Negative Interest Rates Effect Economic Stability

Bachelor Thesis in National Economics NAA303 15 hp

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“You can read Adam Smith, you can read John Keynes, you can read anybody and you can’t find a word to my knowledge on prolonged zero interest rates. That is a phenomenon nobody dreamed would ever happen” furthermore ”Doesn’t mean world will end, but nobody knows full implications”

Warren Buffett in an interview, 29/4-2016
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Abstract

Today’s monetary policy is a historic one, where the introduction of negative interest rates has started a new “age” of unconventional monetary policy and some argue that there is a need for further unconventional monetary tools. The purpose of this thesis is to analyze negative interest rates, how they came to be, what long-term effect they have on economic stability and if its possible to get out. We do this by analyzing existing theoretical and empirical research, including a theoretical model based on household consumption, a cost of money function and an illustration of the liquidity trap. Thereby the thesis concludes that the short term positive effects of negative interest rate policy get exhausted in the long-term as the negative effects increase over time, thus creating an environment of excessive borrowing both by consumers and governments that might lead to instability and economic downturn in the long-term. Furthermore, the negative interest rate policy is creating a difficulty of getting out of the negative interest rate environment because the consumers and the firms have gotten used to the ”cheap money” and might have hard time financing day to day operations in normal interest rate world.

**Key words:** negative interest rates, central banks, monetary policy.
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<td>NIRP</td>
<td>Negative Interest Rate Policy</td>
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<td>ECB</td>
<td>European Central Bank</td>
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<td>DNB</td>
<td>Denmark National Bank</td>
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<td>SNB</td>
<td>Swiss National Bank</td>
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<td>SRB</td>
<td>Swedish Riksbank</td>
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<td>BOJ</td>
<td>Bank of Japan</td>
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<td>MNB</td>
<td>Magyar Nemzeti Bank</td>
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<td>FOMC</td>
<td>Federal Open Market Committee</td>
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<td>QE</td>
<td>Quantitative Easing</td>
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<td>CBI</td>
<td>Central Bank Independence</td>
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<td>ELB</td>
<td>Effective Lower Bound</td>
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Chapter 1

Introduction

No one will lend at a negative interest rate, potential creditors will simply choose to hold cash, which pays zero nominal interest. — Ben Bernanke, Former Chairman of the Federal Reserve, 2009

Despite being an uncharted form of monetary policy and for some economist being something unthinkable, six Central Bank’s around the world have resorted to NIRP (Negative Interest Rate Policy), in response to the deep recession of 2008. Denmark’s National-bank (DNB), being the first to implement NIRP, was followed by European Central Bank (ECB), Swiss National Bank (SNB), Swedish Riksbank (SRB), Bank of Japan (BOJ) and, most recently Hungary’s Magyar Nemzeti Bank (MNB). However, the motivations of the unconventional monetary policy implementation across all six central banks is consistent with those of trying to stabilize the currency, ”fight the inflation” (ie. fight the low inflation) and/or increase growth (Jurksas, 2017). Additionally, there exists a motive for small countries to implement NIRP, in an effort to depreciate their respective currencies and gain a competitive edge and boosting exports.

Central banks’ monetary policy decisions are guided by the targeting of inflation and unemployment. Those targets are accomplished by variations in money supply, interest rate adjustment and most recently QE (Quantitative easing)\(^1\). These monetary tools are used in an attempt to reach optimal levels of inflation and unemployment (P. Giannoni and Woodford, 2003). The effects of NIRP are highly debated among the academic society and a conclusion seems far from reached, because of the many transmission channels by which this monetary policy influences the economy. For instance, isolating the impact of monetary policy on the economic output is difficult since there exists a lag between monetary policy and the effect on economic output (Gruen et al. 1999: Batini and Nelson, 2001).

