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Forecasting the Volatility of an Optimal Portfolio using the GARCH(1,1) Model

by

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Abstract

In this thesis, we have built an optimal portfolio using five assets from the Japanese market. We have investigated the use of GARCH(1,1) when forecasting the volatility of our optimal portfolio. Different time periods have been considered for optimizing our results. An equally-weighted portfolio has been used as a benchmark. Our results show that the optimal portfolio we constructed is more efficient than the equally-weighted portfolio in all chosen situations.

Keywords: Volatility forecasting, Optimal portfolio, GARCH(1,1), Stock market, Investing

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Introduction

1.1 Motivation

In recent years, a lot of studies have been conducted in search of the best model to predict volatility [8] [21] [25]. In our study, we aim to predict the volatility of an optimal portfolio. However, building an optimal portfolio is theoretically possible but not an easy task practically. A crucial characteristic of an optimal portfolio is diversification. The notion of diversification is nothing new to investors. Even from the Shakespearean times, diversification was a known term, and it was indeed used in one of Shakespeare's plays [15]. Notwithstanding the fact that we are aware of diversification, recent studies have helped us grasp the knowledge behind a portfolio in a better way, in an attempt to assist us with portfolio optimization. But this raises an important question: "Is it even possible to build an optimal portfolio?" Our motivation behind the effort to create an optimal portfolio lays upon the fact that more and more people nowadays are investing in the markets. Harry Markowitz in 1952 [16], has given us the foundations of portfolio management. With the help of other books, such as Francis (2013) [11] and Murphy (2008) [19], we have been given improved models upon the subject, that help us construct a portfolio that will give us the lowest risk possible for a given return.

This leads us to another important part of our study, which is risk. Although there are many types of risks, the one that we are focusing on here is market risk, which is measured by volatility. Volatility clustering is an important term to all investors, as it refers to sudden changes in the prices of assets on the market. But apart from volatility clustering, being able to predict volatility can preserve a lot of potential losses or even help investors choose the best time period to invest on the market. An article by Engle (1993) [10] highlighted the importance of forecasting in financial markets as part of asset management.

Of course, in order to forecast volatility, there must be a model that predicts it. In recent studies [8] [21] [25], there have been various volatility models that are used when modeling volatility. These models can be split into three main families [18]:

- GARCH family models,
- Stochastic Volatility (SV) family models and

- Regime-Switching family models.

Apart from these, we can even mention the use of Artificial Intelligence (AI) or Machine Learning (ML) algorithms.

The initial model which was proposed by Engle (1982), is Autoregressive Conditional Heteroskedasticity (ARCH) model. It is a simple model, but the problem is that it requires many parameters [9]. A significant extension of the ARCH model, is the Generalized ARCH (GARCH) model, introduced by Bollerslev (1986). It has the same key properties as the ARCH model, but it requires far less parameters, which makes the process easier [1]. Other extensions of the GARCH model came later into play to take away some unrealistic assumptions the initial models were based on. Our motivation behind using the GARCH model for predicting volatility of a portfolio is simply because GARCH model is one of the most famous models used for volatility. Will the GARCH model bring adequate results, or would it be better to use another model in order to make a better volatility prediction for our study?

1.2 Problem Statement

Different theoretical models have been used to predict the volatility of an optimal portfolio. Our problem therefore arises into two parts:

- The first one is to be able to build an optimal portfolio and
- The second one is to use the best fitting model to predict its volatility.

These problems matter to the investors, as they are the ones who try to maximize their profit and minimize their risk.

As any other model, GARCH model, which is the one used in this thesis, has its limitations. There is a chance that ultimately we will not be able to predict the volatility in the right way, or maybe our results will be insignificant. In either case, investors always feel the pressure and they are the ones facing the consequences in case they make a wrong choice. Our main goal is to construct an optimal portfolio. We will do this using assets from the Japanese market and we are considering a five year period. Finally, we analyse the volatility of our portfolio with the use of GARCH model.

After this introduction, we continue in chapter 2 presenting the theoretical background, with detailed explanations on optimal portfolio, volatility and time series forecasting. In chapter 3, we describe our chosen data. In chapter 4, we forecast volatility using our model. Our analyses and interpretations are presented in chapter 5, and we finish by giving our conclusions in chapter 6.

1.3 Some Important Definitions

For the better understanding of this thesis, there are certain definitions that need to be defined early. First and foremost, we need to understand what a *market* is.

Definition 1.3.1. A market is defined as a place where buyers and sellers interact with each other, for the purpose of exchanging goods or services [19].

A market of course, is not only the physical place that most of us are aware of. The type of market that we are focusing on in this study is an online market, specifically the financial market, where goods are represented by assets. But what exactly is an asset?

Definition 1.3.2. An asset is a resource controlled by the entity as a result of either current or past events and from which future economic benefits are expected to flow to the entity [4].

There are several categories of assets. The two main types are current and non-current assets. Some common examples of current assets (short-term) include cash equivalents and inventory, whereas examples of non-current assets (long-term) include tangible and intangible fixed assets, such as equipment, land, trademarks and copyrights. Another common type of assets is investments assets. These are assets from which the investor earns money through rent or appreciation in value. Examples include rental properties or antiques. In contrast to assets, there exist liabilities, which investors try to avoid, since they have the opposite effect. Assets can become liabilities based on some external factors. A good example is Covid-19, which had a massive effect on asset pricing.

Definition 1.3.3. A liability is a probable future sacrifice of economic benefits arising from present obligations of a particular entity to transfer assets or provide services to other entities in the future as a result of past transactions or events [4].

Liabilities are separated into short-term, long-term and contingent. Short-term liabilities usually have to be paid within a year. Examples include income taxes, bills and short-term loans. Long-term liabilities are due to over a year later. Examples include mortgage, bonds payable and long-term loans. Contingent liabilities are possible liabilities depending on the outcome of a future event. Examples include lawsuits and product warranties.

The interactions of buyers and sellers in the market are the ones which establish the prices of assets. It is essentially the willingness of a buyer to pay a specific price for a good in comparison with the willingness of a seller who demands a specific price for the same good. Of course, if there are more sellers than buyers (higher supply than demand), the prices tend to go down. If there are more buyers than sellers (higher demand than supply), the prices tend to go up [19].

Definition 1.3.4. A portfolio p is defined as a collection of N different assets. Each asset i has a value S_i and the sum of all assets' values is equal to the value of the whole portfolio, S_p.

$$S_p = \sum_{i=1}^{N} S_i.$$

Each asset i in a portfolio has a proportion known as the *weight*, w_i. The weight w_i is calculated as the ratio between the value S_i of the asset and the value of the whole portfolio S_p:

$$w_i = \frac{S_i}{S_p}.$$

As it is understandable, the more weight an asset has, the more its changes affect the value of a portfolio, either positively or negatively [11]. If an investor manages to build a portfolio with maximised returns on its assets and minimising the risk, then the investor will possess an *optimal portfolio*. Characteristics of an optimal portfolio include maximum return of an asset for a given risk as mentioned above, and well defined diversification, which is explained in more detail in its corresponding section.

Chapter 2

Theoretical Background

The theoretical framework is provided in this chapter. The important aspects of an optimal portfolio, as well as volatility and the GARCH(1,1) model are covered, which are useful and relevant for the rest of the thesis.

2.1 Optimal Portfolio

Portfolio theory has been devised by Harry Markowitz in his seminal paper: Portfolio Selection in 1952 [16]. In his paper, he explains the way portfolios are analyzed in order to give the investor the biggest expected return possible for a given risk. For this statistical analysis, there are some important inputs, which are:

- Expected return,
- Standard deviation and
- Correlation.

In order to understand expected return, we first need to show the definitions of a random variable and expected value [11].

Definition 2.1.1. A random variable X is defined as a variable with unknown value, or a function whose values are numerical outcomes of a random phenomenon.

Definition 2.1.2. The expected value $E(X)$ of a discrete random variable X is given by:

$$E(X) = \sum_{s=1}^{N} p_s x_s = p_1 x_1 + p_2 x_2 + \ldots + p_s x_s, \tag{2.1}$$

where p_s is the probability state s will occur and x_s is the outcome when state s occurs. In case of a continuous random variable, the expected value is:

$$E(X) = \int_{\infty}^{\infty} x f(x) dx, \tag{2.2}$$

where $f(x)$ is the probability density function.

Definition 2.1.3. The expected value calculation is used to find the expected return of an asset i, denoted by $E(r_i)$. If this random variable represents the return r_i of an asset or a security, then the expected return is given by:

$$E(r_i) = \sum_{s=1}^{N} p_s r_{i,s}. \tag{2.3}$$

Definition 2.1.4. We define standard deviation σ_X of a random variable X as the dispersion of outcomes around the mean of X [11]. Mathematically, standard deviation is the squared root of the variance, and the variance is computed as:

$$\sigma_X^2 = E[X - E[X]]^2 = \sum_{s=1}^{N} p_s[x_s E(X)]^2, \tag{2.4}$$

or

$$\sigma_i^2 = E[r_i - E[r_i]]^2 = \sum_{s=1}^{N} p_s[r_{is} E(r_i)]^2, \tag{2.5}$$

in case our random variable is the return on an asset i.

Definition 2.1.5. Now, considering two assets i and j, we define the covariance σ_{ij} between the returns as [11]:

$$\sigma_{ij} = E[r_i - E(r_i)][r_j - E(r_j)] = \sum_{s=1}^{N} p_s[r_{is} - E(r_i)][r_{js} - E(r_j)], \tag{2.6}$$

where r_{is} and r_{js} represent the returns on securities i and j when state s occurs.

The covariance can be either positive or negative, meaning that the assets tend to move either to the same or in different directions.

Definition 2.1.6. The correlation ρ_{ij} between two securities i and j is given by:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}, \quad -1 \leq \rho_{ij} \leq 1, \tag{2.7}$$

where σ_{ij} is the covariance between i and j. Also, σ_i and σ_j represent the standard deviations of securities i and j respectively.

In portfolio theory, there are some basic assumptions that we need to take into consideration [11]:

- Investors apprehend every possible opportunity of investment
- The risk of an investment is measured by the standard deviation
- Investors care only about the expected return and risk in an investment.
- All investors prefer the highest expected return possible for a given level of risk.

2.1.1 Financial Risk

Risk is oftentimes related to the danger of loss. This loss can be current, or a fear for a hypothetical future loss. In risk management, risk is connected to the loss of our profit. There are many types of risks, and some of them are presented below [19]:

- **Market Risk** - Market risk is related to asset pricing change. If, for instance, we have 10 shares of a stock worth 50 dollars last night but tonight its worth is down to 48 dollars, we have made a loss of 20 dollars. Of course, one must consider the exchange rate as well, in case the currencies are different between the country one resides and the country that the stock is from.
- **Currency Risk** - Currency risk usually occurs with companies that expand their business internationally. Currency pricing change present risks for losses in case an asset is worth less to another country due to exchange rate.
- **Interest Rate Risk** - Interest rate is an extra amount of money charged on top of the original loan to a borrower by a lender. Interest rate risk included the risk in case of interest rate change. For example, if we have an stable income coming from an asset, the increase of interest rate will decrease our profits.
- **Credit Risk** - An expected positive cashflow in the future that is failed to be met. If someone owns us money that failed to give it to us by the date of our contract, then we have a credit loss.
- **Liquidity Risk** - Liquidity demands arise when one has to pay cash to someone else. When one is unable of fulfilling this task, we call it liquidity risk.
- **Operational Risk** - Direct or indirect loss from a failed process, system failure, a person mistake or other external factors. Examples include fraud, damage to physical assets, or even software/hardware failures.

Expected Return of a Portfolio

Considering a portfolio p of N assets, the expected return of the portfolio $E(r_p)$ is given by [11]:

$$E(r_p) = E\left[\sum_{i=1}^{N} w_i r_i\right] = \sum_{i=1}^{N} w_i E(r_i); \tag{2.8}$$

where w_i is the weight of asset i and $E(r_i)$ is the expected return of asset i.

Risk of a Portfolio

The risk of our N-security portfolio represented by the variance is given by:

$$\begin{aligned} \sigma_p^2 &= E[r_p - E(r_p)]^2 \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \rho_{ij} \sigma_i \sigma_j. \end{aligned} \tag{2.9}$$

2.1.2 Diversification And Utility Analysis

On a two-dimensional space, a lot of pairs of expected returns and risks would form a curved shape that represents the investment opportunities on this space. Inside the shape of the curve, there is the opportunity set, with all the portfolios available. On the edge of this curve, however, there exist optimal portfolios. The curve with the optimal portfolios is called the efficient frontier [11].

