediately took place is why Markowitz chooses the point where the risk free interest rate touches the efficient frontier, and why not the point where the derivative tends to ∞ . In order to answer that question one have to think logical, and say that if that portfolio where chosen then one could only invest in a risk free interest rate and have a higher expected return.

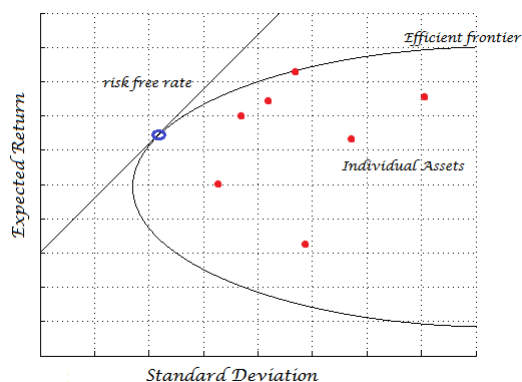


Fig. 6: efficient frontier according to Markovits

The figure above shows the efficient frontier, where we can see the individual assets as red dots and the efficient frontier where we find all the optimal weights of a portfolio on the upper half of the line. The most optimal according to the Markovitz model is the blue circle, where the risk free rate touches the efficient frontier[7].

4 Methodology

In order to optimize portfolios according to mean-variance and omega we will first have to choose assets, the assets will be chosen according to Sharpe criteria ratio and the omega function. It is also important to diversify the portfolio in order to decrease the risk, and that will be considered when choosing assets, where the stress is to choose the best performed asset from each industry according to each measurement, following categories are chosen.

- Consumer Discretionary
- Finance
- Health
- industry
- IT

Both portfolios will be optimized using the modified program of M.C. Bartholomew-Biggs. In the first portfolio assets are chosen according to Sharpe criteria where the risk free interest rate is set to be 4% and they will be optimized according to the efficient frontier. The second portfolio where the assets are chosen according to their omega value when the threshold of the returns is 0.04 will be optimized according to S.J. Kane and M.C. Bartholomew-Biggs paper "Optimizing Omega"[5]. The third portfolio will consist of the worst performed assets will

be optimized according to both Markowitz model and Omega. In order to allocate optimal weights the in-built simplex model `fminsearch` will be used.

4.1 Framework

The collected data is limited to Nasdaq OMX Nordic where the chosen assets are small cap stocks from Scandinavia[8]. The time period of the data is limited between 2005-01-01 and 2007-05-19 i.e 600 trading days, the financial crisis is not considered during the chosen time period. The great collapse within the financial crisis affected the returns with higher volatility and lower expected return, mostly with negative values. A transaction will be presented in this thesis in order to get closer to a real world example. The transaction cost is a percentage and it will be set to 2%. The calculations in this thesis will be done in the profound program of Matlab. Short selling will not be included when optimizing weights in this thesis and in order to avoid such events to happen a penalty function is created which is described in section 3.1.

4.1.1 Choosing assets

25 random assets will be chosen within 5 industries in order to diversify the portfolios. Assets will be formed in 5 groups where each industry represents each group. The chosen groups are based on assets which are defined in previous section. The best performed assets in each branch will be chosen according to each measurement and that will be case for the worst performed assets as well. In order to rank the assets Matlab will be used and the calculations are done with discrete variables instead of functions.

When it comes to choosing assets according to the Omega function we can see that the choice is dependent on the threshold value.

