Modeling the Dynamics of Opinion Change when Individuals may have Different Overt and Covert Opinions

by

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Abstract

This thesis aims to present and explore a mathematical model of attitude change within society. Building on the foundations set up by Lave and March [2] and Eriksson and Strimling [1] we propose a model which includes the possibility of having an overt as well as covert opinion by any individual on a given issue. We lay out the verbal and mathematical specifications for our model, interpret them in the framework of discrete-time dynamical systems and simulate the behavior using Matlab programming. Finally, we propose three conjectures based on the results of our simulations and proceed to subject two of them to further analytical treatment.
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1 Introduction

1.1 Mathematical Modeling of Attitude Change

Attitude change is a research area especially interesting within social psychology, wherein many theories about this process have been developed. Most theories begin with a definition of attitude and share a common denominator of communication. Now, the key parts of this communication are the source, message, receiver and effects. The heuristic-systematic model of information processing [2] is one of those theories. It is a dual process model as it treats two depths in the processing of attitude change; systematic processing (high-involvement in processing information) and heuristic processing (shortcuts such as emotions and gut-feelings). Another of these models is the elaborate likelihood model (ELM) which shares many characteristics with Chaiken’s model; mainly that they are both dual process models that depend on mental stimulation on the one hand, and emotions on the other. The ELM considers cognitive processing as the central route and affective/emotion processing as the peripheral route. Another model of attitude change is the cognitive dissonance theory, developed by Festinger [9]. This theory rests on the idea that people experience a sense of uneasiness when two linked cognitions are inconsistent, such as when there are two conflicting attitudes about a topic. This relates to attitude change since a person experiencing cognitive dissonance is motivated to reduce said dissonance by changing her attitude.

Once mathematicians started showing interest in this topic, a transition from the above verbal models took place, arriving at mathematical models.

1.2 Discussion of Lave and March

In their book "An Introduction to Models in the Social Sciences" [2], Lave and March acquaint the reader with model building in the social sciences. The writers establish an environment of critical thinking and urge the readers to get involved in building the models. In one of the later sections called "diffusion" the authors examine social diffusion as in how and why an item spreads through a society, what are the factors that determine the rate and pattern of diffusion, how to make it diffuse faster or slower and similar questions. The authors first define a few basic concepts that facilitate the spread of things in a society. They do that by introducing an example about the spread of rumor. Firstly, some information about those who know the rumor initially is needed, as they will act as a source or an agent of contagion. Then, we will need some knowledge of the communication patterns in the society, as the spread of rumor happens by communication between an agent of contagion and someone who doesn’t have the information. After that, a descriptive term is presented for the item that is spreading, which can be information, emotion, beliefs, attitude. We call that the item of diffusion which in the case of rumor is information. It should also be mentioned that when talking about diffusion we should consider the medium through which the item of diffusion is being spread, i.e. contact patterns. Spread of emotion for example places greater importance upon visual communication (sight) whereas the spread of rumor places greater value upon verbal communication. The authors mention how these exposure patterns vary substantially due to the emergence of mass media, telephones and television. Here, the authors distinguish between different types of exposure:
symmetrical, if I am exposed to you, you are exposed to me, and asymmetrical, a significant number of people are exposed to the President of the United States for example, but he cannot possibly be exposed to (or aware of) all of them. Some of the factors affecting the chance that a transmission will occur are for example the relativity of the item, whether it is deemed important or not, whether or not it is easy to spread (think here of the conscious decision you have to make every time you spread a rumor versus how a beard will basically spread itself in any face-to-face contact). Some of the factors affecting the chance that a transmission will be accepted are for example the status of the sender. The higher the status of the sender (in the sphere of the object) the more likely that it will be accepted. The trustworthiness of the sender is also a factor, as bias leads to exaggeration in the portrayal of the item, and we all tend to estimate the credibility of the source. Another factor is how the attributes of the item relate to the characteristics of the recipient; a transmission will have higher chances of being accepted if it is consistent with the recipient’s beliefs. The model we just presented is based on person-to-person transmission where the object of diffusion is transmitted through contact between those who have it and those who do not. However, the authors also present what is called a Broadcast model, in which transmission arises in a different matter, for example newspapers and television broadcasts. In this model, there is only one source of information and that information is not spread on by any new sources. This is in contrast to the first model we presented, where every person in possession of the information will turn into a new source.

In my thesis, I will have a model of the first type, based on person-to-person transmission.

1.3 Discussion of Eriksson and Strimling

In their paper "Group differences in broadness of values may drive dynamics of public opinion on moral issues" [3], Kimmo Eriksson and Pontus Strimling propose the idea that an audience’s receptiveness to various arguments depends on the audience’s set of values. This proposition came as a possible explanation to the phenomenon that moral opinions have been converging towards a liberal view with no corresponding increase in liberals to explain it. They develop a hypothesis about the psychological theory of moral foundations which states that there are five psychological foundations of morality; harm, fairness, loyalty, respect and purity. Liberal morality rests on the first two while conservative morality rests on all five. Eriksson and Strimling (2015) assume an infinite population of liberals and conservatives and introduce three parameters that govern the position-change process. After defining the parameters, they later move on to constructing a mathematical model that when simulated for different values of the different parameters displays the dynamics of opinion change. Some of the main results obtained from analysis of the dynamical system are: if a position is present at all in the population, it will be present in both groups but in different proportions; the long-term proportions do not depend on the initial proportions; the long-term proportions are governed by the more biased group; one group’s biases influence the long-term proportions of positions in both groups.
1.3.1 The Mathematical Model of Eriksson and Strimling (2015)

The mathematical model constructed by Eriksson and Strimling [3] serves as inspiration for my model and is therefore presented in the following section.

1.3.2 Introduction of Parameters

Three parameters govern the process by which positions are acquired:

- **Group-exposure bias**: how much less likely it is for an individual to be exposed to (and care about) the arguments of someone from the other political orientation than of someone from the same political orientation (i.e. a Lib is more likely to listen to another Lib.)

  \[ G_L = \frac{\text{Prob(a Lib is exposed to a Con’s argument)}}{\text{Prob(a Lib is exposed to a Lib’s argument)}} \]

  Where \( G_L \) represents group-exposure bias for the liberal population and \( G_C \) its counterpart for the conservative population.

- **Position-change bias**: measures how the difference in moral foundation profiles between the political orientations makes it less likely for an individual to be swayed by the arguments for one position than for the other (i.e., a Lib is less easily swayed to the "against" position than to the "for" position.)

  \[ P_L = \frac{\text{Prob(a Lib who is FOR is swayed when exposed to an arg. AGAINST)}}{\text{Prob(a Lib who is AGAINST is swayed when exposed to an arg. FOR)}} \]

  Where \( P_L \) represents poition-change bias for the liberal population and \( P_C \) its counterpart for the conservative population.

- **Influentiality coefficient**: a measure of how often others are allowed to influence an individual’s position. The parameters \( I_L \) and \( I_C \) are introduced as previous research suggests that conservatives tend to exhibit less openness than liberals [8].