In this investment opportunity space, it is now more understandable and easier to distinguish the different kinds of investors. Generally, there are investors who like to take more risky decisions than others. The marginal utility of wealth, describing the change in the happiness of an individual with one change unit of the utility function, is the first derivative of the utility function with respect to wealth. Mathematically, we can write the marginal utility of wealth as

$$U'(W) = \frac{\partial U}{\partial W} > 0,$$

where W is wealth, and U is the marginal utility.

Now, to understand whether the marginal utility is rising or falling, we have to examine the second derivative of the utility function, which can be positive, negative or equal to zero. These values represent a risk-lover, a risk-averse and a risk-neutral investment behaviour respectively. A risk-lover would have a positive utility function

$$\frac{\partial^2 U(W)}{\partial W^2} = U''(W) > 0,$$

whereas a risk-averse would have a negative utility function

$$\frac{\partial^2 U(W)}{\partial W^2} = U''(W) < 0.$$

The reason why this happens, is because with the increase or decrease of the marginal utility, the investor has more or less wealth and therefore can afford to take more risks, or not. In the case that the second derivative of the marginal utility is equal to zero, the investor does not care to whether they should take a risky decision or not.

Markowitz explains the concept of diversification as spreading an investment's assets, in a way that the correlation between the assets is as low as possible. This allows the overall risk of the portfolio to decrease. Moreover, Markowitz suggests that the assets an investor chooses should be from different industries, since then the chance that the correlation between the assets is negative is more probable [16].

2.1.3 Portfolio Optimization

Convex Optimization

A portfolio optimization is often constructed as a minimization function and some constraints relative to the portfolio. These functions are usually convex, and this is why it is important to understand convex optimization.

Definition 2.1.7. A subset C of R^n is a convex set, if for any $\lambda \in [0;1]$ and $x;y \in C$, it holds that $\lambda x + (1-\lambda)y \in C$.

Definition 2.1.8. Let f be a function defined on an interval I of the real line. The function f is called convex if and only if the inequality

$$f(\lambda x + (1-\lambda)y \leq \lambda f(x) + (1-\lambda)f(y)$$

is satisfied for any x and $y \in I$ and for any $\lambda \in [0;1]$.

Now, if we let $f : C \to R$ and $g_k : C \to R; k = 1;\dots;m$ be convex functions, then we can consider the optimization problem [13]

$$\begin{aligned} &\text{minimize} \quad f(x) \\ &\text{subject to} \quad g_k(x) \leq g_{k;0}; k = 1;\dots;m; \end{aligned} \tag{2.10}$$

where $g_{k;0}$ are constants.

Proposition 1. *Equation (2.10) is a convex optimization problem.*

Proof. In order to verify this, we need to show two things. First, that the set of points x satisfying $g_k(x) \leq g_{k;0}$ is a convex set. We can simply do this by taking $\lambda \in [0;1]$ and two points x and y such that $g_k(x) \leq g_{k;0}$ and $g_k(y) \leq g_{k;0}$. Then, we have that

$$g_k(\lambda x + (1-\lambda)y) \leq \lambda g_k(x) + (1-\lambda)g_k(y) \leq g_{k;0};$$

which shows that the set x is convex.

The second thing we have to show is that the intersection between two convex sets $C_1;C_2$ is a convex set. We can say that an empty set is convex, and therefore we can say that the intersection of two convex sets is probably nonempty. If we take $\lambda \in [0;1]$ and $x;y \in C_1 \cap C_2$, and therefore $\lambda x + (1-\lambda)y \in C_k$ for $k = 1;2$, it means that $\lambda x + (1-\lambda)y \in C_1 \cap C_2$. We have proved that (2.10) is a convex optimization problem. □

We are now trying to find the conditions giving the optimal solution to (2.10).

Proposition 2. *Suppose that f and g_k are convex and differentiable, while there exist $x \in C$ and $\lambda \in R^m$ satisfying*

1. $\nabla f(x) + \sum_{k=1}^{m} \lambda_k \nabla g_k(x) = 0$,
2. $g_k(x) \leq g_{k;0} \quad k = 1;\dots;m;$
3. $\lambda_k \geq 0 \quad k = 1;\dots;m;$
4. $\lambda_k(g_k(x) - g_{k;0}) = 0 \quad k = 1;\dots;m;$

then x is the optimal solution to (2.10) [13].

Proof. Let us define L as the Lagrangian. The Lagrangian belongs from C to R by

$$L(x) = f(x) + \sum_{k=1}^{m} \lambda_k(g_k(x) - g_{k;0}).$$

Condition 1 implies that x is a global minimum point of L. Condition 2 implies that x is a feasible solution to (2.10). Condition 3 shows that L is convex and differentiable. Condition 4 shows that $f(x) = L(x)$ and since $L(x) \leq L(y)$, the condition holds true. □

Maximizing Expectation and Minimizing Variance

Considering equation (2.10), we can create an investment problem in a minimization problem as we saw earlier, or a maximization problem. Here, we will focus on the minimization problem, since we are more interested in it, and we can prove its solution with the help of proposition 2. So let us present the variance minimization problem given a lower bound on the expected value [13]

$$\begin{aligned} \text{Minimize} \quad & \frac{1}{2}\sum_{i=1}^{N} w_i^T \sigma_{ij} \sum_{i=1}^{N} w_i \\ \text{subject to} \quad & \sum_{i=1}^{N} w_i r_f + \sum_{i=1}^{N} w_i^T \mu \geq \mu_0 V_0 \\ & \sum_{i=1}^{N} w_i + \sum_{i=1}^{N} w_i^T 1 \leq V_0, \end{aligned} \tag{2.11}$$

where we assume that $\mu \neq r_f 1$, V_0 is the value of the initial investment, and μ_0 represents the expected return.

Proposition 3. *The solution to (2.11) is given by*

$$w = V_0(\mu_0 - r_f)\frac{\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{ij}(\mu - r_f 1)}{(\mu - r_f 1)^T \sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{ij}(\mu - r_f 1)}. \tag{2.12}$$

Proof. From proposition 1, we have the following linear equations

$$\begin{aligned} \sigma_p^2 \sum_{i=1}^{N} w_i - \lambda_1 \mu + \lambda_2 1 &= 0, \\ \lambda_1 r_f + \lambda_2 &= 0, \\ \sum_{i=1}^{N} w_i^T \mu + \sum_{i=1}^{N} w_i r_f &= \mu_0 V_0, \\ \sum_{i=1}^{N} w_i^T 1 + \sum_{i=1}^{N} w_i &= V_0. \end{aligned}$$

Combining the first two equations, while inserting w to the third and fourth equations, and solving for λ_1 yields to [13]

$$\lambda_1 = \frac{\mu_0 - r_f}{(\mu - r_f 1)^T \sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{ij}(\mu - r_f 1)}$$

Thus, if $\mu_p - E(r_p) > 0$ and $\lambda_1 > 0$, then

$$w = V_0(\mu_0 - r_f)\frac{\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{ij}(\mu - r_f 1)}{(\mu - r_f 1)^T \sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{ij}(\mu - r_f 1)}.$$

□

Lagrange Multipliers

If we would like to present the minimization problem in a simpler and more comprehensive way, following the theory from (2.11), but also the equations (2.1) up to (2.9), we have [11]

$$\begin{aligned} \text{Minimize} \quad & \sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \\ \text{subject to} \quad & \sum_{i=1}^{N} w_i E(r_i) = E(r_p), \\ & \sum_{i=1}^{N} w_i = 1. \end{aligned} \tag{2.13}$$

The two constraints above are called *Lagrange constraints*. Combining the last three equations, allows us to form the Lagrange objective function

$$\text{Minimize } L = \frac{1}{2}\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} + \lambda\left[E(r_p) - \sum_{i=1}^{N} w_i E(r_i)\right] + \gamma\left[1 - \sum_{i=1}^{N} w_i\right]. \tag{2.14}$$

The $\frac{1}{2}$ doesn't affect the results, it only simplifies the solution. The letters λ and γ are called *Lagrange multipliers* since they are multiplied by the two Lagrange constraints.

By taking the partial derivatives and setting them equal to zero, we will be able to find the minimum-risk portfolio. The system of equations is presented as follows [11]

$$\begin{aligned} \frac{\partial L}{\partial w_1} &= w_1\sigma_{11} + w_2\sigma_{12} + \dots + w_n\sigma_{1n} - \lambda E(r_1) - \gamma = 0 \\ \frac{\partial L}{\partial w_2} &= w_1\sigma_{21} + w_2\sigma_{22} + \dots + w_n\sigma_{2n} - \lambda E(r_2) - \gamma = 0 \\ &\vdots \\ \frac{\partial L}{\partial w_n} &= w_1\sigma_{n1} + w_2\sigma_{n2} + \dots + w_n\sigma_{nn} - \lambda E(r_n) - \gamma = 0 \\ \frac{\partial L}{\partial \lambda} &= w_1 E(r_1) + w_2 E(r_2) + \dots + w_n E(r_n) - E(r_p) = 0 \\ \frac{\partial L}{\partial \gamma} &= w_1 + w_2 + \dots + w_n - 1 = 0 \end{aligned} \tag{2.15}$$

2.2 Volatility of a Portfolio

2.2.1 Risk and Volatility

Performing forecasting in financial markets is not the simplest of objectives. Analysts all over the world face complications, which can come down to some of the following reasons. Forecasting depends heavily on past data. If this data is old and no longer applicable, there is a chance the predicting results will be wrong. Another reason forecasting is difficult, is because of financial shocks, which are unpredicted big changes in stock prices. A final reason is due to some assumptions and trends, which make the forecasting models less realistic and therefore reduce their chances of producing useful results for the analysts or investors.

The risk of losing money is obviously the biggest worry of all investors and the main concern of financial management. There exist different types of risk in portfolio theory. In our study, volatility is the main focus, and by volatility we mean the market risk which is measured by variance or standard deviation. The general rule is that the higher the standard deviation, the higher the volatility. Notwithstanding the fact that the terms volatility and standard deviation are similar in the field of finance, they are not exactly the same. Their main difference lays upon the concept of time. Standard deviation shows how far away we are from the mean in general, whereas volatility shows how far we are from the mean on a specific period of time.

2.2.2 Understanding Volatility

Even though we cannot know how exactly volatility will change in the future, there are definitely some factors that can make an impact on it and help us understand it better, such as the gross national product (GNP), unemployment and inflation [8]. From that, we understand that high volatility would mean high risk as the prices on the market can fluctuate drastically in a short period of time.

There are a couple of measurements to show volatility. A very important one is beta(b). The factor beta shows the sensitivity on the market from the returns of a security compared to the whole market. But before we explain how b is used, we need to understand what sensitivity is. Sensitivity captures the amount of a security's reaction generated by a movement in a risk factor. As a partial derivative, the sensitivity is the gradient of the security function's reaction over some market factor

$$\frac{DV}{DS};$$

where V is the value of the security's reaction, and S is the market factor's value at a specific point [19]. If this market factor were to change from point S_1 to S_2, sensitivity could be estimated by

$$\frac{DV}{DS}(S_2 \quad S_1):$$

Beta is used in the CAPM theory, which describes the relationship between risk and expected return on the market. We can describe beta(b) for an asset i, with the following equation [11]

$$b_i = \frac{s_{i;M}}{s_M^2};$$

where *M* represents the market.

If beta is less than one, then a specific security is less volatile than the whole market, while if beta is more than one, then the security is more volatile than the market. Another important measurement of volatility is the Volatility Index, also referred to as the Fear Index. The Volatility Index predicts the fluctuations in prices in the S&P500 Index, a market pricing list with the top 500 companies in the U.S., over the next 30 days.

2.2.3 Volatility Modeling

As we understand, being able to predict volatility can be extremely useful when one deals with option pricing or building an optimal portfolio. In such case, one can build a portfolio which will bring profit to the investor. The main problem with volatility however, is that it's not directly observable on the market [3]. It can be obtained through some statistical measurements. It's important to notice though that nothing is certain, as we are talking about probabilities. A typical method of forecasting volatility is through historical data. We can observe the trend of the prices from a previous period of time, and in that way predict its future trend. If for example the historical volatility has been high, we can expect changes in the future as well, whereas if the volatility has been low, we can expect a future volatility with not many changes. This is also known as volatility clustering or positive autocorrelation, as the return series depends on the history of its own data [19]. However, this method is not the most promising for many, as it isn't forward-looking. One of the most common ways to predict volatility is implied volatility (IV) [5]. This allows the investors to see how the volatility will change in the future with a certain probability. Generally, markets that go upwards with low volatility are called bullish, and markets that are down-trending with sudden big volatility changes are called bearish.