\(^1\)Quantitative easing is a monetary policy where central banks buy government bonds and financial instruments in an attempt to stimulate the economy.
Furthermore, FED (Federal Reserve System of US) is a major central bank that has not introduced NIRP but has introduced QE worth of 2 trillion USD in asset buying since the late 2013 as a response to the crisis (Blot, et al, 2015). Since the US economy is arguably the biggest in the world, any action by the FED is newsworthy and needed to be analyzed by its trading partners, in terms of assessing the impact of FED’s decisions on its own economy. Since then, the FED has started increasing the interest rate at an accelerated rate reaching 1.75 percent as of March 21, 2018\(^2\), meaning that the cycle of low interest rates seems to be over in the US and that the monetary ”normalization” cycle has begun for the US. Additionally, Blake and Gros, (2015) argue that the relationship between ECB and FED is strong in terms of setting similar interest rates in times of crises. This suggests that the FED seems to be leading the ECB in economic crises. Blake and Gros also note that the relationship most of the time is symmetric. In summary, the period of negative interest rates might be coming to an end if the ECB and the other smaller central banks follow the FED’s ”normalization” policy.

Popular believes are that NIRP and the QE program were successful and that the central banks are fully in control and aware of the variables within the respective economies. This notion has been largely disapproved by Fligstein, et al, (2014). Fligstein, et al, (2014) tries to evaluate and account for the build-up of the 2008 crisis by analysing the Federal Reserves main decision-making body, the Federal Open Market Committee (FOMC)\(^3\). Fligstein, et al, conclude that the different issues\(^4\) in the wake of the 2008 crisis, modelled by FOMC transcripts, remained a separate discussion and were never connected together with the crisis. This brings forward an impression of the central banks lacking the ability to see ahead and make sensible monetary decisions, making the Bernanke (2009) quote at the beginning of the chapter, insisting that no one would lend at negative interest rates, highly questionable and proven inaccurate in hindsight.

\(^2\)Federal Reserve Economic Data FRED — St. Louis Fed.
\(^3\)The Federal Reserve controls the three tools of monetary policy–open market operations, the discount rate, and reserve requirements. The Board of Governors of the Federal Reserve System is responsible for the discount rate and reserve requirements, and the Federal Open Market Committee is responsible for open market operations. Using the three tools, the Federal Reserve influences the demand for, and supply of, balances that depository institutions hold at Federal Reserve Banks and in this way, alters the federal funds rate. The federal funds rate is the interest rate at which depository institutions lend balances at the Federal Reserve to other depository institutions overnight. More information can be found at www.federalreserve.gov

\(^4\)Primarily being the housing market and more specifically sub-prime mortgage backed securities, which were derivatives of the US housing market. That traded globally and created a global banking crises when they imploded in 2008.
1.1 Purpose and Research Question

The background of this thesis is an interesting macroeconomic topic that has been widely debated. This thesis suggests research on negative interest rate, since interest rates impact all sectors directly or indirectly and the world has never seen this type of monetary policy on this scale before. It becomes important to know the full extent of negative interest rates, especially since it carries many unknown factors that can become either positive or negative impacts on the economic productivity in the long-run. Therefore, this thesis suggest in depth research of negative interest rates, how they came to be, what it’s effects are on the economic stability and how to possibly get out in the future.

- How do negative interest rates affect/impact long-term economic stability?
  - How did negative interest rates come to be?
  - How to get out the negative interest rates?

1.2 Method and Limitations

1.2.1 Limitations

Interestingly, so far there is no academic literature (to our knowledge) that fully describes NIRP and its long-term effects to a satisfactory extent. In this thesis, we mainly focus on NIRP and briefly mention QE to get a broader sense of the monetary policy and the central banks actions. NIRP is a new phenomenon and it might be hard to draw a full conclusion of the effect from a short time span because of the time lagging indicators and variables. Furthermore, Hungary is omitted from the research since they have only recently implemented NIRP and there is not enough data to do a through analysis.