1.3.3 The dynamical system

Proportions of FOR and AGAINST in the Lib population at a certain time are denoted by \( q_l \) and \( 1 - q_l \) respectively. The authors then set up the events in which a change occurs and denote \( \frac{\Delta q_l}{\Delta t} \) as the change over a small time step \( \Delta t \) in the proportion of FOR in the Lib population. According to the authors, four types of events contribute to change:

- A Lib who is currently against may be swayed from exposure to a Lib who is for. This event happens with rate \( (1 - q_l)q_lI_l \).

- A Lib who is currently against may be swayed from exposure to a Con who is for. Because of group-exposure bias, this happens only with a rate of \( (1 - q_l)q_cG_lI_l \).
• A Lib who is currently for may be swayed from exposure to a Lib who is against. Because of position-change bias, this happens only with a rate of $q_l (1 - q_l) P_I I_l$.

• A Lib who is currently for may be swayed from exposure to a Con who is against. Because of combination of group-exposure bias and position-change bias, this happens only with rate $q_l (1 - q_c) G_l P_I I_l$.

The change equation:

$$\frac{\Delta q_l}{\Delta t} = \left[ + (1 - q_l)(q_l + q_c G_l) - q_l P_l ((1 - q_l) + (1 - q_c) G_l) \right] I_l.$$

The same process is done to obtain the analogous counterparts for the Con population.

1.4 Introduction to Our Problem

This problem was suggested by my supervisor, Kimmo Eriksson, who wondered how we can approach modeling if people had both overt and covert opinions.

Within social sciences, the term "social desirability bias" is used to denote a person’s tendency to over-report on good behavior and under-report on bad one when faced with a survey, all in the purpose of evading possible consequence (for example punishment) and conveying a more likable character. This creates the dichotomy of having opposing overt and covert opinions, meaning that one avoids to voice their actual (covert) opinion in a medium that does not support that opinion, thus acquiring an overt opinion that one chooses to voice instead, in order to seem more socially likable. This concept directed our curiosity towards the consequences of such a dichotomy when studying a population on the micro level.

Now, assuming there are two levels of opinions (overt and covert), and two types of interactions (argument and sanction), then by setting rules and probabilities for how these different interactions occur, we should be able to empirically investigate various social dynamics.

We do that by constructing a mathematical model that somewhat realistically reflects the reactions of human subjects using various parameters.

1.5 Problem Formulation

Based on the defined concept in the introduction, we can assume a population with two levels of opinion (overt and covert), and two types of interactions (argument and punishment). Our main question is, what parameter values matter for which outcomes we obtain? More specifically, when do we get the majority of the population holding different overt and covert opinions? Kimmo Eriksson suggested the following process.

1.6 Verbal model

In our model, we will have four types of people, denoted $q_{00}, q_{10}, q_{01}$ and $q_{11}$. Where the inner index represents the covert opinion, and the outer index represents the overt opinion, and where 0 is against and 1 is for. Here we name $q_{00}$ and $q_{11}$ consonant states, and $q_{01}$ and $q_{10}$
as dissonant states. Where dissonance is defined in social psychology as a state of discomfort experienced by those who simultaneously hold two opposing beliefs or values [10]. The process that determines the interactions is as follows:

- **Step 1:** Agents are paired up. For the time being, let this happen by uniform drawing of pairs.

- **Step 2:** When a pair of agents, A and B, interact they follow this scheme:
  - A observes B’s overt opinion
  - A may punish B. This happens only if B’s overt opinion is different from A’s. The probability of punishment depends on whether A’s covert opinion is the same as her overt opinion (high probability of punishment) or different (low probability of punishment). If B is punished there is some probability of B changing her overt opinion.
  - A may make an argument for her overt opinion. The probability of A making an argument depends on whether A’s covert opinion is the same as her overt opinion (high probability of argument) or different (low probability of argument). If an argument is made there is some probability of B changing her covert opinion (in case it was different from A’s overt opinion).

### 1.6.1 Objective of the thesis

Now that we have our verbal model, we can proceed to developing our mathematical model and study the dynamics of various interactions and how they are influenced by various model parameters. This study will be carried out using simulations and, in interesting special cases, analytic methods.
2 Mathematical Setup of the Problem

In this section we want to develop the verbal model into a mathematical one and define the parameters described by Eriksson in mathematical terms.

Observe here that we have two parameters governing the opinion-change process, namely; action-type bias and position-change bias. Action-type bias can be defined as the probability of a person taking an action, either argument or punishment. In this thesis, action-type bias depends on whether the person holds consonant or dissonant views, as dissonant has a smaller tendency to take action so as not to invite further scrutiny into their dichotomy. We denote action-type bias by \( P_{XX}(\text{pun}) \) and \( P_{XX}(\text{arg}) \) and distinguish between four different biases for argument \( (P_{00}(\text{arg}), P_{10}(\text{arg}), P_{01}(\text{arg}) \text{ and } P_{11}(\text{arg})) \), and their counterparts for punishment. It may be worth mentioning here that in this thesis we treat consonant states as equals, and dissonant states as equals, i.e. \( P_{00}(\text{arg}) = P_{11}(\text{arg}) \) and \( P_{01}(\text{arg}) = P_{10}(\text{arg}) \), such that all probabilities always satisfy \( 0 \leq P \leq 1 \).

Position-change bias is bias experienced when switching between states, i.e. consonant to dissonant and vice versa. This also depends on the harmony in the person’s views, as a person is more inclined to switch away from a dissonant state than a consonant one, seeking to reduce dissonance [10]. Position-change bias is denoted as \( P_{XX} \rightarrow XY \) for switching from consonant states, and \( P_{XY} \rightarrow XX \) for switching from dissonant states. We notice here that there are two variants for each initial state as \( P_{00} \) can turn into either \( P_{01} \) or \( P_{10} \), this means we will have eight different variants of this parameter. However, we wish to maintain a uniformity among values that start at consonant states and end at dissonant, and a uniformity among values that go the other way around. More clearly noted:

\[
P_{00 \rightarrow 01} = P_{00 \rightarrow 10} = P_{11 \rightarrow 10} = P_{11 \rightarrow 01} = P_{\text{CON} \rightarrow \text{DIS}}
\]

and likewise for the dissonant \( \rightarrow \) consonant counterparts. These probabilities are defined such that \( 0 \leq P \leq 1 \).

Note here that the probability of punishment arises only when there is a contrast in the agents’ overt opinions, while probability of argument is always present.

Now that we have the verbal model, we can proceed to putting it in mathematical terms. After that, we have specific questions that interest us and that we would like answered. To do that, we simulate our mathematical model and derive conjectures that we later on analyze.