Another way to model volatility is through exponentially moving average models. Recent data can affect volatility in a stronger way than older data, and this model allows the recent data to have a stronger impact to volatility, hence improving the results [17]. Apart from the exponentially moving average models, one more way to model volatility is via autoregressive models. Being an extension of random walk, autoregressive models allow the past data to work as the predictors for the future data in a stochastic manner. The difference between the two is that autoregressive models will take into account all past shocks of the given data, in contrast to the moving average models which will only take the last shock into account. From these two models, the ARMA model is formed [8], which is explained in more detail in the next section.

A good volatility model must present certain characteristics that are presented below [23]:

- *Long memory persistence*. Volatility is highly persistent, and as we explained above, volatility clustering is a frequent event, where small changes are followed by other small changes in the prices of assets and the opposite. It is also suggested that the volatility models, because of their persistence, could be significant for the changes in the volatility over the next year. That is to say, changes now could be relevant and useful even for the next year.

- *Mean reversion*. At some point, volatility clustering, either positive or negative, will return to a more normally volatile market. This is referred to as *mean reversion*.

- *Leverage effects*. For many volatility models, there has been an assumption implemented that states that the conditional volatility of an asset is symmetrically affected by either positive or negative changes. The symmetry assumption is not true, and leads to unrealistic results. Since then, there have been many models that take into account other parameters and implementing some innovations, creating an asymmetric impact on volatility. This is known as the *leverage* or *risk premium effect*.

- *Excess kurtosis*. In financial time series, stock returns distribution present fatter tails than normal distribution. This is the fourth moment, or as it is called, *kurtosis*, which for a normal distribution is around 3, but for financial time series it is estimated from 4-50. This is definitely outside the normal distribution range.

- *Volatility co-movements*. Price movements in one market can influence the same movements in another market. This suggests the importance of the incorporation of multivariate models in volatility modelling.

2.3 Time Series Forecasting

2.3.1 Time Series

The prices of assets can either go up or down after being invested. This depends on the *return on an asset*. We have different types of returns such that

$$r_i = \frac{(p_i - p_{i-1}) + d_i}{p_{i-1}};$$

or the logarithmic rate or return

$$r_i = ln(\frac{p_i}{p_{i-1}});$$

where r_i is the return on the asset, p_{i-1} is the price of the asset in the beginning of the holding period, p_i is the price of the asset at the end of the holding period, and d_i is the dividends of the asset which we consider to be zero in our case [11].

In order to understand ARCH and GARCH family models, which are a crucial tool for forecasting volatility, we need to understand the concept of time series analysis. This analysis is based upon important theories, such as stationarity and autoregressive moving average (ARMA) models.

Definition 2.3.1. A time series is defined as a sequence of observations that appear on a successive order and on a specific time period [12].

There exist different types of time series example in real life. For instance, stock prices are an example of a time series. We can see if they generally go upwards or downwards. Another example is weather forecasting. The temperatures create a time series, and it is helpful to predict what the temperature will be in a following period.

2.3.2 Components of a Time Series

Time series are based upon some components. There are four main components of time series analysis. These are the trend, the seasonality, the cyclical component and the irregular (random) component. Each of these are explained below [12]

- *Trend* - The trend is the long-term observation pattern of a time series. In simple words, the trend component shows if the series is generally going upwards or downwards. It can happen that a time series can neither go up or down, then we say that the series is stationary around the mean.
- *Seasonality* - If a time series shows a regular fluctuation pattern for a specific time period over a year, then we call that the seasonal component. An example of seasonal fluctuations are the Christmas Holidays. People usually spend more around that time of the year and therefore sales go up. After the holidays are over, sales go back to normal rates.
- *Cyclical Component* - This cyclical component is formed when there is an up and down fluctuation movement around a trend. These two semicircles that are formed, complete a somewhat full circle. The extend of this circle depends on the circumstances of a given situation.
- *Random Variation* - If a time series is unpredictable and we cannot really know what is going to happen in a following time period, then this is a random component. Every time series has a random variable in it, as we cannot be always certain of the outcomes of the future.

On figure 2.1 we show an example that includes some of the time series components. This particular example includes the Microsoft daily stock prices from 1^{st} of January 2021, until the 1^{st} of January 2022.

The trend is the easiest component to observe. We can see that in this time-scale of one year there is an upward trend, where the prices go from approximately 200 to 325. Moreover, we can also observe a random component. This can be observed around October 2021, where before that we had a steady upward slope and then suddenly there is a fall.

2.3.3 Stationarity

Stationarity is a significant notion of time series. A time series is stationary when its statistical properties remain constant over time. The most important statistical properties that we care about in this study is mean and variance. Stationarity is a key principal for many models and statistical tools examining time series. It's important to notice that there exists both a *strict* and a *weak* stationarity. We show the definitions below [12].

Definition 2.3.2. If the process $X(t)$ has vectors $(X_1, \ldots, X_k)'$ and $(X_{1+h}, \ldots, X_{k+h})'$ with the same joint distribution, then the process is *strictly stationary*.

Definition 2.3.3. The process $X(t)$ is *weakly stationary* if:

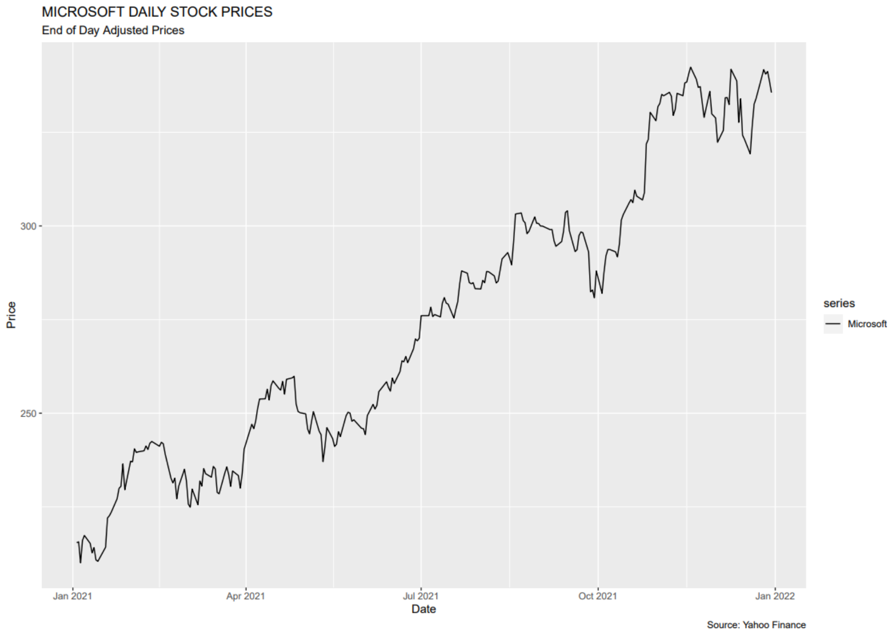


Figure 2.1: Microsoft Daily Stock Prices

- $E(X_t^2) < ¥$
- $E(X_t) = m$
- $Cov(X_t; X_{t+h}) = gc(h)$,

where $gc(\ )$ is the autocorrelation function of $X(t)$.

A simple example of weak stationarity, or *second-order stationarity* as it is also called, is *white noise*. We explain the notion of white noise below [12].

Definition 2.3.4. For a positive constant s^2, the process (e_t) is called *weak white noise* if:

- $E(e_t) = 0$
- $E(e_t^2) = s^2$
- $Cov(e_t; e_{t+h}) = 0$.

However, for the last point of the definition, if we explicitly say that the variables e_t and e_{t+h} are independent and identically distributed, then the process (e_t) is called *strong white noise*.

There are various tests that can be done to check the stationarity of a time series. In this thesis, we care about two tests, which will also be used in chapter 3, where we take a look at our data. For now, we focus on the theoretical aspects of the *Autocorrelation Function (ACF) plots* and the *Dickey-Fuller test*.

Autocorrelation Function (ACF) Test

Autocorrelation shows how similar the values of some variables are with the variables themselves over successive time intervals [20]. Graphically, we can study autocorrelation through a *correlogram*, and numerically through the *Durbin-Watson test*.

Starting with the correlograms, the *lag* is the most important factor on a ACF plot. The lag shows the order of correlation, which means that the numbers in the time series, that are one apart, are correlated. Of course, correlation is between -1 and 1 as explained before, so any correlation values above zero indicate positive correlation, and if they are below zero they indicate negative correlation. Stationary time series will degrade quickly to zero, while non-stationary time series will decrease to zero in a slower rate [20].

Numerically, the Durbin-Watson test is a good way to test for stationarity. It is used to detect if autocorrelation exists in the residuals of a regression analysis. The null hypothesis of the test is that there is no first order correlation, while the alternative hypothesis states that there is. The test statistic is given with the following formula [14]

$$d = \frac{\sum_{t=1}^{n}(e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2},$$

where d is the value of the test, e_t are the residuals from the least squares regression. In other words, e_t is the difference between the observed and predicted value of individual t.

The results of the test can be from 0 to 4. Any value smaller than 2 indicates positive autocorrelation, while from 2 to 4 indicates negative autocorrelation. The value of 0 indicates no autocorrelation [14].

Augmented Dickey-Fuller (ADF) Test

An appropriate way of testing for a unit root is through the augmented Dickey-Fuller test (ADF). A unit root is a feature in stochastic processes that can make time series non-stationary. However, we don't want that, as we are trying to prove stationarity. Dickey and Fuller, in 1979, constructed the ADF model [2]

$$y_t = \phi y_{t-1} + u_t,$$

where y_t is the stochastic variable of the regression line, t is the observation number, and u_t is the disturbance term. Also, $\phi = 1$ for the null hypothesis and $\phi < 1$ for the alternative hypothesis. The null hypothesis states that the time series contains a unit root while the alternative states that it does not.

Dickey-Fuller tests are also known as t-test and can be conducted for an intercept or deterministic trend with the following model

$$y_t = \phi y_{t-1} + \mu + \lambda t + u_t,$$

but since a test of $\phi = 1$ is equal to a test of $\psi = 0$

$$\Delta y_t = \psi y_{t-1} + \mu + \lambda t + u_t.$$

Now, the test statistics for the Dickey-Fuller test are defined as

$$T = \frac{\hat{\psi}}{SE(\hat{\psi})},$$

where we can reject the null hypothesis in favor of the alternative if T is less than the critical value.

Nevertheless, the tests are valid if u_t is assumed to be white noise, and specifically not autocorrelated. There could be autocorrelation in Δy_t however, and for that reason, the true size of the test could be higher than the usual 5% nominal size. We can avoid this problem by using p-lags in the following model [2]

$$\Delta y_t = \psi y_{t-1} + \sum_{i=1}^{p} a_i \Delta y_{t-i} + u_t, \tag{2.16}$$

in which the dynamic structures present on Δy_t are taken away by the lags, therefore making u_t non-autocorrelated.

2.3.4 ARIMA(p,d,q) Model

The ARMA family models are well-known to be used for predictions of stochastic processes. The "AR" part means *autoregressive* meaning that past data play a role on how future data will trend. The AR model has one parameter, p, and it is given by:

$$X_t(p) = \varepsilon_t + \sum_{i=1}^{p} c_i X_{t-i},$$

where ε_t is the linear innovation process of X_t, and c_i is a sequence of coefficients which represent the parameters.

The "MA" part refers to a *moving-average* process given by:

$$X_t = \varepsilon_t + \sum_{i=1}^{\infty} c_i \varepsilon_{t-i},$$

with

$$\varepsilon_t = X_t - E(X_t | H_c(t-1)),$$

where ε_t is a linear innovation process, and $H_c(t-1)$ is the Hilbert space. The Hilbert space allows us to define a space that may not have a definite dimension.

The Moving Average (MA) model has one parameter, namely as q, and the process is given by:

$$X_t(q) = \varepsilon_t + \sum_{i=1}^{q} c_i \varepsilon_{t-i}.$$

Now, the ARMA process has two parameters, p and q and it's given by:

$$X_t + \sum_{i=1}^{p} a_i X_{t-i} = c + \varepsilon_t + \sum_{j=1}^{q} b_j \varepsilon_{t-j}, \tag{2.17}$$

where p and q are integers, c, a and b are real coefficients, and ε_t is the linear innovation of (X_t). For the ARMA process, p is the "AR" order and q is the "MA" order. The difference between ARMA(p,q) and ARIMA(p,d,q) process is the parameter d, which is the order of integration. The ARIMA process is given by a (X_t) process that for $k = 0, \ldots, d-1$, the process $(\Delta^k X_t)$ is not second-order stationary, and the process $(\Delta^d X_t)$ is an ARMA(p,q) process. Random walk is the most simplified version of the ARIMA process, namely as ARIMA(0,1,0), and it is given by [12]:

$$X_t = \varepsilon_t + \varepsilon_{t-1} + \cdots + \varepsilon_1 + X_0, \quad t \geq 1, \tag{2.18}$$

where ε_t is weak white noise.