The countries that were analyzed from the viewpoint of ”zombie firms”\textsuperscript{5} were Greece, Spain, Ireland, Portugal, Slovenia, Germany and France, meaning that in terms of ”zombie firms” this thesis can only speak about those countries. Since those seven countries belong to the euro area they were also analyzed more broadly by Jurksas (2017) and

\textsuperscript{5}A firm that is heavily indebted and is only able to pay interest on the debt but unable to repay the principle. These companies are only able to survive in a low interest rate environment or negative interest rate environment.
included in our empirical study. Additionally, the data gathered by Jurksas, (2017) omits in some cases Japan and there cannot be anything said about the effect of NIRP on that specific sector in Japan.

1.2.2 Outline

The thesis will start by introducing the problem and briefly mention relevant information about the subject. Thereafter, the thesis introduces the purpose of this thesis and the research questions before introducing extensive, relevant and interesting literature. The literature review presents the current conditions of monetary policy, how it got to this point and some of the suggested solutions by Bernanke and Rogoff. Additionally, the literature review presents other views on the topic so to get a broader understanding of the topic and the different viewpoints, such as famous ideas by Milton Friedman and Friedrich Hayek.

The third chapter of this thesis contains a theoretical section with some recently developed models by Walsh. He tries to develop mathematical formulations and models that explain household consumption, cost of money function and liquidity trap that get changed within a NIRP environment. The theoretical part will be used as a helping tool for better understanding of NIRP and interpretation from the results in the empirical section.

The fourth chapter of the thesis contains an empirical section that illustrates the different economic sectors in graph form and points out where the NIRP was introduced and how that sector has performed before the introduction and after the introduction of NIRP. A big critique to this method is that it is largely dependent and based on research from Jurksas, (2017), Storz et al. (2017) and their data gathering, approximations and calculations. Moreover, if the authors (Storz et al. and Jurksas) made some mistakes in the calculations, data gathering, data processing and other errors, that could lead to misinformed conclusion in this thesis. Because this thesis builds on the research of the authors (Storz et al. and Jurksas) and if their data is flawed we will ultimately make a wrong interpretation of their findings and make a misinformed conclusion based on that data.

\footnote{Jurksas (2017) and Storz et al. (2017) came to their findings based on data gathered from Thomson Reuters, Statistical Data Warehouse, IMF Financial Soundness Indicator, and Amadeus/Orbis database respectively.}
The conclusion summarizes the findings from empirical, theoretical model and literature review and concludes the thesis. This chapter will focus on answering the purpose and research questions, in such a manner that it’s easy to follow and understand.
Chapter 2

Literature Review

Negative interest rates hurt banks’ balance sheets, with the ‘wealth effect’ on banks overwhelming the small increase in incentives to lend. — Joseph Stiglitz, April 13, 2016

This part of the thesis will focus and place weight on existing research in the field of NIRP. The first part of this chapter will present current conditions of monetary policy, followed by additional monetary policy tools, proposition as viable monetary policy tools and finally other views so that the literature review receives a broad viewpoints on the subject.

Firstly, let us make a clear definition of the meaning of negative interest rates. Having a negative interest means that the policy rate of the central bank is negative. This means that the depositors will have to pay interest for the privilege of storing the money in a private bank or for big institutions and private banks to pay a central bank interest for keeping their money with that central bank. The design of negative interest is to incentivize banks to lend/invest more freely, private people to invest, lend and spend more freely so not to pay a fee on the money in bank and a purpose may also be to depreciate the countries exchange rate.