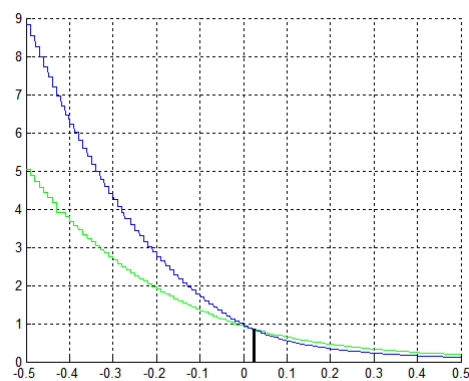


Fig. 7: comparizon between two assets Omega function

Figure(7) gives a clear picture of two assets omega function, where the blue lined asset will be chosen if the threshold is less than approximately 0.02 and if the threshold are chosen

to be greater, then the green lined asset will be chosen. A problem is that when comparing assets according to the omega function, the result depends on the threshold. We have the same problem with Sharpe ratio as well, but since the Sharpe ratio is linear and depends on the risk free interest rate it will be easier to find the inference point, since it will depend on the risk free interest rate.

4.1.2 Selecting portfolios

In order to optimize the portfolio where the assets are chosen according to Sharpe, the mean-variance model will be applied. The portfolio will be constructed when expected return is maximized and variance is minimized on the efficient frontier.

In order to calculate weights within the portfolio the constructed program from "optimizing omega" by M.C. Bartholomew Biggs will be modified and used in this thesis. When we reach the optimization of the portfolio where the assets are chosen according to the omega function, we will use the formula given in "Optimizing Omega", where the weights are decided according to the threshold. The values are continuous when optimizing portfolios. The portfolios are going to be re-balanced once a month so that new weights will be applied within the portfolios. We will also investigate worst case portfolios, where the content of the portfolios are based on the worst performed assets, that portfolio will be denoted by wc portfolio. When it comes to the optimization of the wc portfolio it will be optimized both with Markowitz model and also with Omega. That is to investigate how the optimizations theories work for the cases where the assets have a history of bad performance.

4.2 Methods of calculations

The calculations are done within the profound program of Matlab, variables are calculated with discrete figures when analysing the assets and continuous when optimizing portfolios. When calculating the data series a few problems are confronted. Some data points are missing since trade did not take place on those days. In order to avoid problems like that NaN occurring in Matlab, a small construction is made by the help of using the built-in function `isnan`. That problem is worked out in a way so that every NaN is changed to the previous data point so that the increment will be equal to zero. Another problem that was detected is that the data series of the stocks were not adjusted. That problem was handled manually since the downloaded series was Excel files. Some of the data points in the excel sheets actually contained a value but sometime that value was zero, a for loop was added into the code in order to avoid prices equal to zero.

4.3 Description of the constructed programs

The constructed programs are supposed to optimize portfolios according to Omega and Sharpe/Markovitz. In order to make the program work faster and more effective we have been constructing a few subprograms so that the main program does not load or calculate values over and over again. The following two subsections will describe how the programs are constructed in order to get the proper weights according to each description. The first five rows in the constructed program are the same for both measurements and those are:

1. The program starts by reading the excel file with all the data points. The downloaded excel file of one asset contains a matrix where the columns are defined as: Date, Highest price, Lowest price, Closing price, Average price, Total volume. The rows represent days where each row is based one transaction day.
2. Third column which is the closure price will be used in this thesis.
3. In order to avoid data points consisting of NaN the program will change those values into the previous value.
4. The same problem occurs sometimes when the data points have the value 0 in this case the data point will be equal to the previous data point.
5. The daily increment is now calculated and based on the closure price. The vector will represent the x-axis.

4.3.1 Sharpe/Markovitz

6. The expected return and standard deviation is calculated.
7. Sharpe value will now be calculated by using the formula given in section 2.2.
8. Top assets of each industry will now be chosen according to calculations in 7, in order to create a portfolio.
9. A vector called z is created. The vector z is the weights of the assets in the portfolio where the initial values are equal and the sum is equal to 1.
10. `fminsearch` starts in order to find optimal values of the vector z . The built-in function will loop from 11 to 13 until optimal values are found.
11. In order to avoid short selling a punishment function is created where the returns will be punished if weight becomes negative.
12. The covariance matrix is calculated and multiplied with the weights and if a weight is negative it will now be visible since the result will be affected by a huge number.