2.3.5 GARCH(p,q) Model

As explained in the previous section, volatility is represented through variance or standard deviation. The GARCH process is similar to ARCH process, with the only difference that GARCH has a moving average term together with the autoregression component, which allows a more accurate result. The "CH" part stands for conditional heteroskedasticity. This means that if we have a set of random variables, there are some variables within the set that have different variances from the rest. The second moment of GARCH process is given by [12]:

$$\sigma_t^2 = \omega + \alpha(B)\varepsilon_t^2 + \beta(B)\sigma_t^2, \tag{2.19}$$

where α and β are polynomials of degrees q and p respectively, and B is the standard backshift operator. The backshift operator is important when working with forecasting, as it helps with the shift of data to previous periods for the calculation of volatility.

From equation (2.19), if we remove the third term, then we have the $ARCH(q)$ process, which is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2, \tag{2.20}$$

since the backshift operator gives $B^i \varepsilon_t^2 = \varepsilon_{t-i}^2$, and $\alpha(B) = \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$.

Apart from the above $GARCH(p,q)$ process, there is even the strong $GARCH(p,q)$ process, which is the most common and allows a more explicit result. If we denote η for a probability distribution with mean 0 and variance 1, then the process (ε_t) is called strong $GARCH(p,q)$ with respect to η if:

$$\varepsilon_t = \sigma_t \eta_t,$$

and

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{2.21}$$

where w is a positive constant, and a_i, b_j are non-negative constants. The lags are represented by i and j. Coefficient a is the lagged squared return error term and represents how fast the model reacts to changes in the market. Coefficient b is the lagged conditional variance term and represents the persistence of volatility [12].

Exponential GARCH

The following GARCH extensions are not of our interest, but they are provided for any readers that would like to get provided with some extra information upon this topic. As any other model, GARCH models have limitations, and this is why there have been multiple variations of them that can improve them. The main limitation of simple GARCH models is that they consider a symmetric upper and lower change in volatility, which is not realistic. Change in volatility is asymmetric, meaning that the extend of volatility going down, does not necessarily mean that it will be the same extend going upwards when volatility changes again positively [12]. Exponential GARCH (EGARCH) works upon the assumption that negative shocks can have a bigger impact on volatility than positive shocks, therefore creating a realistic asymmetry. If we denote h for a probability distribution with mean 0 and variance 1, then the process (e_t) is called $GARCH(EGARCH(p;q))$ with respect to h if:

$$e_t = s_t h_t;$$

and

$$\log s_t^2 = w + \sum_{i=1}^{q} a_i g(h_{t-i}) + \sum_{j=1}^{p} b_j \log s_{t-j}^2; \tag{2.22}$$

where

$$g(h_{t-i}) = q h_{t-i} + V(|h_{t-i}| - E|h_{t-i}|);$$

and w, a_i, b_j, V and q are real numbers.

Threshold GARCH

The conditional variance can be specified with the positive and negative parts of past innovations. This is a usual way to define asymmetry, specifically by $h_t = h_t^+ + h_t^-$. If we denote h for a probability distribution with mean 0 and variance 1, then the process (e_t) is called threshold $GARCH(p;q)$ with respect to h if:

$$e_t = s_t h_t;$$

and

$$s_t^2 = w + \sum_{i=1}^{q} a_{i;+} e_{t-i}^+ - a_{i;-} e_{t-i}^- + \sum_{j=1}^{p} b_j s_{t-j}; \tag{2.23}$$

where w, $a_{i;+}$, $a_{i;-}$, and b_i are real numbers.

Other Extensions of GARCH

Other important variation of these models include NGARCH, which has an extra parameter that allows correlation between the stocks and the volatility processes, and absolute value GARCH (AGARCH), in which the volatility is linear in the absolute value of returns. This makes a significant difference as the simple GARCH models are non-linear [8].

2.3.6 Forecasting With GARCH(1,1)

In equation (2.21), with $p = q = 1$, we have

$$s_t^2 = w + a_1 e_{t\ 1}^2 + b_1 s_{t\ 1}^2 :$$

Our next period's forecast is a combination of the last period's forecast and the last period's squared return. Through some substitutions, we can obtain the variance forecasting for k steps ahead. However, we first need to find the unconditional variance of e_t [22]

$$\begin{aligned}
Var[e_t] &= E[e_t^2] \quad (E[e_t])^2 \\
&= E[e_t^2] \quad 0 \\
&= E[s_t^2 h_t^2] \\
&= E[s_t^2] \\
&= w + a_1 E[e_{t\ 1}^2] + b_1 s_{t\ 1}^2 \\
&= w + (a_1 + b_1) E[e_{t\ 1}^2] :
\end{aligned}$$

Since e_t is a stationary process, $Var[e_t] = Var[e_{t\ 1}] = E[e_{t\ 1}^2]$ and so

$$Var[e_t] = \frac{w}{1 \quad a_1 \quad b_1} :$$

Furthermore, since $e_t = s_t h_t$

$$E[e_t^2] = E[s_t^2] = \frac{w}{1 \quad a_1 \quad b_1} :$$

Having these properties now in mind, we can forecast for the k-step variance

$$\begin{aligned}
s_t^2 &= w + a_1 e_{t\ 1}^2 + b_1 s_{t\ 1}^2 \\
\hat{s}_{t+1}^2 &= w + a_1 E[e_t^2 j I_{t\ 1}] + b_1 s_t^2 \\
&= w + a_1 s_t^2 + + b_1 s_t^2 \\
&= w + (a_1 + b_1) s_t^2 \\
&= s^2 + (a_1 + b_1)(s_t^2 \quad s^2) \\
&\vdots \\
\hat{s}_{t+k}^2 &= w + (a_1 + b_1) \hat{s}_{t+k\ 1}^2 \\
&= s^2 + (a_1 + b_1)(\hat{s}_{t+k\ 1}^2 \quad s^2) \\
&= s^2 + (a_1 + b_1)^k (s_t^2 \quad s^2);
\end{aligned}$$

where we have substituted the unconditional variance found previously. Also, I_{t-1} is the information given at time $t-1$. From the forecasting equations, we observe that as the steps ahead increase, the variance converges to the unconditional variance of ε_t [22].

Maximum Likelihood Estimation

The most common way to estimate the GARCH family models is through the Maximum Likelihood (ML) method. This method provides the density of the probability distribution as a function of its parameters. To start with, the log-likelihood function of the normal distribution, where the mean function is $\varepsilon_t = \sigma_t \eta_t$, is given by [21]

$$L_T = -\frac{1}{2}\sum_{t=1}^{T}[\ln(2\pi) + \ln(\sigma_t^2) + \eta_t^2]; \tag{2.24}$$

with T representing the number of observations.

To estimate the significance of the statistical tests, sometimes the *t-distribution*, or *Student's t-distribution* as it is also called, is preferable. It is similar to the normal distribution as it has a bell shape, but it differs in the way that it has fatter tails, therefore the chance for extreme values are higher. For Student's t-distribution, the log-likelihood function is given by [21]

$$L_T = \ln\left[\Gamma\left(\frac{\nu+1}{2}\right)\right] - \ln\left[\Gamma\left(\frac{\nu}{2}\right)\right] - \frac{1}{2}\ln[\pi(\nu-2)] - \frac{1}{2}\sum_{t=1}^{T}\left[\ln\sigma_t^2 + (1+\nu)\ln\left(1+\frac{\eta_t^2}{\nu-2}\right)\right]; \tag{2.25}$$

where Γ is the gamma function and ν are the degrees of freedom.

Chapter 3

Data

In this chapter, the data being used for this study is presented. First, information about the Nikkei 225 index is provided, and after that, an analysis of the data is given.

3.1 Nikkei 225

Nikkei 225 is the leading index of Japanese stocks and was established in 1950 in Tokyo, a few years after the second world war. It includes the 225 largest companies in Japan, which are updated every year in the beginning of October. The currency is Japanese Yen, and the trading hours are between 9:00 - 11:30, and 12:30 - 15:00 (GMT+09:00). The ticker of Nikkei 225 can be found in Yahoo Finance as JP225 [6].

The portfolio that we have built consists of five stocks, all from the Japanese market. The portfolio includes both large and small Japanese companies. Some large companies are trusted by many investors as they tend to do well, and then there are small companies that one can choose if one believes that they will keep on growing. Below we present more details about them:

- **Secom (SOMLF)**: Secom is a security services provider.
- **Tokyo Electron (TOELF)**: An electronics and semiconductor company.
- **Nintendo (NTDOF)**: A very famous video-game developer company.
- **Fast Retailing (FRCOF)**: A clothing designer company for both men and women.
- **Fanuc Corporation (FANUF)**: A robotics and computer system company.

The stock prices were collected from *Yahoo Finance*. In our thesis, we are examining two time periods. The chosen in-sample period is between the 1^{st} of January 2017 until the 1^{st} of January 2022. We will call this **period 1**. The other in-sample period is between the 1^{st} of April 2020 until the 1^{st} of January 2022. We will call this **period 2**. The out-of-sample period is the same for both period 1 and 2, and it's divided into two periods again. First, we are examining the first week of January 2022, and then we are examining the first two weeks of January 2022. It is interesting to observe this time-scale as *Covid-19* began at the end of 2019 and created a lot of sudden changes in the stock prices, known as volatility clustering. This is the reason why we also have **period 2**, since it begins when the effects of the pandemic stopped for many companies.

3.2 Descriptive Statistics

In the tables below, we present some daily logarithmic return statistics for our portfolio stocks. The results show that our data are not normally distributed, due to the fact that skewness and kurtosis are not close to zero and three respectively. There will be more normality tests later in this chapter.

Table 3.1: Presentation of the descriptive statistics (Period 1)

Asset List					
	SOMLF	TOELF	NTDOF	FRCOF	FANUF
No. Obs.	1258	1258	1258	1258	1258
Mean	-0.000040	0.001472	0.000642	0.000367	0.000192
Std. Dev.	0.011451	0.022368	0.021480	0.020017	0.023461
Maximum	0.082592	0.161003	0.128385	0.177548	0.192272
Minimum	-0.143109	-0.160881	-0.078875	-0.144489	-0.11203
Skewness	-3.054717	0.368934	0.324321	0.869525	0.170170
Kurtosis	54.675324	11.297885	1.924374	16.4437099	5.155244

Table 3.2: Presentation of the descriptive statistics (Period 2)

Asset List					
	SOMLF	TOELF	NTDOF	FRCOF	FANUF
No. Obs.	442	442	442	442	442
Mean	-0.000126	0.002588	0.000419	0.000844	0.001121
Std. Dev.	0.013175	0.025230	0.019699	0.021101	0.024751
Maximum	0.082592	0.116534	0.073926	0.141564	0.112634
Minimum	-0.12601	-0.084162	-0.061066	-0.144489	-0.097535
Skewness	-1.922362	0.443027	0.101201	0.626424	0.006162
Kurtosis	29.541968	2.907700	0.790761	13.751081	1.332998

3.3 Analysis of the Data

In figures (3.1) and (3.2), it can be clearly seen that there have been sudden changes in the stock prices of many companies during 2020 for period 1, when Covid-19 started having its effect. However, before and after the pandemic period, even though it's not really over yet, a relatively calm volatility market is observed. Of course, it is easier to see this effect on the larger companies, in contrast to the smaller ones.