2.1 Mathematical Model

Now that we have established our parameters, we are one step closer to being able to draw change equations likewise the ones derived by Eriksson et al. Before we get there, however, we need to identify all possible interactions between our agents, i.e. what is described as Step 1 in our verbal model. We found it best and simplest to introduce those interactions through tree diagrams, where each sub-population has its own tree diagram that displays the results of its interaction with other subpopulations. Tree diagrams are usually used to illustrate a series of events, each node on the diagram represents an event [6]. The root of the diagram is
called the parent event and the probability associated with a node is the chance of that event occurring after the parent event occurs. The probability that the series of events leading to a particular node will occur is equal to the product of that node and its parents’ probabilities. Note here that we only include interactions that result in a change in the subpopulation, i.e. we chose to ignore affirmative interactions between \( q_{00} \) and \( q_{00} \) for example as they do not result in a change in any of the subpopulations. However - and this was quite counter-intuitive to figure out - interactions between \( q_{01} \) and \( q_{01} \) might actually lead to a change, and that is to be included in the tree diagram of \( q_{01} \).
Interactions with \( q_{00} \)

By XX meets YY we indicate that we are studying the effect of XX on possible actions of YY, where YY is the active agent. Note here that we see the indices independently, where the first number signifies covert opinion and the second is the overt opinion.

![Diagram](image)

Figure 1: Interactions with \( q_{00} \).
Interactions with $q_{01}$

In the first two cases, we notice that no punishment takes place. This is because there is an agreement in the overt opinions of the interacting agents. However, the agreement does not intercept with the probability of argument, as a person might still argue for his case to seek affirmation.

Figure 2: Interactions with $q_{01}$. 

an agreement in the overt opinions of the interacting agents. However, the agreement does not intercept with the probability of argument, as a person might still argue for his case to seek affirmation.
Interactions with $q_{10}$

Figure 3: Interactions with $q_{10}$. 

$P_{11}(\text{pun}), P_{10 \rightarrow 11}$

$1 - P_{11}(\text{pun}), P_{10 \rightarrow 11}$

$10$ meets $11$

$P_{11}(\text{pun}), P_{10 \rightarrow 11}$

$1 - P_{11}(\text{pun}), P_{10 \rightarrow 11}$

$11$ meets $01$

$P_{01}(\text{arg}), P_{10 \rightarrow 11}$

$1 - P_{01}(\text{arg}), P_{10 \rightarrow 11}$

$11$ meets $00$

$P_{00}(\text{arg}), P_{10 \rightarrow 00}$

$1 - P_{00}(\text{arg}), P_{10 \rightarrow 00}$

$00$ meets $10$

$P_{10}(\text{arg}), P_{10 \rightarrow 00}$

$1 - P_{10}(\text{arg}), P_{10 \rightarrow 00}$

$00$ meets $10$
Interactions with $q_{11}$

Figure 4: Interactions with $q_{11}$. 

11 meets 00

$P_{10}(\text{pun}). P_{11\rightarrow 10}$  
$1 - P_{10}(\text{pun}). P_{11\rightarrow 10}$

10  
00  
10  
00  
10

11  
01  
11  
01  
11

11 meets 10

$P_{10}(\text{pun}). P_{11\rightarrow 10}$  
$1 - P_{10}(\text{pun}). P_{11\rightarrow 10}$

10  
00  
10  
00  
10

11  
01  
11  
01  
11

$P_{10}(\text{arg}). P_{10\rightarrow 00}$  
$1 - P_{10}(\text{arg}). P_{10\rightarrow 00}$  
$P_{10}(\text{arg}). P_{11\rightarrow 01}$  
$1 - P_{10}(\text{arg}). P_{11\rightarrow 01}$
From the previous probability trees we can derive the following change equations in our populations of interest. We do that by collecting all influx to and outflow from a single population and assembling them in a single change equation.

\[ \frac{\Delta t_{00}}{\Delta t} = -q_{00} \cdot q_{11} \cdot P_{11}(pun) \cdot P_{00\rightarrow01} \cdot P_{11}(arg) \cdot P_{01\rightarrow11} \]
\[ -q_{00} \cdot q_{11} \cdot P_{11}(pun) \cdot P_{00\rightarrow01} \cdot (1 - P_{11}(arg) \cdot P_{01\rightarrow11}) \]
\[ -q_{00} \cdot q_{11} \cdot (1 - P_{11}(pun) \cdot P_{00\rightarrow01}) \cdot P_{11}(arg) \cdot P_{00\rightarrow10} \]
\[ -q_{00} \cdot q_{01} \cdot P_{01}(pun) \cdot P_{00\rightarrow01} \cdot P_{01}(arg) \cdot P_{01\rightarrow11} \]
\[ -q_{00} \cdot q_{01} \cdot P_{01}(pun) \cdot P_{00\rightarrow01} \cdot (1 - P_{01}(arg) \cdot P_{01\rightarrow11}) \]
\[ -q_{00} \cdot q_{01} \cdot (1 - P_{01}(pun) \cdot P_{00\rightarrow01}) \cdot P_{01}(arg) \cdot P_{00\rightarrow10} \]
\[ +q_{10} \cdot q_{00} \cdot P_{00}(arg) \cdot P_{01\rightarrow00} + q_{10} \cdot q_{10} \cdot P_{10}(arg) \cdot P_{10\rightarrow00} \]
\[ +q_{01} \cdot q_{10} \cdot P_{10}(pun) \cdot P_{01\rightarrow00} + q_{01} \cdot q_{00} \cdot P_{00}(pun) \cdot P_{01\rightarrow00} \]
\[ +q_{11} \cdot q_{00} \cdot P_{00}(pun) \cdot P_{11\rightarrow10} \cdot P_{00}(arg) \cdot P_{10\rightarrow00} \]
\[ +q_{11} \cdot q_{10} \cdot P_{10}(pun) \cdot P_{11\rightarrow10} \cdot P_{10}(arg) \cdot P_{10\rightarrow00} \]

\[ \frac{\Delta t_{01}}{\Delta t} = +q_{00} \cdot q_{11} \cdot (1 - P_{11}(pun) \cdot P_{00\rightarrow01}) \cdot P_{11}(arg) \cdot P_{00\rightarrow10} \]
\[ +q_{00} \cdot q_{01} \cdot (1 - P_{01}(pun) \cdot P_{00\rightarrow01}) \cdot P_{01}(arg) \cdot P_{00\rightarrow10} \]
\[ -q_{10} \cdot q_{11} \cdot P_{11}(pun) \cdot P_{10\rightarrow11} - q_{10} \cdot q_{01} \cdot P_{01}(pun) \cdot P_{10\rightarrow11} \]
\[ -q_{10} \cdot q_{00} \cdot P_{00}(arg) \cdot P_{10\rightarrow00} - q_{10} \cdot q_{10} \cdot P_{10}(arg) \cdot P_{10\rightarrow00} \]
\[ +q_{11} \cdot q_{00} \cdot P_{00}(pun) \cdot P_{11\rightarrow10} \cdot (1 - P_{00}(arg) \cdot P_{10\rightarrow00}) \]
\[ +q_{11} \cdot q_{10} \cdot P_{10}(pun) \cdot P_{11\rightarrow10} \cdot (1 - P_{10}(arg) \cdot P_{10\rightarrow00}) \]