2.1 The Current Conditions for Monetary Policy

The current economic and monetary conditions, left after the deep recession of 2008, forced the central banks into a sharp monetary policy easing following and during the recession. The recession left the problems of low nominal interest rates, low inflation, low growth and the questioning of central bank independence (CBI) (Bernanke, 2017). The main reason for the currently low nominal interest rate is central bank’s response to the recession of 2008, where the central banks lowered the interest rate to historic lows and some central banks even went into negative territory. The most considerable problem was that the central banks were not able to reduce short-term interest rate
"enough" below zero, without creating the circumstance of a major cash withdrawal. This creates a problem of limited capabilities on the part of central banks in coping with the next recession. The conditions of 2008 created a need for further and more decisive action by the central banks, hence the introduction of QE and NIRP by some of the world’s biggest central banks as mentioned previously. Bernanke, (2017) argues that the conventional monetary policy now faces challenges and that unconventional tools like QE and forward guidance, "open mouth operations", can provide a significant boost to the already existing monetary policy tool kit. Furthermore, Bernanke, (2017) and Rogoff, (2017) argue that "open mouth operations" can influence the future market expectations and affect financial conditions today.

The QE program and NIRP have been widely controversial and its implementation has been divergent in the sense that different central banks have different rules governing them. The FED by law was able to purchase only Treasury securities and mortgage-backed securities. The other central banks, like ECB, were able to buy a wide range of securities ranging from corporate bonds and as far as equity (Bernanke, 2017). The FED has in turn received several congressional letters raising the concerns about QE program effect on currency and inflation. Additionally, the Brazilian and German finance ministers criticized the QE program, but the Brazilian finance minister Guido Mantega stating that QE program is designed to affect the exchange rate market and cheapen the currency across the board and saying that the FED had initiated a "Currency war". "We are in the midst of an international currency war, a general weakening of currency. This threatens us because it takes away our competitiveness," Brazilian finance minister Guido Mantega, (2010).

2.1.1 Additional Monetary Policy Tools

Having additional tools, such as NIRP, QE and forward guidance, may still be insufficient to offset a severe economic downturn (Bernanke, 2017: Rogoff, 2017). Rogoff, (2017) argues that despite all the "stability", the centre of monetary policy has never seen this level of intellectual turmoil. Not even at the brake-up of the Bretton Woods System\(^1\) that coincided in the early 1970’s, as the monetary policy went off the gold

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\(^1\)Bretton Woods System was a system where countries exchange rate was pegged to the US dollar and the dollar was convertible into gold at an exchange rate of 35 dollars/oz. The system was regulated by the International Monetary Fund (IMF) and lasted from 1945 to 1971.
Moreover, Rogoff (2017) argues that central banks have already initiated planning, studies and experimentation of expanding the monetary policy kit, suggesting it would be counterproductive to introduce a radical new monetary policy in a brewing recession. Especially since the new monetary policy may need inclusion of effective forward guidance so not to lose the confidence and create more panic.

There is substantial amount of empirical evidence and models on the recessionary periods from the mid-1950s until now. That suggest that FED has cut its key interest rate on average by 5.5 percent to fight past recessions (Yellen, 2016). Taking into account that this is the second longest expansion on record\(^2\), and that the US interest rate is currently at 1.5-1.75 percent while the ECB and the other central banks are still in negative territory, making it difficult to cut key interest rate if a recession should occur in the near future. Rogoff (2017) argues that the lower the starting interest rate when a recession hits, the higher the probability that central banks need to go further below the zero bound. He claims that if central banks go to negative it will cross a threshold where the flight to paper currency will increase since paper money does not pay interest, positive or negative. Vinals, Gray and Eckhold (2016) argue that the tipping point where people would move into paper currency would come in the range of minus 0,75 percent to minus 2 percent. Additionally, very few banks within the NIRP regions have introduced negative deposit rates, either unwilling or unable to do so in fear of capital outflow which would lead to net interest margins\(^3\) being crushed.