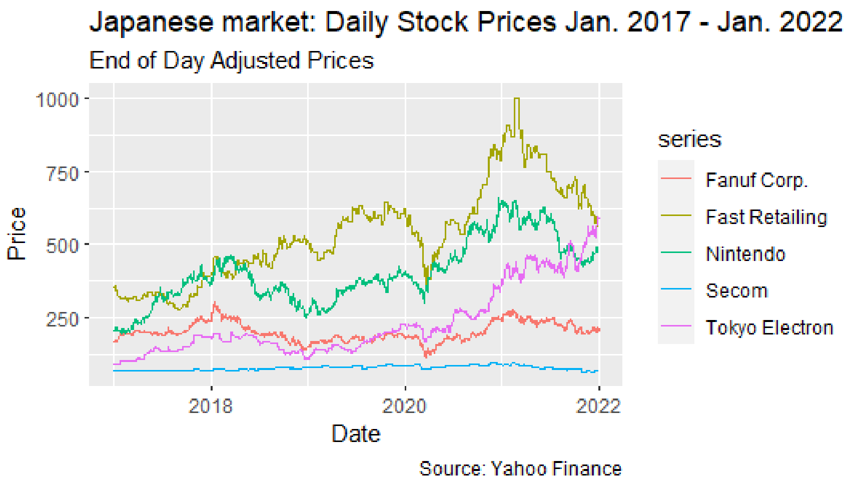


Figure 3.1: Daily portfolio stock prices (Period 1)

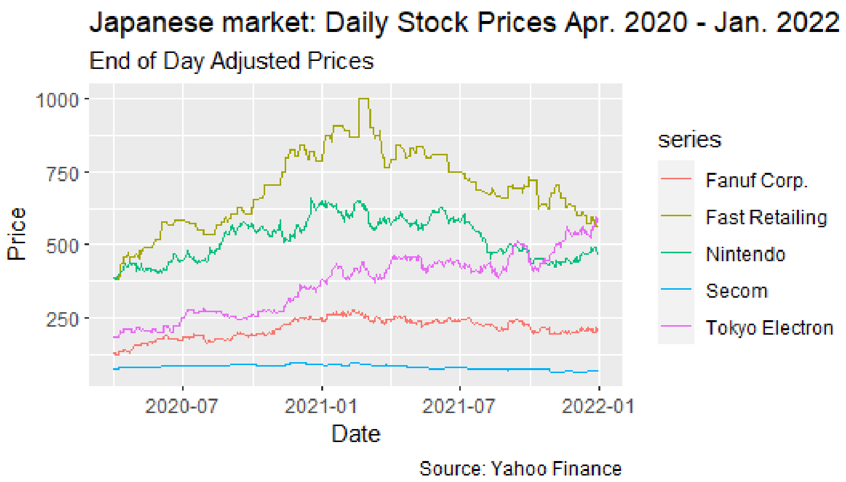
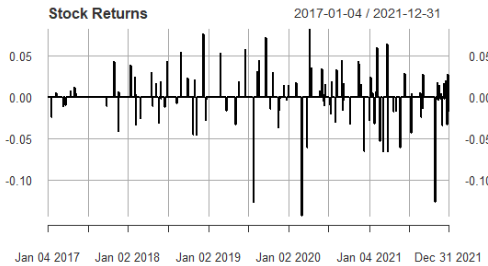
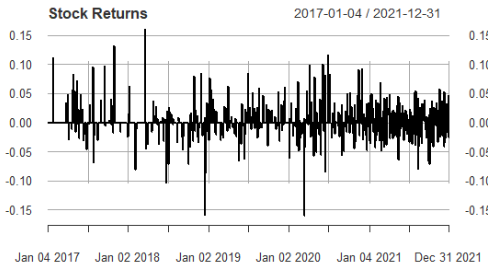


Figure 3.2: Daily portfolio stock prices (Period 2)

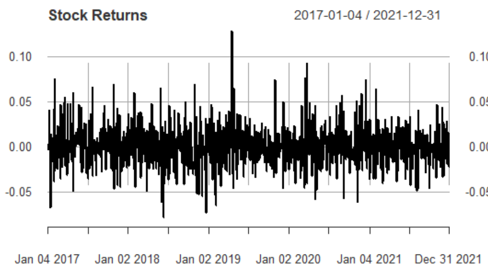
The effects of Covid-19 are also noticeable in figure (3.3), which shows the returns of our stocks. From the middle of 2019 up until the middle of 2021 there has been several cases of sudden volatility change. Nevertheless, before 2019 and after the middle of 2021, things were looking more stable, just like in figure (3.4). Investors prefer the observation of stock price changes through returns because of positive autocorrelation's main fact, which is that future trends depend on their own past data.



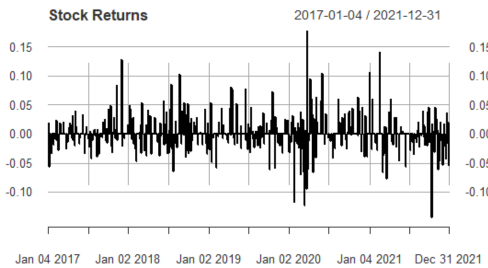
(a) Secom



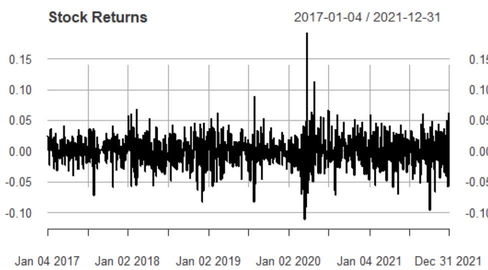
(b) Tokyo Electron



(c) Nintendo

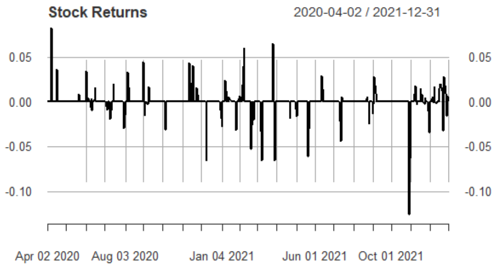


(d) Fast Ret.

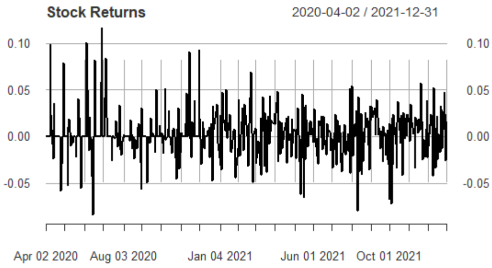


(e) Fanuf Corp.

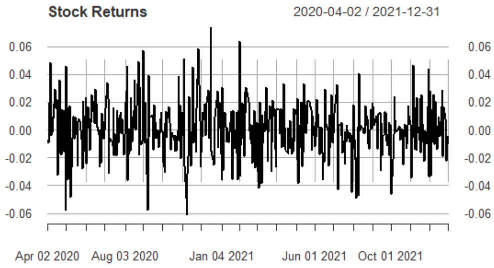
Figure 3.3: Stock returns (Period 1)



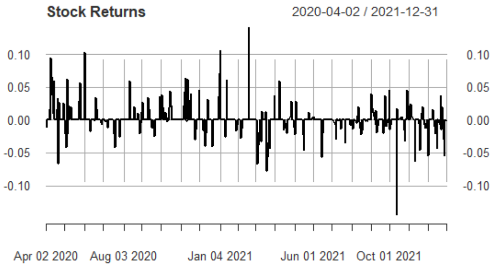
(a) Secom



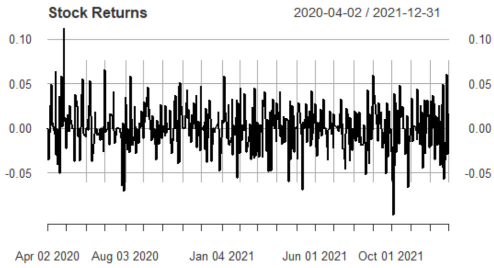
(b) Tokyo Electron



(c) Nintendo



(d) Fast Ret.



(e) Fanuf Corp.

Figure 3.4: Stock returns (Period 2)

It is important that we check the normality of the returns. This can happen through a quantile-quantile plot. First, we plot the returns with normal distribution, and as it observable in figure (3.5) and (3.6), the tails fall far from the line, which would normally reject normality. However, we choose to do another test to check for normality and be more certain of our interpretations.

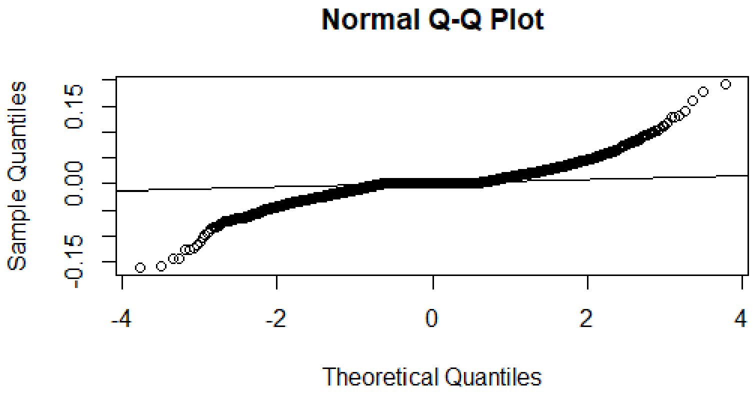


Figure 3.5: Q-Q plot with normal distribution (Period 1)

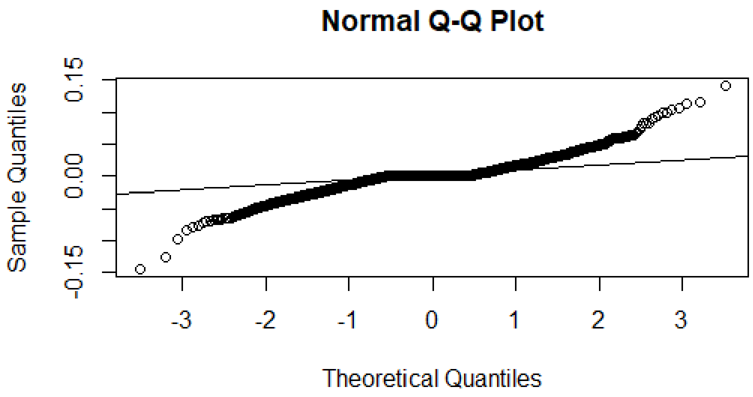


Figure 3.6: Q-Q plot with normal distribution (Period 2)

The *Shapiro-Wilk Test* was conducted to give us an ultimate answer on whether the returns follow normality or not. The test statistic is formulated as follows [24]:

$$W = \frac{(\sum_{i=1}^{N} a_i x_{(i)})^2}{\sum_{i=1}^{N}(x_i - \bar{x})},$$

where $x_{(i)}$ is the i^{th} order statistic, a_i's are constants generated from the sample size N, and $\bar{x}$ is the sample mean.

Furthermore:

- H_0: The sample N being tested comes from a normal population.
- H_1: The alternative hypothesis is true if the p-value is less than the alpha level.

Our sample gives us a p-value of 0.1, which rejects the null hypothesis of normality.

Chapter 4

Volatility Forecasting

In this chapter, the minimization problem is implemented with relation to our data and the optimal weights of our portfolio are obtained. An equally-weighted portfolio is also constructed and compared to the optimal portfolio, in order to show the efficiency of the latter one. Lastly, the GARCH(1,1) model is used to forecast the volatility of each asset.

4.1 Computing the Optimal Weights

Building an optimal portfolio is mainly finding the optimal weights to be assigned on our considered assets. By using equation (2.13) and our data, the objective function to be minimized is as follows

$$\text{Minimize } \sigma_p^2 = \sum_{i=1}^{5}\sum_{j=1}^{5} w_i w_j \sigma_{ij}.$$

The covariance matrices are presented in tables (4.1) and (4.2) below

Table 4.1: Covariance matrix of portfolio securities (Period 1)

	Secom	T.Electron	Nintendo	Fast Ret.	Fanuf
Secom	0.0329	0.0017	0.0018	0.0004	0.0029
T.Electron	0.0017	0.1252	0.0119	0.0087	0.0196
Nintendo	0.0018	0.0119	0.1153	0.0003	0.0080
Fast Ret.	0.0004	0.0087	0.0003	0.1001	0.0200
Fanuf	0.0029	0.0196	0.0080	0.0200	0.1376

Table 4.2: Covariance matrix of portfolio securities (Period 2)

	Secom	T.Electron	Nintendo	Fast Ret.	Fanuf
Secom	0.0433	-0.0009	0.0058	-0.0001	0.0050
T.Electron	-0.0009	0.1591	0.0124	0.0101	0.0271
Nintendo	0.0058	0.0124	0.0970	-0.0019	-0.0039
Fast Ret.	-0.0001	0.0101	-0.0019	0.1113	0.0124
Fanuf	0.0050	0.0271	-0.0039	0.0124	0.1531

In the covariance matrices given in table (4.2), we observe that some of the covariances are negative. These negative values lead us to minimized portfolio variance and therefore standard deviation, which is our goal. We remind that when optimizing a portfolio, we aim for minimizing variance with a fixed expected returns level.