13. The barrier parameter multiplied with the logarithm of each weight is now deducted from the value calculated in 12. The smaller the weight the greater the deduction will be. Since the logarithm is negative for every weight smaller than 1 then it will not be a deduction but a addition.

This process will continue until the optimal values are found, i.e the point where we have the smallest standard deviation and the highest expected return.

4.3.2 Omega

6. In order to get the omega value for a certain assets we need to start from the distribution function where we first create a vector which is supposed to represent the y-axis. The vector contains values that are equal and the sum of the vector is 1.
7. In order to calculate the area under the distribution function we simply multiply each incremented value from the y- and x-axis. A vector containing different areas for different data points are now created.
8. After deciding a threshold value the constructed program will search for every value that is smaller or equal to the inputted value by the built-in function `find` and a vector is created. The program will create another vector containing values that are greater or equal to the inputted value. Now the program will take the highest value from the first vector and the lowest value from the second vector and choose the value that is closest to the decided threshold.
9. We will now use the threshold value in order to divide the cumulative distribution function according to the Omega formula. We will use the discrete areas of the cumulative distribution function from 7 and add them together to get the two areas that are of interest to us.
10. The two areas according to the omega formula will now be divided to each other and the omega value as a result. From this point the selection will be made and those assets that have the highest value will be chosen to be optimized in the portfolio.
11. To start the optimization of the portfolio a vector is created with equal weights as initial values.
12. `fminsearch` will now loop until the optimal weights are found.
13. The cumulative distribution function will be called upon and the threshold will be decided according to the highest return where we have the lowest risk.
14. The modified program will now divide the returns from 5 into 200 pieces in order to create continuous areas from the distribution function.

15. The areas is now calculated and a continuous omega value is visible This procedure will run until optimal values for omega portfolio are found, by optimal meaning the highest return with respect to the lowest risk are found, by risk meaning the ratio between the loss and profit area.

4.3.3 Transaction Costs

When buying stocks in real life there is often a transaction cost, the price of that cost depends on where the stocks are bought. The whole idea of transaction costs make the trading more expensive, thus an analysis is made in this thesis to see wether if it is optimal to re-balance in this case or not. Transaction costs are assumed to be percentage in this thesis.

1. The constructed program starts from point 5 (from main description of the program) where we can see the returns of the assets. We will also use the matrix consisting of the re-balanced value for each stock at each time period.
2. The cumulative prices of each stock are calculated i.e the price, where we have an initial value as an input variable. Prices will be multiplied by corresponding weights.
3. From the previous step we can now find the price of each stock at a certain time. We are interested to find the value of stocks for every 30 days since the transaction cost is a percentage.
4. The increments from the matrix consisting of re-balanced weights are considered and each value that are negative, i.e those stocks that are sold will be equal to 0 since there will be no cost for selling stocks.
5. The price of each stock at re-balancing time i.e every 30 days in this thesis will be multiplied by corresponding stocks bought(which is calculated in step 4).
6. Every value in the matrix from step 5 will be added together and it will result as the total cost of transactions.

5 Analysis

The table below shows 25 different assets from 5 different branches, where the assets are randomly chosen. Risk free interest rate is 4% and threshold is 0.04. When analyzing the table consisting of Omega and Sharpe values it is easy to notice that many omega values is less than 1 and many Sharpe values are negative. That means in other words that the probability of loosing is higher than gaining which is by definition inefficient stocks. It is then important to keep in mind that every asset will not be considered in the portfolio and also to realize that

this is an example taken from existing assets.