\[ \frac{\Delta t_{01}}{\Delta t} = +q_{00} \cdot q_{11} \cdot P_{11}(pun) \cdot P_{00\rightarrow01} \cdot (1 - P_{11}(arg) \cdot P_{01\rightarrow11}) \]
\[ +q_{00} \cdot q_{01} \cdot P_{01}(pun) \cdot P_{00\rightarrow01} \cdot (1 - P_{01}(arg) \cdot P_{01\rightarrow11}) \]
\[ -q_{01} \cdot q_{11} \cdot P_{11}(arg) \cdot P_{01\rightarrow11} - q_{01} \cdot q_{01} \cdot P_{10}(pun) \cdot P_{01\rightarrow00} \]
\[ -q_{01} \cdot q_{00} \cdot P_{00}(pun) \cdot P_{01\rightarrow00} - q_{01} \cdot q_{01} \cdot P_{01}(arg) \cdot P_{01\rightarrow11} \]
\[ +q_{11} \cdot q_{00} \cdot (1 - P_{00}(pun) \cdot P_{11\rightarrow10}) \cdot P_{00}(arg) \cdot P_{11\rightarrow01} \]
\[ +q_{11} \cdot q_{10} \cdot (1 - P_{10}(pun) \cdot P_{11\rightarrow10}) \cdot P_{10}(arg) \cdot P_{11\rightarrow01} \]
Here we notice that our change equations may be simplified further, but we choose to keep them in this form in order to maintain their narrative value; this way we can directly discern the link between each line in the equations and its corresponding node in the tree diagrams.

2.2 Discussion of Discrete Time Dynamical Systems

Dynamical systems is the study of the long-term behavior of evolving systems [5]. It is a mathematical construction used to describe a system whose state evolves with time over a state space according to a fixed rule [14]. The theory of dynamical systems as we know it today was born at the end of the 19th century along with the struggles to answer the fundamental questions concerning the stability and evolution of the solar system. These questions led to the formulation of a vast and powerful scientific field, with a variety of applications to mathematics, physics, biology, chemistry, engineering, economics, social sciences, medicine, and many more [5].

One definition of a dynamical system could be stated as follows:

Definition

Given a state space $X$, a set of times $T$, and a rule (also called a map) $R$, a dynamical system is one whose state changes with time $t \in T$ over $X$ in a manner dictated by $R$ [13]. In particular, a time-independent discrete-time system would be defined by the equation $x(t + 1) = R(x(t)) \in X$.

We may notice that this is a very general description, and according to such definition we could talk about a dynamical system on virtually any space and using multiple different approaches to time measurement. In practice, however, the space will usually have additional structure, such as being a metric space or a smooth manifold [5]. Still, it is important to remember that the definition itself allows any sort of space and time to be considered.
There are two main types of dynamical systems, based on the character of the time variable: discrete and continuous. In this paper we analyze a discrete system, which can in principle be presented as the iteration of a function. One well-known example of such a system would be a logistic map. It highlights the possibility of complex, chaotic behavior arising from fairly simple initial conditions [14]. Continuous systems, on the other hand, are usually described by a differential equation [3]. An example of such a system is the visually pleasant Lorenz system (also known as the Lorenz attractor), well known for its chaotic solutions that can, when plotted, resemble a butterfly or figure eight [5].

For my model, the state space is

\[ X = \{(q_{00}, q_{01}, q_{10}, q_{11}) : q_{00}, q_{01}, q_{10}, q_{11} \geq 0 \text{ and } q_{00} + q_{01} + q_{10} + q_{11} = 1\} \]

Once a time step \( \Delta t \) is set, \( R \) is given by

\[
R(q_{00}, q_{01}, q_{10}, q_{11}) = (q_{00} + \frac{\Delta q_{00}}{\Delta t} \cdot \Delta t, q_{01} + \frac{\Delta q_{01}}{\Delta t} \cdot \Delta t, q_{10} + \frac{\Delta q_{10}}{\Delta t} \cdot \Delta t, q_{11} + \frac{\Delta q_{11}}{\Delta t} \cdot \Delta t) \]

One important idea we need to mention in relation to dynamical systems, which is especially significant in light of our work in this paper, is the concept of an equilibrium (also known as the fixed point). Equilibrium is, to put it shortly, the simplest possible solution to a dynamical system. For discrete systems, which are of special interest to us, it is a solution in which the state variable is constant [12].

**Definition**

\( \hat{x} \in X \) is an equilibrium point if

\[ \hat{x} = R(x) \]

Reaching a point of equilibrium means that the system will not change over time anymore; it will remain stable. For more in-depth technical descriptions of such phenomena, the reader can refer to the stability theory, which addresses precisely these topics: stability of solutions of differential equations and of trajectories of dynamical systems [11].
3 Simulations

I have written a MatLab programme that helps us calculate change in the subpopulations using our change equations. This programme requires various inputs such as initial values for all subpopulations and parameter values. The programme iterates the change equations over forty time steps and presents the results in the form of a graph displaying the roads different subpopulations take to get from initial values to final quantities.

We have run many simulations to investigate the behavior of the model and the impact of various parameter values on its stability. First we wanted to check for equilibrium dependence upon initial values, that we did by holding all parameter values at 0.5 and varying the initial values in several simulations. Our results showed a certain dependence upon initial values as the final quantities swayed in relevance to the different starting points. However, it should be noted that the values for the dissonant subpopulations ($q_{01}$ and $q_{10}$) always end up equal no matter the initial values chosen.

The impact that the different parameters have is somewhat intuitive to evaluate, they play the role of a catalyst in the change equations and lead to a faster equilibrium.

3.1 Simulation results

For a script of the MatLab programme that generated these results, see the appendix.

In simulation A, we wanted to generate a general case with all parameter values set to 0.5 and initial values set to $q_{00} = q_{01} = 0.2$, $q_{11} = q_{10} = 0.3$.

![Figure 5: Results of simulation A visualised with a MatLab plot.](image-url)
In simulation B we wanted to make a comparison with simulation A in order to decide whether there is a dependence on initial values, so in this simulation we have all parameter values set to 0.5 like in simulation A, but with different initial values, mainly, \( q_{00} = q_{10} = 0.2, \) \( q_{11} = q_{01} = 0.3 \)

Figure 6: Results of simulation B visualised with a Matlab plot.

In simulation C we wanted to see the effect of having equal dissonant proportions in the population, as we already know from simulations A and B that the subpopulation with the higher dissonant proportion will give an advantage to the corresponding consonant subpopulation. Here we set all parameter values to 0.5 and initial values \( q_{00} = 0.1, \) \( q_{11} = 0.4, \) \( q_{01} = q_{10} = 0.25 \). This simulation demonstrates that once we have equal dissonant values, consonant values will move in parallel.

In simulation D, we are interested in knowing what would happen if the entire population consisted solely of dissonant groups, will there be an emergence of consonant individuals? and will they eventually make a majority? Here, we simulate our programme for parameter values set to 0.5 and initial values as \( q_{00} = q_{11} = 0, \) \( q_{10} = 0.65, \) \( q_{01} = 0.35 \)
Figure 7: Results of simulation C visualised with a Matlab plot.

Figure 8: Results of simulation D visualised with a Matlab plot.
3.2 Conjectures

After obtaining the simulation results, we were able to make three conjectures, these conjectures are supported by simulations B, C and D respectively.