If we then follow equations (2.14) and (2.15), using the Lagrange objective function and Lagrange multipliers, we will be able to obtain the optimal weights. Of course, the more assets a portfolio has, the more difficult it gets to calculate the optimal weights by hand. Thankfully, through the package *PortfolioAnalytics* in R Studio, we are able to calculate the optimal weights given our constraints and our objective. The source code for this is given in appendix (7.1), chapter 7 of our study. The optimal weights are given below

Table 4.3: Optimal portfolio weights (Period 1)

Asset	Optimal Weight
Secom	0.5327
Tokyo Electron	0.1029
Nintendo	0.1332
Fast Retailing	0.1551
Fanuf Corp.	0.0761

Table 4.4: Optimal portfolio weights (Period 2)

Asset	Optimal Weight
Secom	0.4515
Tokyo Electron	0.0929
Nintendo	0.1853
Fast Retailing	0.1735
Fanuf Corp.	0.0968

The optimized weights of **period 1** give us expected returns of $9{:}81712\%$ with a standard deviation (risk) of $0{:}8544\%$.

The optimized weights of **period 2** give us expected returns of $16{:}3092\%$ with a standard deviation (risk) of $0{:}9176\%$

4.2 Studying Stationarity

Stationarity is crucial to be studied as statistical models can be then more effective and precise. Stationarity, as explained in a previous chapter as well, means that properties such as the mean and the variance remain constant through time.

For the stationarity check of each asset, the *Augmented Dickey-Fuller (ADF)* test, and the *Autocorrelation Function (ACF)* test were used. Starting with the ADF test, the null hypothesis shows non-stationarity while the alternative hypothesis shows stationarity of the times-series. The result is depended upon the p-value of the statistical test. If the p-value is less than 0.5, then we can reject the null hypothesis, proving stationarity.

Every asset's returns were proved to be stationary in our portfolio for **both periods**. The p-value was significantly small and therefore was printed as 0.01 in the R program.

Table 4.5: Stationarity test results (Period 1)

Asset	P-value	Lag	Critical Value
Secom	0.01	10	-11.1981
Tokyo Electron	0.01	10	-10.1509
Nintendo	0.01	10	-10.5768
Fast Retailing	0.01	10	-11.5334
Fanuf Corp.	0.01	10	-11.0022

Table 4.6: Stationarity test results (Period 2)

Asset	P-value	Lag	Critical Value
Secom	0.01	7	-7.8946
Tokyo Electron	0.01	7	-7.4126
Nintendo	0.01	7	-7.6061
Fast Retailing	0.01	7	-8.5805
Fanuf Corp.	0.01	7	7.5766

Moving on to the ACF test, we study stationarity through correlograms. We see in the plots below, that there is no serial correlation for each asset and therefore behave as white noise. For **both periods**, every asset's correlation degrades quickly to zero, which shows absence of serial correlation. There are some instances where the correlation of some assets falls outside the two blue doted lines. This means that the correlation at that specific lag cannot be ignored. However, there are not a lot of these instances and therefore don't affect the test as much. The correlograms between the assets are similar. Therefore, below we only show the ones of Secom, however the correlograms of the other assets can be found on appendix (7.4).

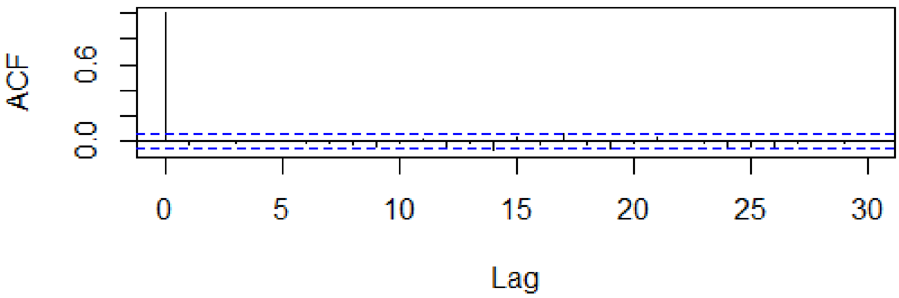


Figure 4.1: Correlogram of Secom (Period 1)

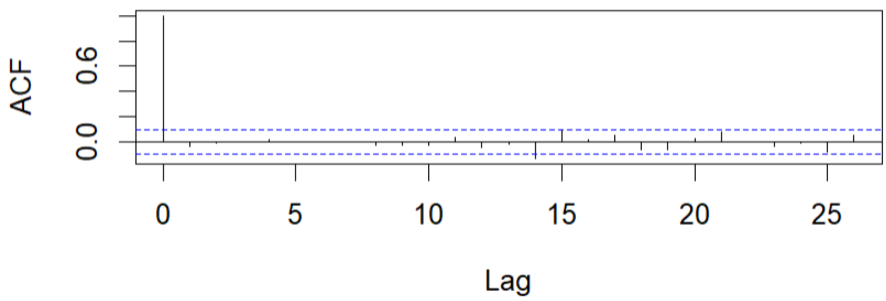


Figure 4.2: Correlogram of Secom (Period 2)

4.3 Equally-Weighted Portfolio Comparison

An equally-weighted portfolio means that each asset has a weight of 0:2, since we have five assets and the total weight must be equal to one. If we compare the results of an equally-weighted portfolio with the results of the optimised-weighted portfolio, we see that the equally-weighted portfolio gives us higher returns, but higher volatility as well. This holds true for **both periods**.

Table 4.7: Portfolio comparisons (Period 1)

Portfolio	Returns(%)	Volatility(%)
Optimal Portfolio	9.81712	0.8544
Equally-weighted Port.	16.59182	16.28275

Table 4.8: Portfolio comparisons (Period 2)

Portfolio	Returns(%)	Volatility(%)
Optimal Portfolio	16.3092	0.9176
Equally-weighted Port.	30.9566	16.6889

4.4 Forecasting Volatility Using GARCH(1,1)

We forecast each asset individually for a time-period of 7 days ahead, and another time-period for 14 days ahead. Below, we present the graphical illustrations of how volatility changes for each asset as well as tables that show the results for the optimal parameters of the GARCH model.

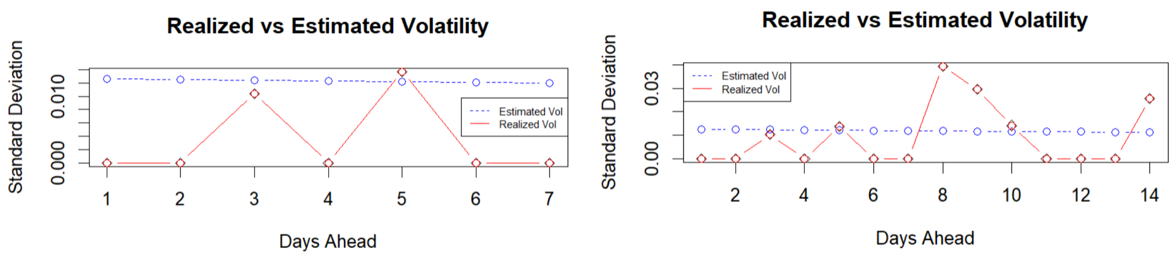


Figure 4.3: Secom forecasting (Period 1)

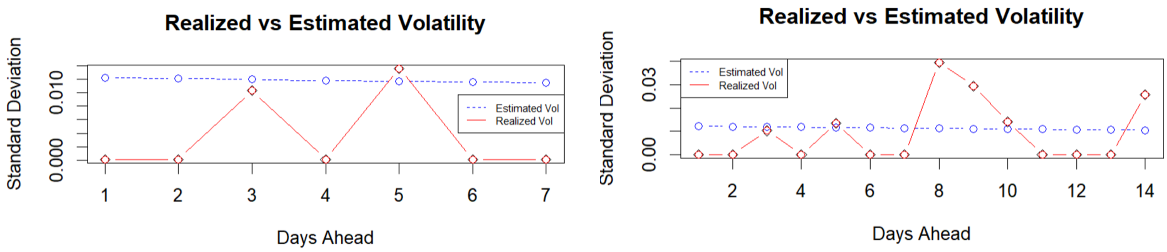


Figure 4.4: Secom forecasting (Period 2)

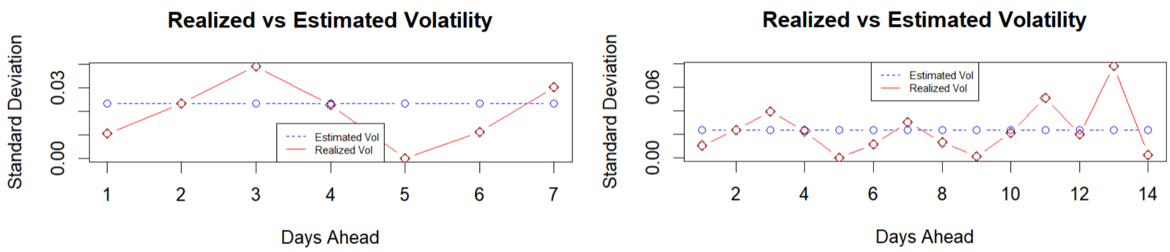


Figure 4.5: Tokyo Electron forecasting (Period 1)

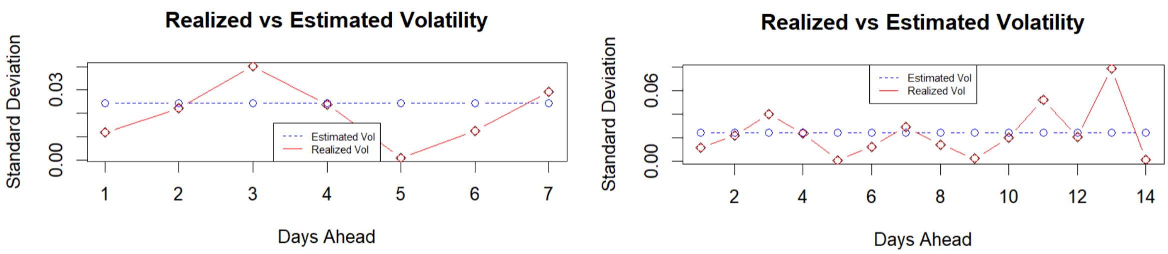


Figure 4.6: Tokyo Electron forecasting (Period 2)

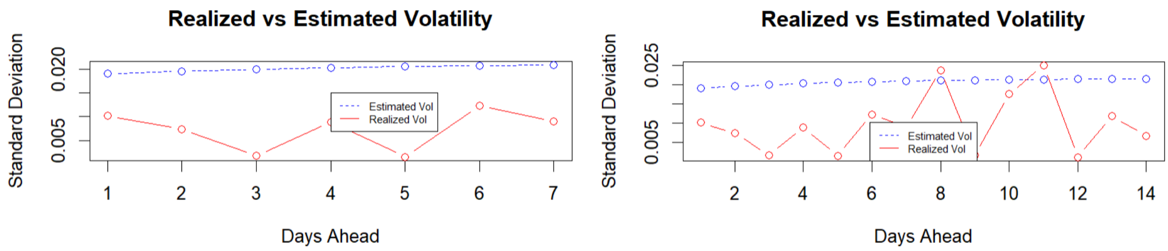


Figure 4.7: Nintendo forecasting (Period 1)

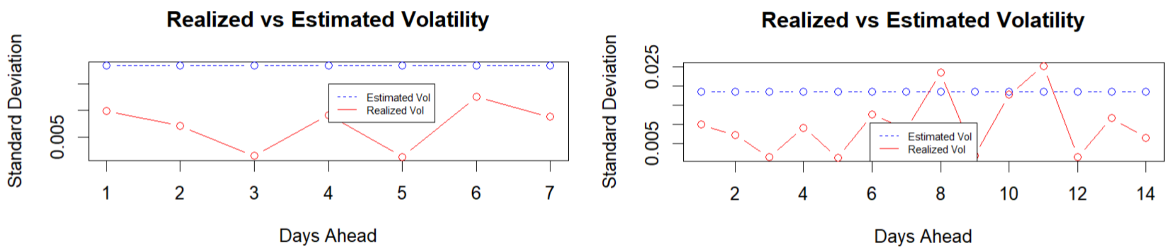


Figure 4.8: Nintendo forecasting (Period 2)