		omega	sharpe
finance	Holdingselskabet af 1958	0,816	0,0047
	Alm. Brand Pantebreve B	0,3464	-0,0242
	Schaumann Properties	0,6213	-0,0323
	sparbank	0,9078	0,0394
	havsfrun Investments B	0,7724	0,0108
industrials	DanTruck	0,7098	-0,0212
	SCF Technologies	0,5305	-0,1479
	BTS Group B	0,9352	0,0682
	DLH B	1,1562	0,0323
	Eitzen Bulk Shipping	0,6056	-0,0499
consumer discretionary	Århus Elite B	0,7307	-0,0744
	AaB	0,7627	-0,0197
	Andersen & Martini B	0,5066	0,0236
	Brandby IF B	0,3781	-0,003
	netonnet	0,5771	-0,053
Health	Diamyd Medical B	1,0534	0,0312
	ChemoMetec	0,6049	-0,0776
	Biohit Oyj B	0,2078	-0,4714
	probi	0,6872	-0,0397
	Genmab	1,0087	0,0924
IT	Novotek	0,5181	-0,0909
	comendo	0,606	-0,1144
	Acando B	0,7277	-0,0023
	note	0,8632	-0,073
	doro	0,748	-0,0315

Tab. 1: shows a table of Omega and Sharpe values

As written before the risk free interest rate is set to be 4% when calculating the Sharpe value and the threshold is set to 0.04 when calculating the Omega value. The difference between the threshold and the risk free interest rate is that the threshold is actually a return and the interest rate is a percentage. So what the omega function actually does is that it separates every return that is greater than 0.04 and divides it with every return less than 0.04.

From the table we can easily choose the top assets from each industry. We can even see that Omega and Sharpe ranks assets differently which gives us information to realize the difference between the measurements. As we can see Omega chooses Sparbank, DLH B, AaB, Diamyd Medical B and Note to be the five best performed assets in each industry. While Sharpe chooses Sparbank, BTS group B, Anders & Martini B, Genmab, and Acando B to be the most efficient assets in each industry. The two portfolios containing the best performed

assets from each industry according to each measurement will be compared with each other. The third worst case portfolio contains the worst performed assets which in this case are the following: Alm. Brand Pantebreve B, SCF technologies, Århus Elite AB, Biohit Oyj B and Novotek. The wc portfolio will be compared to its self when optimizing with different techniques.

When analyzing the transaction costs and compare them with portfolios that had not been re-balanced we get different indications and that depends on how volatile the stocks are. The following graphs show unbalanced portfolios which are green lined and re-balanced that are blue lined. The graphs does not show the transaction cost.



Fig. 8: Price of two portfolios with same stocks, dependent of Sharpe ratio

From Figure 8 we can actually see that the blue lined portfolio which is the re-balanced portfolio have higher return than the unbalanced portfolio. The difference as seen is about 10kr in 600th day, and with a transaction cost set to 2% we get a total cost of 7 kr which is a quite close to 10kr. If the transaction cost had been 3% then we would gain more money by not re-balancing, since the total transaction cost in that case had been higher than 10 kr.

Another interesting property is the Sharpe ratio on the balanced and unbalanced portfolios, the green-lined (unbalanced) portfolio have a higher Sharpe qouta than the blue-lined. The omega value on the other hand ranks the blue-lined portfolio higher.

When comparing the unbalanced omega portfolio with the balanced we obtain figure 9.

The figure shows the difference between the unbalanced portfolio which is the green-lined and balanced portfolio which is the blue-lined. Both portfolios end up in with almost the same amount on the 600th day. The total transactions costs is about 37 kr, which indicates that it is much better to not re-balance in this case, since we will almost have the same amount at the 600th day.

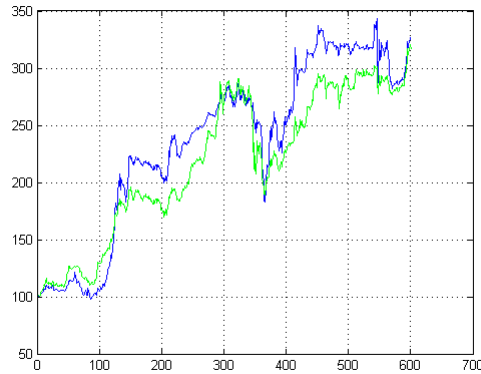


Fig. 9: Price of two portfolios with same stocks, dependent of Omega value

6 Result and Conclusion

After analyzing the table and choosing five of the best performed assets according to each measurement another program will run in order to calculate the optimal weights within the portfolios. The optimal weights are re-balanced within a time period of 30 days so that the complete data series consisting of 600 days are re-balanced 20 times. This is because of the irregularity within the properties of the stocks.