Both Bernanke (2017) and Rogoff (2017) suggest and entertain the possibility and fall-out of raising the inflation target to 4 percent as a new monetary mandate, instead of the current 2 percent. Bernanke (2017) proposes temporary price-level targeting approach, meaning that the inflation would be calculated on an average. Under recessionary time, the mandate would be to raise the inflation target and under expansionary time lower inflation target, therefore, making the inflation an average of 2 percent over time. Bernanke (2017) also recognizes that there are difficulties with this proposition such as communication policy and need of making changes to the current framework of the FED. Rogoff (2017) second suggestion approach to solving the shortcoming of existing monetary policy tools with going further into negative territory with interest rates. Both of those suggestions being radical and controversial, since the effects of these

\(^2\)Federal Reserve Economic Data — FRED — St. Louis Fed

\(^3\)Net interest margin is a measure of how successfully a bank/firm is at lending in comparison to their cost.
monetary policies on peoples psychology and the threshold where people lose confidence in the central bank is still unknown and therefore creating problems (Rogoff, 2017). Additionally, he recognizes the difficulties in implementing large negative interest rates. It is not something done overnight with all the barriers facing the lower zero bound. He goes on to summarize that deep NIRP would create stress on the commercial banks. Since the banks would have an ever-increasing difficulty of transmitting the cost of negative interest on the wholesale consumer, as illustrated in Europe, where banks have been reluctant to do so for the small depositors.

2.1.2 Propositions as Viable Monetary Policy Tools

The first proposition from Bernanke, (2017) and Rogoff, (2017) is to raise target inflation from 2 to 4 percent. The idea being that central banks would perhaps have 2 extra percent of nominal interest rate to cut in a recessionary period. Proposing the rising inflation target is plausible and viable but it creates several problems. Firstly, over the past two decades central banks have convinced the public of the benefit of 2 percent inflation, it would then raise a question of what is stopping the central banks at 4 percent and central bank would suffer loss in confidence (Rogoff, 2017). Subsequently, he suggests that there is a fundamental difference between 2 and 4 percent inflation on psychology, since the working people would need to do more in terms of protecting against inflation than before when the inflation was at 2 percent.

The second proposition from Rogoff, (2017) is an attempt to adding additional tools to the monetary policy toolbox, such as going deeper into negative interest rates. He goes on to suggest that its much easier to implement negative interest today than in Keynes\textsuperscript{4} time. With all the electrical advantages and efficiency’s surrounding the paper currency that exist today that did not exist back then. The problem of dealing with run into paper currency can easily be avoided if a central bank needs to go deeper into negative territory. The first involves getting rid of all large notes from the paper money circulation which creates more costs for large institutions that need to transport, store and insure larger amount of small denominated currency. The second, creating a pegged exchange rate between electronic currency and paper currency, which might be little

\textsuperscript{4}John Maynard Keynes was an economist and a philosopher that lived 1883-1946. Mostly known for his contributions to economic science with his General Theory. Where Keynes arguments for fiscal policy combined with monetary policy which conflicted with economic views of his time.
harder to implement and sell to the public. The third, moving into a cashless society which makes it easy to pay interest (positive or negative) on electronic currency since there is no alternative currency. The fourth being a technological solution to interest payment (positive or negative) on paper currency (Rogoff, 2017). In conclusion, Rogoff, (2017) states: "Eliminating the zero bound will not make an aging economy young, nor will it transform an economy with low productivity growth into a powerhouse of innovation. But effective negative interest rate policy can help monetary authorities in fighting deep recessions”.

2.2 Other Views

Bernanke (2017) and Rogoff, (2017) as discussed previously, evaluate the difficulties and constrains on monetary policy tool with NIRP and try to develop further tools in preparing for the next financial slowdown. Some authors discuss and reach a conclusion of a specific part of the NIRP problem, like reaching the zero-lower bound (ZLB) and problems that occur when finally breaking it, like effective lower bound (ELB) (Agarwal and Kimball, 2015: Gust et al. 2015: Williamson 2017: Angrick and Nemoto, 2017). The authors agree with the perspectives from Bernanke, (2017) and Rogoff, (2017) in respect to going deeper into negative interest rate. On the other hand, they take it one step further in evaluating the difficulties, positive and negative aspects, feasibility and peoples psychology in response to ELB and prolonged ZLB.