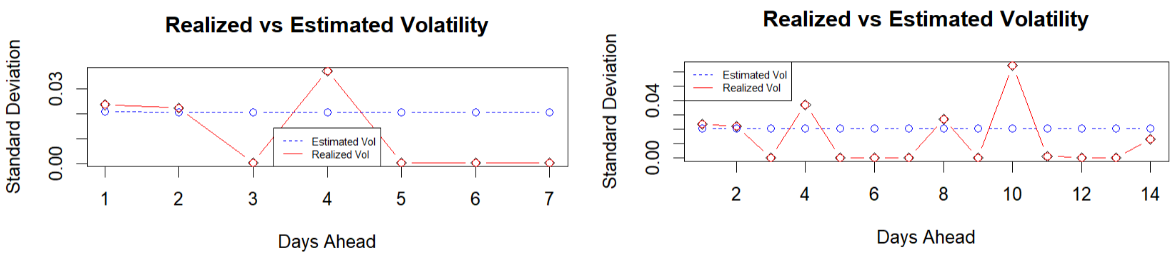


Figure 4.9: Fast Retailing forecasting (Period 1)

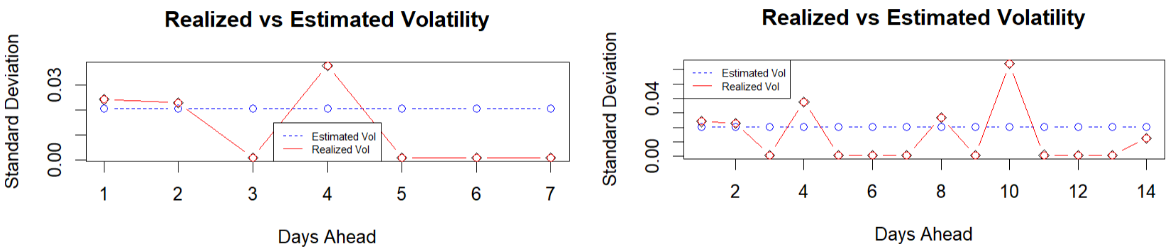


Figure 4.10: Fast Retailing forecasting (Period 2)

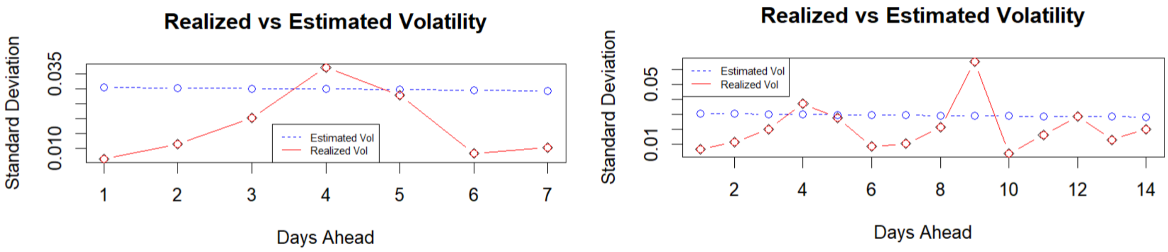


Figure 4.11: Fanuf forecasting (Period 1)

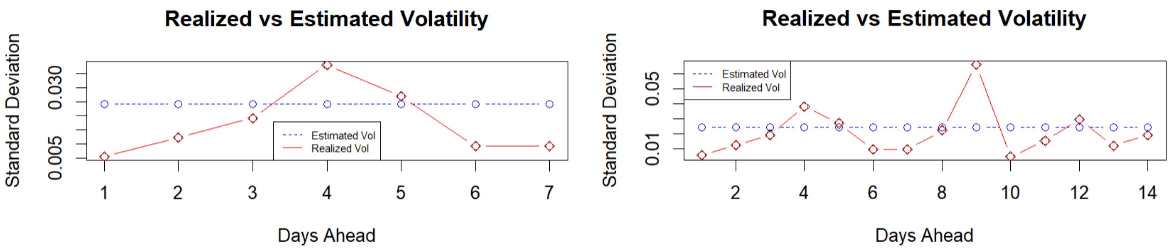


Figure 4.12: Fanuf forecasting (Period 2)

Table 4.9: Optimal parameters for the forecasting model of our optimal portfolio (Period 1)

	Estimate	Std. Error	t value	Pr(>|t|)
w	0.000019	0.000006	2.90750	0.003643
a_1	0.077192	0.014307	5.39545	0
b_1	0.889378	0.021373	41.61172	0

Table 4.10: Optimal parameters for the forecasting model of our optimal portfolio (Period 2)

	Estimate	Std. Error	t value	Pr(>|t|)
w	0.000005	0	1051.5	0
a_1	0.000010	0.000493	0.020252	0.98384
b_1	0.991968	0.001058	937.4	0

Table 4.11: Optimal parameters for the forecasting model of an equally-weighted portfolio (Period 1)

	Estimate	Std. Error	t value	Pr(>|t|)
w	0.000001	0.000001	1.11172	0.26626
a_1	0.077199	0.014781	5.22295	0
b_1	0.889390	0.015619	56.94228	0

Table 4.12: Optimal parameters for the forecasting model of an equally-weighted portfolio (Period 2)

	Estimate	Std. Error	t value	Pr(>|t|)
w	0	0.000001	0.30533	0.76011
a_1	0.007697	0.005991	1.28476	0.19888
b_1	0.984247	0.005590	176.08263	0

Chapter 5

Analysis of the Model

Interpretation from the results of the model are presented in this chapter. Each section in chapter 4 is examined individually in order for an answer to be given to the research questions stated in the beginning of the thesis. Can we build an optimal portfolio? Is the GARCH model efficient enough when forecasting volatility?

5.1 Portfolio Analysis

In section 4.1, we computed the optimal weights of our portfolio for **both periods**. Our results show that Secom has the biggest weight, with approximately 50% in both cases. It is interesting to notice that Secom is the asset with the smallest market value. Nevertheless, stock prices do not matter in the way optimal weights are created. The optimal weights are decided by return on investment (ROI), which we used with the help of R Studio.

Connected to section 4.1, section 4.3 compares the returns and volatility of the optimal portfolio, with the respective ones of the equally weighted portfolio. In **both periods**, the equally weighted portfolio gives higher returns but also higher volatility, therefore risk. What we notice however is that **period's 2** portfolios give higher returns for approximately the same volatility. It seems that the shorter period of data, together with the fact that it was after the Covid-19 effect, give better results to a portfolio. However, how can we know which portfolio is better? The choice between the optimal portfolio and the equally-weighted portfolio is subjective. As we said before, some people will take higher risks than others, in an attempt to obtain more profit. In order to answer this question, we need to consider portfolio performance evaluation. There are several tests that can be used here, but we are using the *Sharpe's ratio test*. This is a model that analyses empirical statistics of different portfolios and the one that will give the higher ratio, is the one that is more efficient. The formula of the model is given by [11]

$$S_p = \frac{r_p \quad r_f}{s_p};$$

where r_p is the returns of the portfolio, r_f is the risk-free rate of the portfolio, and s_p is the standard deviation of the portfolio. Note that the risk-free rate in this instance is the Japan government bond which is considered risk-free.

With this formula, we can now plot the numbers that we obtained in section 4.3, and see which portfolio is better. Nevertheless, apart from the numbers that we will be getting from the two portfolios, we need to have a benchmark point as well. This benchmark is the capital market line (CML), and in this occasion in it given by the Japanese stock index *Nikkei*. We have not calculated the returns and standard deviation of the Japanese stock index, but its monthly Sharpe ratios can be found easily online, so considering our two periods, we can compute the average of the Sharpe ratios and use them directly for comparison with the two portfolios. Below, we present the results

Table 5.1: Sharpe's ratios (Period 1)

Portfolio	Sharpe's Ratio
Optimal Portfolio	11.4701
Equally-weighted Port.	1.0183
Japanese stock index	0.6883

Table 5.2: Sharpe's ratios (Period 2)

Portfolio	Sharpe's Ratio
Optimal Portfolio	17.7153
Equally-weighted Port.	1.8517
Japanese stock index	0.8863

From the results above, it is clearly observable that both the optimal and the equally-weighted portfolio are above the benchmark line, but the optimal portfolio gives a far larger ratio, making it more preferable and efficient. This high ratio comes as a result of two things. The first one is a very low risk-free rate. The Japanese government bond has fallen from approximately 2% to 0.1% in the last 20 years. The other reason is the low standard deviation that we obtained from our optimal portfolio. However, an unrealistic assumption is that we don't consider transaction costs. If we were to do so, the ratio would be much smaller, but still fair to say that it would be higher than the one of the equally-weighted portfolio.

5.2 Model Analysis

The most important and significant part of our study comes on section 4.4, where we use our GARCH model to predict the volatility of the assets that we have in our portfolio. The first thing we notice is that the forecasted volatility is close to a straight line. This is because we are using the GARCH(1,1) model. In case we were to use the GARCH(2,2) model or higher, we would get a more accurate result, where the line is curved. Furthermore, the line is straight as it represents the average of the estimated future volatility. Another explanation for this phenomenon has to do with the historical volatility. If an asset hasn't been very volatile in the past, it is expected not to be in the future either. This is why the predicted volatility line has minimal slope. In case we had several sudden changes in the historical volatility, we would expect a more curved line in the first place. For some assets, the forecasted line may not intersect the realized volatility as much in 7 days ahead, but it's more noticeable that it is indeed the average of the realized volatility through the graphs of 14 days ahead. Nevertheless, the predicted volatility line of Nintendo does not represent the average of the realized volatility in an adequate way.

Another thing we observe is that the forecasting lines are very similar in **both periods**. This is logical, as we are using the same model to forecast the volatility of our data. However, one can notice slight differences in the forecasting lines between the two periods in the assets of Nintendo and Fanuf.

To interpret the accuracy of our forecasting model, the use of MSE and MAE is needed. These are error estimators that can be used in various models, just like the one in this study. More specifically, MSE stands for Mean Squared Error, and it measures the variance of the residuals. It is given by [7]

$$MSE = \frac{\sum (y_i - y_i^{\prime})^2}{n}, \tag{5.1}$$

where y_i is the i^{th} observed value, $y_i^{\prime}$ is the i^{th} predicted value, and n is the number of observations.

MAE stands for Mean Absolute Error and it measures the average of the residuals. It is given by [7]

$$MAE = \frac{\sum |y_i - y_i^{\prime}|}{n}. \tag{5.2}$$

Below, we can see the tables for the error estimations of our model.

Table 5.3: Errors for 7 days ahead (Period 1)

Asset	MSE	MAE
Secom	0.00010731	0.00921893
Tokyo E.	0.00016403	0.01025211
Nintendo	0.00017958	0.01283025
Fast R.	0.00027551	0.01458745
Fanuf	0.00026980	0.01456528

Table 5.4: Errors for 14 days ahead (Period 1)

Asset	MSE	MAE
Secom	0.00017435	0.01151456
Tokyo E.	0.00042542	0.01525620
Nintendo	0.00017060	0.01171194
Fast R.	0.00039749	0.01712955
Fanuf	0.00031111	0.01481886

Table 5.5: Errors for 7 days ahead (Period 2)

Asset	MSE	MAE
Secom	0.00009873	0.00886641
Tokyo E.	0.00016277	0.01025146
Nintendo	0.00014254	0.01127046
Fast R.	0.00026584	0.01452826
Fanuf	0.00016458	0.01166749

Table 5.6: Errors for 14 days ahead (Period 2)

Asset	MSE	MAE
Secom	0.00017215	0.01135235
Tokyo E.	0.00043336	0.01554042
Nintendo	0.00013019	0.01027248
Fast R.	0.00038717	0.01700422
Fanuf	0.00025601	0.01263527

We also present the errors estimations for an equally-weighted portfolio below:

Table 5.7: Errors for 7 days ahead (Period 1)

Asset	MSE	MAE
Secom	0.000015662	0.002216102
Tokyo E.	0.00018592	0.00810090
Nintendo	0.000019257	0.00265632
Fast R.	0.00012805	0.00617467
Fanuf	0.00016902	0.00750191

Table 5.8: Errors for 14 days ahead (Period 1)

Asset	MSE	MAE
Secom	0.00021848	0.00981881
Tokyo E.	0.00076366	0.01989124
Nintendo	0.00011375	0.00802750
Fast R.	0.00043058	0.01383656
Fanuf	0.00057296	0.01838814

Table 5.9: Errors for 7 days ahead (Period 2)

Asset	MSE	MAE
Secom	0.00015408	0.00215160
Tokyo E.	0.00018809	0.00818345
Nintendo	0.00001426	0.002419425
Fast R.	0.00013766	0.00608776
Fanuf	0.00013186	0.00607249

Table 5.10: Errors for 14 days ahead (Period 2)

Asset	MSE	MAE
Secom	0.00022014	0.0097436
Tokyo E.	0.00078460	0.02019585
Nintendo	0.00009613	0.007455454
Fast R.	0.00044148	0.01352634
Fanuf	0.00048685	0.01562496

From the error tables of our optimal portfolio model, we draw the following conclusions:

- For almost all assets, forecasting for 14 days ahead gives us higher errors than forecasting for 7 days ahead. This is logical, as the further in the future we try to predict, the larger the uncertainty due to unprecedented changes. It is worth noticing that this was not the case for Nintendo.
- **Period 2** gives us lower errors than **period 1**. This is also expected for the argument given previously.