The result of the comparison between simulations A and B indicates that dissonant groups converge to the same value. This led us to our first conjecture:

- In equilibrium, the proportions of dissonant groups will end up at the same value.

Conjecture 1: if \( q = (q_{00}, \ldots, q_{11}) = R(q) \), then \( q_{01} = q_{10} \)

The result from simulation C reveals that the consonant groups will move in parallel once initial values for the dissonant ones are set equal. This led to our second conjecture:

- In dynamics, if we set equal the initial values of the dissonant groups, the consonant groups will move in parallel.

Conjecture 2: if \( q_{01} = q_{10} \), then \( \frac{\Delta q_{00}}{\Delta t} = \frac{\Delta q_{11}}{\Delta t} \)

The result from simulation D led us to the following conjecture:

- Starting with the entire population consisting of dissonant groups solely, the equilibrium will have a majority of consonant groups.

Conjecture 3: if \( q_{00} = q_{11} = 0 \), then, at equilibrium \( q_{00} + q_{11} > q_{01} + q_{10} \)

After making these conjectures, we wanted to systematically verify them. We did that by writing a programme in MatLab that varied the initial values in a thousand simulations.

Verifying Conjecture 1 meant setting action-type bias equal for all groups \( P_{CON}(arg) = P_{DIS}(arg) = P_{CON}(pun) = P_{DIS}(pun) = 0.5 \). Position-change bias was set to 0.7 for \( P_{DIS \rightarrow CON} \) and 0.5 for \( P_{CON \rightarrow DIS} \). Initial values were varied over a thousand simulations.

Conjecture 2 was verified by analytic treatment.

Conjecture 3 was verified by setting all action-type bias to 1 \( P_{CON}(arg) = P_{DIS}(arg) = P_{CON}(pun) = P_{DIS}(pun) = 1 \). Position-change bias was set to 0.5 for both \( P_{DIS \rightarrow CON} \) and \( P_{CON \rightarrow DIS} \) and initial values for consonant groups were set to 0 \( (q_{11} = q_{00} = 0) \). The program was run over 300 time steps with the initial values varied over a thousand simulations.

Programme results were consistent and all diversification of initial values resulted in verification of our three conjectures.
4 Analytic Treatment

In this section, we plan to present mathematical proofs of our conjectures and why they hold.

4.1 Conjecture One

"In equilibrium, the proportions of dissonant groups will end up at the same value."

In order to prove Conjecture 1, we use the contrapositive method, i.e., we negate both terms of our conditional statement and reverse the inference. This conjecture is equivalent to saying that as long as \( q_{01} \neq q_{10} \) then we are not at equilibrium.

We begin by looking at the change equations for \( q_{01} \) and \( q_{10} \).

\[
\frac{\Delta q_{01}}{\Delta t} = +q_{00} \cdot q_{11} \cdot (1 - P_{11}(\text{pun}) \cdot P_{00\rightarrow01}) \cdot P_{11}(\text{arg}) \cdot P_{00\rightarrow10} \\
+q_{00} \cdot q_{01} \cdot (1 - P_{01}(\text{pun}) \cdot P_{00\rightarrow01}) \cdot P_{01}(\text{arg}) \cdot P_{00\rightarrow10} \\
-q_{10} \cdot q_{11} \cdot P_{11}(\text{pun}) \cdot P_{10\rightarrow11} - q_{10} \cdot q_{01} \cdot P_{01}(\text{pun}) \cdot P_{10\rightarrow11} \\
-q_{10} \cdot q_{00} \cdot P_{00}(\text{arg}) \cdot P_{10\rightarrow00} - q_{10} \cdot q_{10} \cdot P_{10}(\text{arg}) \cdot P_{10\rightarrow00} \\
+q_{11} \cdot q_{00} \cdot P_{00}(\text{pun}) \cdot P_{11\rightarrow10} \cdot (1 - P_{00}(\text{arg}) \cdot P_{10\rightarrow00} \\
+q_{11} \cdot q_{10} \cdot P_{10}(\text{pun}) \cdot P_{11\rightarrow10} \cdot (1 - P_{10}(\text{arg}) \cdot P_{10\rightarrow00} \\
\]

(5)

\[
\frac{\Delta q_{10}}{\Delta t} = +q_{00} \cdot q_{11} \cdot P_{11}(\text{pun}) \cdot P_{00\rightarrow01} \cdot (1 - P_{11}(\text{arg}) \cdot P_{01\rightarrow11} \\
+q_{00} \cdot q_{01} \cdot P_{01}(\text{pun}) \cdot P_{00\rightarrow01} \cdot (1 - P_{01}(\text{arg}) \cdot P_{01\rightarrow11} \\
-q_{01} \cdot q_{11} \cdot P_{11}(\text{arg}) \cdot P_{01\rightarrow11} - q_{01} \cdot q_{01} \cdot P_{10}(\text{pun}) \cdot P_{01\rightarrow00} \\
-q_{01} \cdot q_{00} \cdot P_{00}(\text{pun}) \cdot P_{01\rightarrow00} - q_{01} \cdot q_{01} \cdot P_{01}(\text{arg}) \cdot P_{01\rightarrow11} \\
+q_{11} \cdot q_{00} \cdot (1 - P_{00}(\text{pun}) \cdot P_{11\rightarrow10}) \cdot P_{00}(\text{arg}) \cdot P_{11\rightarrow01} \\
+q_{11} \cdot q_{10} \cdot (1 - P_{10}(\text{pun}) \cdot P_{11\rightarrow10} \cdot P_{10}(\text{arg}) \cdot P_{11\rightarrow01}) \\
\]

(6)

If we set equal all parameter values we may simplify the two change equations. Position-change parameter can become \( P \) and Action-type parameter \( A \). Then we get the following simplified equations:

\[
\frac{\Delta q_{01}}{\Delta t} = +q_{00} \cdot q_{11} \cdot (1 - A \cdot P) \cdot A \cdot P \\
+q_{00} \cdot q_{01} \cdot (1 - A \cdot P) \cdot A \cdot P \\
-q_{10} \cdot q_{11} \cdot A \cdot P - q_{10} \cdot q_{01} \cdot A \cdot P \\
-q_{10} \cdot q_{00} \cdot A \cdot P - q_{10} \cdot q_{10} \cdot A \cdot P \\
+q_{11} \cdot q_{00} \cdot A \cdot P \cdot (1 - A \cdot P) \\
+q_{11} \cdot q_{10} \cdot A \cdot P \cdot (1 - A \cdot P) \\
\]

(7)
\[
\frac{\Delta q_{10}}{\Delta t} = +q_{00} \cdot q_{11} \cdot A \cdot P \cdot (1 - A \cdot P) + q_{00} \cdot q_{01} \cdot A \cdot P \cdot (1 - A \cdot P) - q_{01} \cdot q_{11} \cdot A \cdot P - q_{01} \cdot q_{01} \cdot A \cdot P - q_{01} \cdot q_{00} \cdot A \cdot P - q_{10} \cdot q_{10} \cdot (1 - A \cdot P) \cdot A \cdot P + q_{11} \cdot q_{00} \cdot (1 - A \cdot P) \cdot A \cdot P + q_{11} \cdot q_{10} \cdot (1 - A \cdot P) \cdot A \cdot P
\]

(8)

Our conjecture says that at equilibrium, proportions of \( q_{01} \) and \( q_{10} \) are equal. In dynamical systems, we know that at equilibrium, the change equations for these two groups will both be equal to zero, and since they will both be equal to zero, then they will also be equal to each other.