Fats and Summers, (2018) take into account of previous research that global patterns of lower saving and investment and that many of the developed countries have been engaging in an expanding fiscal policy that have had negative impact on growth rates, slower productivity growth and "secular stagnation". However, Vinals et al. (2016) adds that the expanding fiscal policy and unconventional monetary policy might have been plausible in the long term-decreasing nominal interest rate environment that started long before the introduction of NIRP, meaning the cost of financing become less expensive. Similarly, Blanchard and Summers, (2017) argue "firstly that it not surprising, that the crucial role of the financial sector, and the costs of financial crises; second, the complex nature of fluctuations, from the role of non-linearity’s, to the limits of policy, to the persistent effects of shocks; third, the fact that we are and may be, for the foreseeable future, in an environment of low interest rates, an environment which interacts with the
first two factors, and force a rethinking of not only monetary, but also fiscal and financial policies”. Additionally, Rachel and Smith, (2017), show that real rates might be a global phenomenon since both the advanced economies and emerging market are experiencing a decrease in the nominal interest rates. Specially in the advanced economies where the nominal interest rate has been falling for more than three decades. The explanation for falling nominal interest rates is argued by Vinals et al. (2016) to have two factors, the first being that after-tax rate of return has decreased which leads to a decrease in all rates both risky and safe. The second being that the safety premium has increased somewhat, resulting in a lower safe rate. However, Rachel and Smith (2017) ”identified 5 factors that account for around 3.10 out of 4.50 percent fall of long-term real interest rates over the past 30 years and none of them are likely to reverse quickly: a lower trend growth (1.00 percent); worsening demographic trends (0.90 basis points); low investment rates due to the falling price of capital goods (0.50 percent); rising inequality (0.45 percent); savings gluts in emerging markets (0.25 percent).”

Alternatively, Milton Friedman famously argues that the time lag in both received data and reaction in economic variables to policy (monetary policy) is implemented in an attempt to guide the economy in the direction central banks modelled. Milton Friedman extensively argued that those lags are way too long and incredibly unpredictable, meaning that any effort to effectively manage macroeconomic policy is in itself impossible (Blanchard and Summers, 2017). Additionally, Friedrich Hayek, (1945) argues that the key is the free market economic thinking, because the free market ”invisible hand” guides economic development to prosperity much more effectively than central bank planning. The main argument behind Friedrich Hayek’s (1945) reasoning is the assumption that many of the variables that shape an economy can never be captured by a single central plan. This means that the economic complexity cannot be described by a conventional mathematical model, because there will always be a crucial economic variable that will be omitted in the analysis.
Chapter 3

Theoretical Model

"In theory, there is no difference between theory and practice. In practice, there is." — Benjamin Brewster, 1882

The purpose of the theoretical model is a representation of economic behaviour and relationship between selected variables over a specified time period. While doing so, the goal should be to create a model that reflects reality "fairly" well and that allows for predictions of economic behaviour to some degree depending on the models accuracy. The modelling includes: forecasting economic activity to a significantly high accuracy, planning and allocation of capital, for finance such as predictive modelling like trading and for this thesis purpose, modelling the Effective Lower Bound (ELB). This thesis takes into account that models are approximations of important variables and therefore there exists a margin of error.

This section of the thesis presents a mathematical illustration and formulation of the ELB, liquidity traps associated with ELB and conventional policies at ELB. The following mathematical formulations and economic descriptions are summarized from the book Monetary Theory and Policy by Walsh (2017). Walsh, (2017) illustrates and argues that the previous assumptions by standard models which imply that nominal interest is bounded below by zero were wrong. This is backed by the fact of recent experience where the Swiss 10-year bond fell into negative and therefore, Walsh, (2017) argues, that it would be better to discuss ELB than zero lower bound (ZLB). The first section of the of this chapter describes the household budget and what happens when introducing negative interest rates to the equation. The second section of this chapter introduces a function for cost of holding money and how its affected by the negative interest rate policy. The third and final section of this chapter describes in debt liquidity trap and illustrates it in a graph.