Comparing the optimal portfolio error tables, with the respective ones for the equally-weighted portfolio, we can observe that in some cases, the optimal portfolio has smaller errors than the equally-weighted one, while in other cases the opposite is true. However, this does not show the efficiency of the portfolio, but rather the accuracy of the forecasting method. Ultimately, the errors are small for both portfolios, which indicate the validity of our model.

Chapter 6

Conclusion

This thesis aimed to identify the answer to two questions. The first one was if it is possible to build an optimal portfolio, and secondly whether the GARCH(1,1) model works well when forecasting the volatility of a portfolio or not. Based on our results and the analysis of our data, it can be concluded that we were successfully able to obtain sufficient results to both questions. For the construction of the optimal portfolio, through explanation of the necessary formulas needed and the help of R studio, we were able to build a portfolio with our five assets where we get the lowest risk possible for given expected returns.

Considering our forecasting model now, we were able to predict the future volatility of our portfolio assets in a sufficient way, since we got the average of the realized volatility for all assets except Nintendo. We also showed that the errors for **period 2** are smaller, which is what we expected. However, when it comes to investing, it is crucial to be able to predict whether the stock prices will go up or down, and a predicted volatility of minimal upward or downward slope might not be of greatest help. It could be therefore recommended to use GARCH(2,2) model, which gives predicted volatility lines with more curves. In that case it would be easier to observe when the prices of an asset are going towards the benefit of an investor or not.

Another limitation is that we were not able to produce plots for the forecasted volatility of the optimal and equally-weighted portfolios as a whole. Having the plots of the individual assets was helpful, but having the plots of the whole portfolios could have given us a better general idea of where the volatility of the portfolios is predicted to move. In that case, it would be also easier to observe the benefits of the optimal portfolio versus the equally-weighted portfolio.

Overall, the findings of the thesis contribute positively to the efforts of investors in a practical manner. Even though there are plenty of papers examining GARCH models, we did not find a paper where the authors plot the forecasted volatility while at the same time trying to build an optimal portfolio. The GARCH(1,1) model has been used for many years and will continue to be used as it has given us results that helped us find effective answers to our research questions.

6.1 Future Research

Understanding GARCH models in an attempt to forecast volatility of a portfolio is crucial for many investors. With regards to that, we recommend some topics in which further research can be conducted upon:

- Investigating the differences between various GARCH models on how well they perform with volatility forecasting of a portfolio.
- Building an optimal portfolio consisting of cryptocurrencies, while also trying to forecast the portfolio's volatility using the appropriate GARCH models.
- Comparing the Japanese market to other global markets and examining their differences when it comes to volatility forecasting and portfolio profit attainment.

Chapter 7

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Chapter 8

Appendices

8.1 Portfolio Weights Optimization

```
library(PortfolioAnalytics)

install.packages(c("ROI","ROI.plugin.glpk","ROI.plugin.quadprog"))

port_spec = portfolio.spec(names(stocks))
port_spec = add.constraint(portfolio = port_spec, type = 'long_only')
port_spec = add.objective(portfolio = port_spec, type = 'risk',
                    name = 'StdDev')
opt = optimize.portfolio(xts.LOGRETURNS, portfolio = port_spec,
                   optimize_method = 'ROI')

print(opt)

opt_weights = extractWeights(opt)
chart.Weights(opt)
```

8.2 ADF Test

```
stationary_test_returns = ""
for (i in 1:ncol(stocks))
{
 stationary_test_returns = paste(stationary_test_returns,
     names(stocks)[i],
                            "Critical-Value: ",
                            adf.test(xts.LOGRETURNS[,i])$statistic,
                            "Lag: ",
```

```
                                          adf.test(xts.LOGRETURNS[,i])$parameter,
                                           "p-value: ",
                                           adf.test(xts.LOGRETURNS[,i])$p.value,
                                              "\n")

}
writeLines(stationary_test_returns)
plot(as.numeric(xts.LOGRETURNS))
```

8.3 ACF Test

```
par(mfrow=c(1,1))
acf(as.numeric(xts.LOGRETURNS[,1]))
acf(as.numeric(xts.LOGRETURNS[,2]))
acf(as.numeric(xts.LOGRETURNS[,3]))
acf(as.numeric(xts.LOGRETURNS[,4]))
acf(as.numeric(xts.LOGRETURNS[,5]))
```

8.4 Correlograms of Stocks

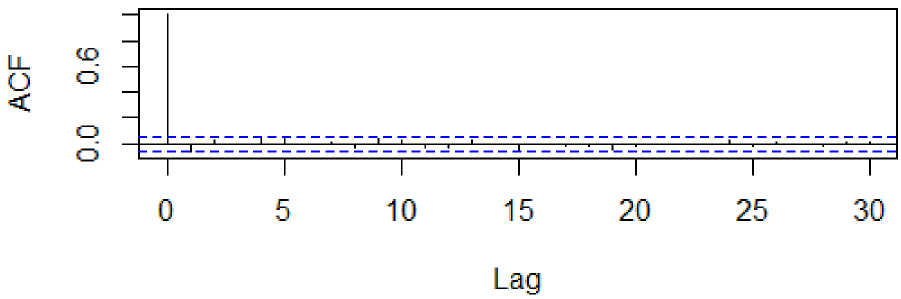

Figure 8.1: Correlogram of Tokyo Electron (Period 1)

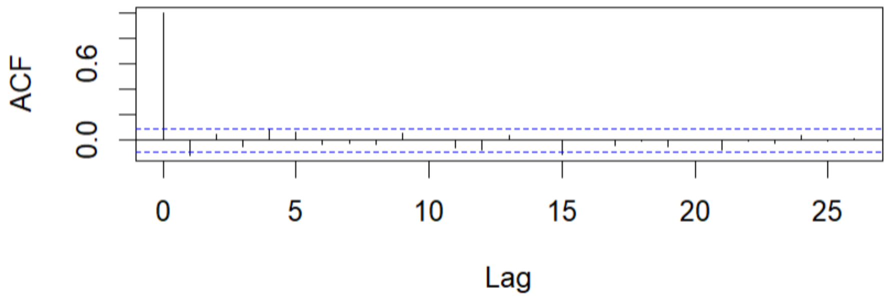

Figure 8.2: Correlogram of Tokyo Electron (Period 2)

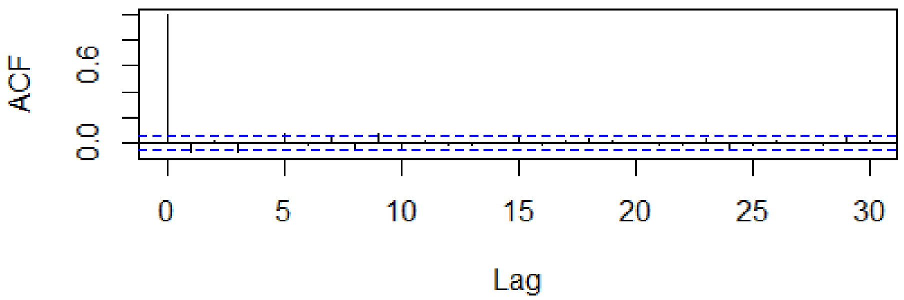

Figure 8.3: Correlogram of Nintendo (Period 1)

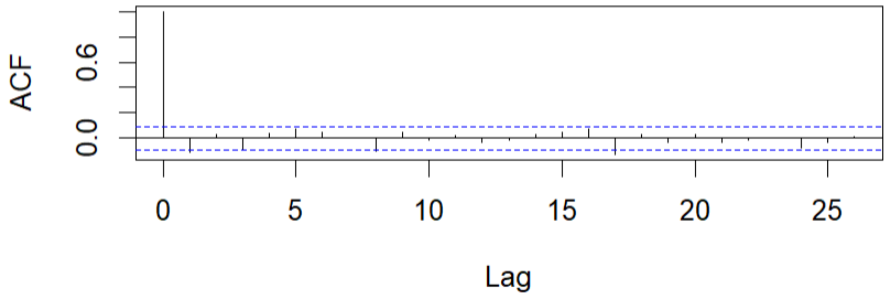

Figure 8.4: Correlogram of Nintendo (Period 2)

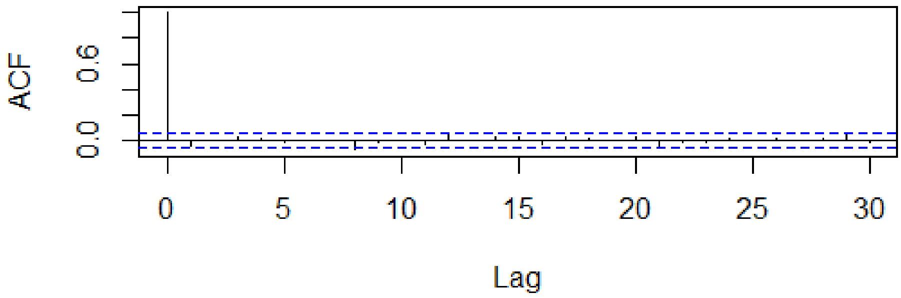

Figure 8.5: Correlogram of Fast Retailing (Period 1)

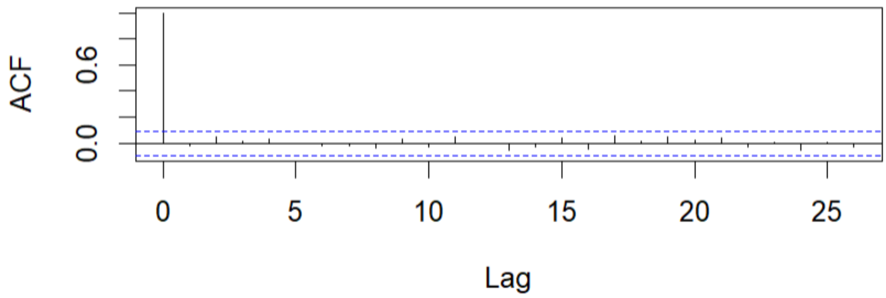

Figure 8.6: Correlogram of Fast Retailing (Period 2)

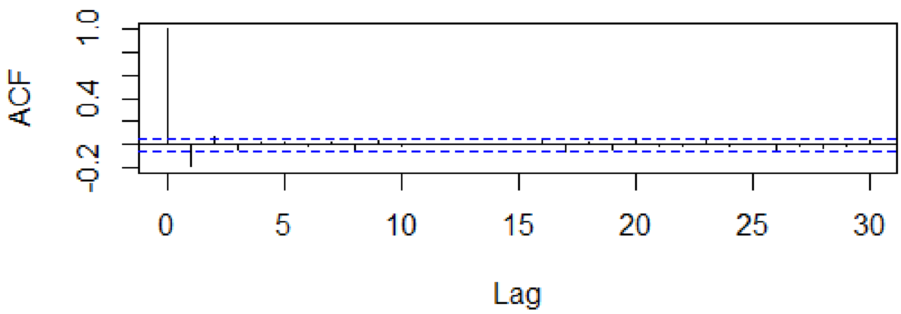

Figure 8.7: Correlogram of Fanuf Corporation (Period 1)

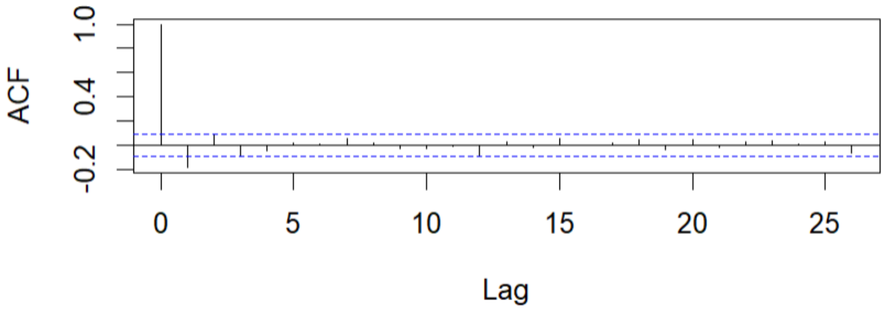

Figure 8.8: Correlogram of Fanuf Corporation (Period 2)