It is straightforward to see that once these two equations are set equal to each other, a few terms will be repeated on both sides of the equal sign, these similar terms will - once transferred to one side of the equal sign - add up to zero. After that simplification, we will have the following equations:

\[
\frac{\Delta q_{10}}{\Delta t} = \frac{\Delta q_{01}}{\Delta t} \tag{9}
\]

\[-q_{10} \cdot q_{11} \cdot A \cdot P - q_{10} \cdot q_{00} \cdot A \cdot P - q_{10} \cdot q_{10} \cdot A \cdot P = -q_{01} \cdot q_{11} \cdot A \cdot P - q_{01} \cdot q_{00} \cdot A \cdot P - q_{01} \cdot q_{01} \cdot A \cdot P
\]

We notice that a common factor in all terms is \( A \cdot P \) and since neither \( A \) nor \( P \) is equal to zero, we may divide all terms on both sides by \( A \cdot P \). This will leave us with

\[-q_{10} \cdot q_{11} - q_{10} \cdot q_{00} - q_{10} \cdot q_{10} = -q_{01} \cdot q_{11} - q_{01} \cdot q_{00} - q_{01} \cdot q_{01}
\]

(10)

It would be a good idea to get rid of all minus signs in this equations and group the corresponding terms together. We get

\[q_{10}^2 + q_{10} \cdot (q_{11} + q_{00}) = +q_{01}^2 + q_{01} \cdot (q_{11} + q_{00}) \tag{11}
\]

Now we can just solve the equation in terms of \( q_{10} \) and \( q_{01} \). We let \( q_{00} + q_{11} = c \), then we get

\[q_{10}^2 + q_{10}c = q_{01}^2 + q_{01}c
\]

\[(q_{01} + \frac{1}{2}c)^2 = (q_{10} + \frac{1}{2}c)^2
\]

Thus we have two solutions:

\[q_{10} = q_{01} \quad \text{or} \quad q_{10} = -q_{01} - c = -q_{01} - q_{11} - q_{00}
\]

(12)

Note that in our model \( q_{01}, q_{10}, q_{11}, q_{00} \in [0, 1] \) and \( q_{10} + q_{01} + q_{11} + q_{00} = 1 \). Thus, the only possible solution is \( q_{10} = q_{01} \), which proves our conjecture.
4.2 Conjecture Two

"In dynamics, if we set equal the initial values of the dissonant groups, the consonant groups will move in parallel."

To prove this conjecture we prove that the change equations of \( q_{11} \) and \( q_{00} \) become identical when we equate \( q_{01} \) and \( q_{10} \) and assume consonant parameters equal and dissonant ones likewise. To clarify; what we do here is unify \( P_{11}(pun) \) and \( P_{00}(pun) \) into \( P_{C}(pun) \), we do the same for \( P_{01}(pun) \) and \( P_{10}(pun) \) which becomes \( P_{D}(pun) \) and so on. We also denote position-change bias as either \( P_{C} \) for the cases where the final state is consonant and \( P_{D} \) for cases when the final state is dissonant.

\[
\begin{align*}
\frac{\Delta q_{00}}{\Delta t} &= -q_{00} \cdot q_{11} \cdot P_{C}(pun) \cdot P_{D} \cdot P_{C}(arg) \cdot P_{C} \\
&- q_{00} \cdot q_{11} \cdot P_{C}(pun) \cdot P_{D} \cdot (1 - P_{C}(arg) \cdot P_{C}) \\
&- q_{00} \cdot q_{11} \cdot (1 - P_{C}(pun) \cdot P_{D}) \cdot P_{C}(arg) \cdot P_{D} \\
&- q_{00} \cdot q_{01} \cdot P_{D}(pun) \cdot P_{D} \cdot P_{D}(arg) \cdot P_{C} \\
&- q_{00} \cdot q_{01} \cdot P_{D}(pun) \cdot P_{D} \cdot (1 - P_{D}(arg) \cdot P_{C}) \\
&- q_{00} \cdot q_{01}(1 - P_{D}(pun) \cdot P_{D}) \cdot P_{D}(arg) \cdot P_{D} \\
&+ q_{10} \cdot q_{00} \cdot P_{C}(arg) \cdot P_{C} + q_{10} \cdot q_{10} \cdot P_{D}(arg) \cdot P_{C} \\
&+ q_{01} \cdot q_{10} \cdot P_{D}(pun) \cdot P_{C} \cdot q_{01} \cdot P_{C}(pun) \cdot P_{C} \\
&+ q_{11} \cdot q_{00} \cdot P_{C}(pun) \cdot P_{D} \cdot P_{C}(arg) \cdot P_{C} \\
&+ q_{11} \cdot q_{10} \cdot P_{D}(pun) \cdot P_{D} \cdot P_{D}(arg) \cdot P_{C} \\
\end{align*}
\]

(13)

\[
\begin{align*}
\frac{\Delta q_{11}}{\Delta t} &= -q_{11} \cdot q_{00} \cdot P_{C}(pun) \cdot P_{D} \cdot P_{C}(arg) \cdot P_{C} \\
&- q_{11} \cdot q_{00} \cdot P_{C}(pun) \cdot P_{D} \cdot (1 - P_{C}(arg) \cdot P_{C}) \\
&- q_{11} \cdot q_{00} \cdot (1 - P_{C}(pun) \cdot P_{D}) \cdot P_{C}(arg) \cdot P_{D} \\
&- q_{11} \cdot q_{10} \cdot P_{D}(pun) \cdot P_{D} \cdot P_{D}(arg) \cdot P_{C} \\
&- q_{11} \cdot q_{10} \cdot P_{D}(pun) \cdot P_{D} \cdot (1 - P_{D}(arg) \cdot P_{C}) \\
&- q_{11} \cdot q_{10}(1 - P_{D}(pun) \cdot P_{D}) \cdot P_{D}(arg) \cdot P_{D} \\
&+ q_{10} \cdot q_{11} \cdot P_{C}(pun) \cdot P_{C} + q_{10} \cdot q_{01} \cdot P_{D}(pun) \cdot P_{C} \\
&+ q_{10} \cdot q_{11} \cdot P_{C}(arg) \cdot P_{C} + q_{10} \cdot q_{01} \cdot P_{D}(arg) \cdot P_{C} \\
&+ q_{00} \cdot q_{11} \cdot P_{C}(pun) \cdot P_{D} \cdot P_{C}(arg) \cdot P_{C} \\
&+ q_{00} \cdot q_{01} \cdot P_{D}(pun) \cdot P_{D} \cdot P_{D}(arg) \cdot P_{C} \\
\end{align*}
\]

(14)

Upon a quick examination of the two change equations we can indeed confirm that they are identical to the extent that it validates our conjecture and proves that \( q_{11} \) and \( q_{00} \) move in parallel.
Conclusions

A qualitative discussion of our conjectures reveals a certain symmetry in the movements of our overt and covert groups. This symmetry is present even regardless of the biases which we have set to 0.5 in order to avoid its effect on our model. However - and this is important to point out - our conjectures hold as long as the biases are set to 0.5, further simulations and inquiries would have to be made before we can generalize our conjectures to include all bias values. Because the symmetry we see between the consonant and dissonant population is not a result of cognitive dissonance, this points us to the dynamics instead, indicating a balance between the depletion of dissonant population to the advantage of the consonant one.