Fig. 10: Comparison between two optimized portfolios

The graph above shows the result where the initial capital is set to be 100 kr. The blue-lined portfolio is based on Omega portfolio optimization which have higher return, but at the same time a higher volatility. The green-lined portfolio have much lower return, and since it is constructed according to the mean-variance model we can clearly see that the volatility is very low compared to the Omega portfolio.

The optimizations of the wc portfolios is as mentioned before done by both techniques and the figure below describes that.

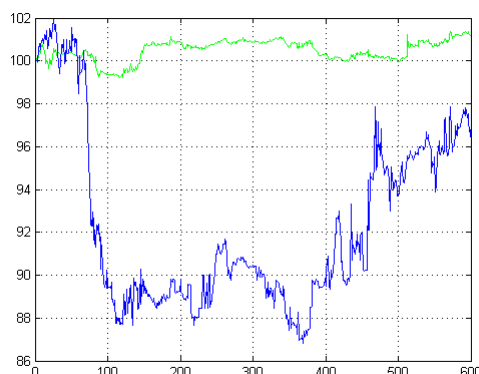


Fig. 11: Comparison of wc portfolio, when using different optimization techniques

The blue lined portfolio is optimized according to Omega while the green lined is optimized according to Markowitz. From the figure above we can clearly see that Markowitz optimize the portfolio in a more efficient way. The difference on the 600th day is close to 5 kr between the portfolios and the total gain will be approximately 1 kr for the Markowitz portfolio. The Omega portfolio results in a loss of about 4 kr on the 600th day.

The results are very low compared the previous figure, and that is because of the poor choice of assets, since we chose the worst performed assets.

We can easily see the differences from the previous picture and conclude that the Markowitz model provides an optimization technique that makes the portfolio less volatile, but on the other hand a lower expected return. Omega does not consider the volatility and therefore the portfolio can be very volatile, but on the other hand it will concentrate on the gain and loss proportions.

In the field of finance there have been some discussions regarding re-balancing the portfolios and the optimal time between re-balancing, since it costs to re-balance your portfolio. This thesis will open up questions concerning re-balancing and transaction costs. We can see through the given examples in this work that sometimes it can be good to re-balance and sometimes not, all dependent on the cost.

In order to conclude this thesis we can say that if we are interested in investing at long term where we want our portfolio to have a small variance as possible it is more interesting to invest according to Markovitz model. For short term investors the Omega function are more efficient to use since it will not consider the volatility but the loss and gain proportion instead and the historical data will be based on a smaller time period.

After completing this thesis I have realized that the Omega function is a powerful tool to use within the area of finance. It has enlightened the field with a modern way of measuring assets. Hopefully this thesis (among others) will be a starting point for other students to illuminate the very existence of such a tool and perhaps to develop and research deeper in the subject.

7 References

- [1]The Omega Measure: Hedge fund portfolio optimization, Alexandre Favre-Bulle and Sebastian Pache, University of Lausanne-Ecole Des Hec.
- [2]An introduction to Omega, Con Keating and William F.Shadwick, The finance development centre, 2002.
- [3]<http://www.investopedia.com/terms/s/sharperatio.asp>
- [4]A Universal Performance Measure, Con Keating and William F.Shadwick, The finance development centre, 2002.
- [5]Optimizing Omega, S.J. Kane and M.C. Bartholomew-Biggs, University of Hertfordshire, Hatfield AL10 9AB, School of Physics Astronomy and Mathematics,2009.
- [6]http://nobelprize.org/nobel_prizes/economics/laureates/1990/markowitz-autobio.html
- [7]http://en.wikipedia.org/wiki/Modern_portfolio_theory
- [8]<http://www.nasdaqomxnordic.com/aktier/historiskakurser/>