In conclusion, we have briefly explored the landscape of attitude change modeling. We gave a brief theoretical introduction and discussed how our work fits into the existing literature. We then described our mathematical model of attitude change, created within the discrete-time dynamical systems framework, which allows for the possibility of covert and overt opinions. We were able to propose and empirically verify through simulation the following three conjectures. Firstly, it could be observed that in equilibrium, the proportions of dissonant groups will end up at the same value. Secondly, in dynamics, if we set equal the initial values of dissonant groups, the consonant groups will move in parallel. Thirdly, we saw that starting with the entire population consisting of dissonant groups solely, the equilibrium will still have a majority of consonant groups. We were able to analytically prove the first two conjectures. There are a few possible directions for expanding on the topic. For instance, it could be interesting to think about mechanisms which would helpfully or partially account for the psychological forces which might act against dissonance of overt and covert opinions, an intricate model would need to be built in order to encompass the complexities and nuances of different ways to process cognitive dissonance. Social psychology states for example that we reduce dissonance by either changing one of our attitudes (which we modelled in this thesis), by acquiring new information that outweighs our dissonant beliefs, or by reducing the importance of the conflicting belief [10]. These details could all be included in the modelling process - albeit adding a considerable amount of complexity to the model. Moreover, seeing as this model has been simplicitive in terms of population generalisations, a future model could take into account individual differences and adjust accordingly, or should we say even more dynamically, in its mappings and simulations.

However, it is important to keep in mind that however much we try to emulate and simulate these interactions, and however much we aim to replicate a real life situation and an exchange of opinions, facts or emotions, we will still find it extremely difficult to accurately predict outcomes at any non-aggregate level. However, we might still be able to predict trends and behavioural patterns of large enough groups of people.
Bibliography

References


Appendix: MatLab code

Code for simulating our dynamical system

```
% action-type bias:
p_00san = 0.5;
p_01san = 0.5;
p_00arg = 0.5;
p_01arg = 0.5;
p_11san = 0.5;
p_10san = 0.5;
p_11arg = 0.5;
p_10arg = 0.5;

% position-change bias:
p_00 = 0.5; % change to consonant states
p_01 = 0.5; % change to dissonant states

% initial values
q00(1) = 0.7;
q10(1) = 0;
q01(1) = 0;
q11(1) = 0.3;
t = 40;

for i = 2:t+1;
    q00(i) = q00(i-1) - q00(i-1) * q11(i-1) * (p_11san * p_01) * (p_11arg * p_00) - ...
        q00(i-1) * q11(i-1) * (p_11san * p_01) * (1 - (p_11arg * p_00)) - ...
        q00(i-1) * q11(i-1) * (1 - (p_11san * p_01)) * (p_11arg * p_01) - ...
        q00(i-1) * q01(i-1) * (p_01san * p_01) * (p_01arg * p_00) - ...
        q10(i-1) * q00(i-1) * p_00arg * p_00 + q10(i-1) * q10(i-1) * p_01arg * p_00 + ...
        q01(i-1) * q10(i-1) * p_10san * p_00 + q01(i-1) * q00(i-1) * p_00san * p_00 + ...
        q11(i-1) * q00(i-1) * p_00san * p_01 * p_00arg * p_00 + ...
        q11(i-1) * q10(i-1) * p_10san * p_01 * p_10arg * p_00;

    q10(i) = q10(i-1) + q00(i-1) * q11(i-1) * (1 - (p_11san * p_01)) * (p_11arg * p_00) + ...
```

28
\[ q_0(i) = q_0(i-1) + q_00(i-1) * q_11(i-1) * p_{11}\text{san}*p_{01} * (1 - (p_{11}\text{arg}*p_{00})) + \ldots \]

\[ q_0(i) = q_0(i-1) + q_01(i-1) * q_10(i-1) * p_{10}\text{san}*p_{00} + \ldots \]

\[ q_1(i) = q_1(i-1) + q_11(i-1) * p_{11}\text{san}*p_{01} * (1 - (p_{11}\text{arg}*p_{00})) + \ldots \]

\[ q_1(i) = q_1(i-1) + q_10(i-1) * p_{10}\text{san}*p_{00} + \ldots \]

```
end
plot(q00, 'red');
hold on;
plot(q01, 'blue');
hold on;
plot(q10, 'yellow');
hold on;
```
Code for verifying the first conjecture

```matlab
%Script modified to check the 1st conjecture
clear all;

%action-type bias:
p_00san = 0.5;
p_01san = 0.5;
p_00arg = 0.5;
p_01arg = 0.5;
p_11san = 0.5;
p_10san = 0.5;
p_11arg = 0.5;
p_10arg = 0.5;

%position-change bias:
p_00 = 0.7; %change to consonant states
p_01 = 0.5; %change to dissonant states

%initial values
% In this iteration of the script we want to vary the initial values.
% For the time being, we will use random numbers as a method of
% variation.
n = 1000;
conj1 = zeros(n,1);
for k = 1:n
    qx = randfixedsum(4,1,1,0,1);
    q00(1) = qx(1);
    q01(1) = qx(2);
    q10(1) = qx(3);
    q11(1) = qx(4);
    t = 400;
    for i = 2:t+1;
        q00(i) = q00(i-1) - q00(i-1)*q11(i-1)*(p_11san*p_01)*(p_11arg*p_00)
            - ... q00(i-1)*q11(i-1)*(p_11san*p_01)*(1 - (p_11arg
            *p_00)) - ... q00(i-1)*q11(i-1)*(1 - (p_11san*p_01))*(p_11arg*p_01)
            - ... q00(i-1)*q01(i-1)*(p_01san*p_01)*(p_01arg*p_00)
            ) - ...
```
\[
q_00(i) = q_00(i-1) + q_01(i-1) * (1 - (p_01san * p_00)) * (p_01arg * p_01) + ...
\]

\[
q_01(i) = q_01(i-1) + q_00(i-1) * q_11(i-1) * (1 - (p_11san * p_01)) * (p_01arg * p_01) + ...
\]

\[
q_11(i) = q_11(i-1) + q_00(i-1) * q_11(i-1) * (1 - (p_00san * p_01)) * (p_00arg * p_00) + ...
\]
q_{11}(i-1)q_{10}(i-1)p_{10\text{san}}p_{01}p_{10\text{arg}}p_{00} - ...
q_{11}(i-1)q_{10}(i-1)p_{10\text{san}}p_{01}(1 - (p_{10\text{arg}}p_{00})) - ...
q_{11}(i-1)q_{10}(i-1)(1 - (p_{10\text{san}}p_{01}))p_{10\text{arg}}p_{01};

\textbf{end}

x(k) = q_{00}(t);
y(k) = q_{11}(t);
plot(x(k), y(k), '*'); hold on

if abs(q_{01}(t) - q_{10}(t)) < 0.000001
conj1(k) = 1;
\textbf{end}

Conjecture_1 = \text{sum(conj1)}/n
Code for verifying third conjecture

clear all;
%Script modified to check the 3rd conjecture
%action-type bias:
p_00san = 1;
p_01san = 1;
p_00arg = 1;
p_01arg = 1;
p_11san = 1;
p_10san = 1;
p_11arg = 1;
p_10arg = 1;

%position-change bias:
p_00 = 0.5; %change to consonant states
p_01 = 0.5; %change to dissonant states

%initial values
% In this iteration of the script we want to vary the initial values.
% For the time being, we will use random numbers as a method of
% variation.
n = 1000;
conj3 = zeros(n,1);
for k = 1:n
    qx = randfixedsum(2,1,1,0,1);
    q01(1) = qx(1);
    q10(1) = qx(2);
    q11(1) = 0;
    q00(1) = 0;
    t = 300;
    for i = 2:t+1;
        q00(i) = q00(i-1) - q00(i-1)*q11(i-1)*(p_11san*p_01)*(p_11arg*p_00) - ...  

        q00(i-1)*q11(i-1)*(p_11san*p_01)*(1 - (p_11arg*p_00)) - ...  
        q00(i-1)*q11(i-1)*(1 - (p_11san*p_01))*(p_11arg*p_01) - ...  
        q00(i-1)*q01(i-1)*(p_01san*p_01)*(p_01arg*p_00) - ...  
        q00(i-1)*q01(i-1)*(p_01san*p_01)*(1 - (p_01arg*p_00)) - ...  
        q00(i-1)*q01(i-1)*(1 - (p_01san*p_01))*(p_01arg*p_01) + ...  
        q10(i-1)*q00(i-1)*p_00arg*p_00 + q10(i-1)*q10(i-1)*p_10arg*p_00 + ...
q01(i-1)\cdot q10(i-1)\cdot p_{10san} \cdot p_{00} + q01(i-1)\cdot q00(i-1)\cdot p_{00san} \cdot p_{01} \cdot p_{00arg} \cdot p_{00} + ... \\
q11(i-1)\cdot q00(i-1)\cdot p_{00san} \cdot p_{01} \cdot p_{00arg} \cdot p_{00} + ... \\
q11(i-1)\cdot q10(i-1)\cdot p_{10san} \cdot p_{01} \cdot p_{10arg} \cdot p_{00}; \\

q10(i) = q10(i-1) + q00(i-1) \cdot q11(i-1) \cdot (1-(p_{11san} \cdot p_{01})) \cdot p_{11arg} \cdot p_{01} + ... \\
q00(i-1)\cdot q01(i-1)\cdot (1-(p_{01san} \cdot p_{01})) \cdot p_{01arg} \cdot p_{01} - ... \\
q10(i-1)\cdot q11(i-1)\cdot p_{11san} \cdot p_{00} - q10(i-1)\cdot q01(i-1)\cdot p_{01san} \cdot p_{00} - ... \\
q10(i-1)\cdot q00(i-1)\cdot p_{00arg} \cdot p_{00} - q10(i-1)\cdot q10(i-1)\cdot p_{10arg} \cdot p_{00} + ... \\
q11(i-1)\cdot q00(i-1)\cdot p_{00san} \cdot p_{01}\cdot (1-(p_{00arg} \cdot p_{00})) + ... \\
q11(i-1)\cdot q10(i-1)\cdot p_{10san} \cdot p_{01}\cdot (1-(p_{10arg} \cdot p_{00})); \\

q01(i) = q01(i-1) + q00(i-1) \cdot q11(i-1) \cdot p_{11san} \cdot p_{01}(1-(p_{11arg} \cdot p_{00})) + ... \\
q00(i-1)\cdot q01(i-1)\cdot p_{01san} \cdot p_{01}(1-(p_{01arg} \cdot p_{00})) - ... \\
q10(i-1)\cdot q11(i-1)\cdot p_{11arg} \cdot p_{00} - q10(i-1)\cdot q01(i-1)\cdot p_{01san} \cdot p_{00} - ... \\
q01(i-1)\cdot q00(i-1)\cdot p_{00san} \cdot p_{00} - q01(i-1)\cdot q01(i-1)\cdot p_{01arg} \cdot p_{00} + ... \\
q11(i-1)\cdot q00(i-1)\cdot (1-(p_{00san} \cdot p_{01})) \cdot (p_{00arg} \cdot p_{01}) + ... \\
q11(i-1)\cdot q10(i-1)\cdot (1-(p_{10san} \cdot p_{01})) \cdot (p_{10arg} \cdot p_{01}); \\

q11(i) = q11(i-1) + q00(i-1) \cdot q11(i-1) \cdot p_{11san} \cdot p_{01} \cdot p_{11arg} \cdot p_{00} + ... \\
q00(i-1)\cdot q01(i-1)\cdot p_{01san} \cdot p_{01} \cdot p_{01arg} \cdot p_{00} + ... \\
q10(i-1)\cdot q11(i-1)\cdot p_{11san} \cdot p_{00} + q10(i-1)\cdot q01(i-1)\cdot p_{01san} \cdot p_{00} + ... \\
q01(i-1)\cdot q11(i-1)\cdot p_{11arg} \cdot p_{00} + q01(i-1)\cdot q01(i-1)\cdot p_{01arg} \cdot p_{00} - ... \\
q11(i-1)\cdot q00(i-1)\cdot p_{00san} \cdot p_{01} \cdot p_{00arg} \cdot p_{00} - ... \\
q11(i-1)\cdot q00(i-1)\cdot p_{00san} \cdot p_{01}(1-(p_{00arg} \cdot p_{00})) - ... \\
q11(i-1)\cdot q00(i-1)\cdot (1-(p_{00san} \cdot p_{01})) \cdot p_{00arg} \cdot p_{01} - ... \\
q11(i-1)\cdot q10(i-1)\cdot p_{10san} \cdot p_{01} \cdot p_{10arg} \cdot p_{00} - ... \\
q11(i-1)\cdot q10(i-1)\cdot p_{10san} \cdot p_{01}(1-(p_{10arg} \cdot p_{00})) - ...
q_{11}(i-1)q_{10}(i-1)(1 - (p_{10_{san}}p_{01}))p_{10_{arg}}
p_{01};

end

if q_{00}(t)+q_{11}(t) > q_{01}(t)+q_{10}(t)
    conj3(k) = 1;
    maj(k) = q_{00}(t)+q_{11}(t);
end

Conjecture_3 = \frac{\text{sum(conj3)}}{n}
min\_majority = \text{min(maj)}