1The Zero Lower Bound Problem (ZLBP) is a situation in which the central bank of a country wants to lower the short-term nominal interest rates, but faces a hindrance when the interest rate reaches or nears zero, and cannot lower it further. The central bank would prefer to lower the interest rate even more to stabilize the economy, but can’t do so not only because the interest can’t get any lower than zero, but also because of various other reasons (macroeconomicanalysis.com).
3.1 What is Happening With Budget Restrictions

Walsh, (2017) goes on to mathematically illustrate why the conventional models assume nominal interest has to be positive. Consider a typical budget constraint of a household and ignoring the capital and labour supply totally while replacing them with periodic endowments $y_t$ that are paid out at time $t$. Walsh, (2017) assumes that $y_t$ is spent on consumption, money and bonds, which are budget constrained in the following form:

$$y_t + \frac{(1 + i_{t-1})b_{t-1} + m_{t-1}}{1 + \pi_t} + \tau_t = c_t + b_t + m_t,$$

where $b_{t-1}$ and $m_{t-1}$ are real value of bond and money holdings, consumption denoted by $c_t$ and $t-1$ indicates that the values are carried forward from previous period. Unlike inflation rate $\pi_t$ and the lump sum transfer $\tau_t$, which are at calculated at time $t$. Lastly, $i_{t-1}$ accounting for nominal interest yield for the bonds. Additionally, for simplification purposes, adding $\left(\frac{i_{t-1}}{1 + \pi_t}\right) m_{t-1}$ on both sides of equation (3.1) we get the following with some algebraic manipulations:

$$y_t + (1 + r_t)d_{t-1} + \tau_t = c_t + \left(\frac{i_{t-1}}{1 + \pi_t}\right) m_{t-1} + d_t,$$

where $1 + r_t$ is replacing $\left(\frac{1 + i_{t-1}}{1 + \pi_t}\right)$ and $m_t + b_t$ becomes $d_t$. Furthermore, when $i_{t-1} \geq 0$ as the upper part of the $\left(\frac{i_{t-1}}{1 + \pi_t}\right) m_{t-1}$ in the equation (3.2), that part of the function represents the cost of holding money expressed in lost interest on that money. Solving the (3.2) equation for $d_{t-1}$ gives following:

$$d_{t-1} = \frac{c_t + \left(\frac{i_{t-1}}{1 + \pi_t}\right) m_{t-1} - \tau_t - y_t}{1 + r_t} + \frac{d_t}{1 + r_t},$$

now rewriting (3.3) one period forward, meaning $d_{t-1+1} = d_t$, gives following:

$$d_t = \frac{c_{t+1} + \left(\frac{i_t}{1 + \pi_{t+1}}\right) m_{t+1} - \tau_{t+1} - y_{t+1}}{1 + r_{t+1}} + \frac{d_{t+1}}{1 + r_{t+1}}.$$
\[ d_{t+1} = \frac{c_{t+2} + \left( \frac{i_{t+1}}{1+\tau_{t+2}} \right) m_{t+1} - \tau_{t+2} - y_{t+2}}{1 + r_{t+2}} + \frac{d_{t+2}}{1 + r_{t+2}}, \] (3.5)

now taking equation (3.4), (3.5) and inserting them into (3.3) a pattern emerges:

\[ d_{t-1} = \frac{c_t + \left( \frac{n-1}{1+\tau_t} \right) m_{t-1} - \tau_t - y_t}{1 + r_t} + \frac{c_{t+1} + \left( \frac{n}{1+\tau_{t+1}} \right) m_t - \tau_{t+1} - y_{t+1}}{(1 + r_t)(1 + r_{t+1})} \]
\[ + \frac{c_{t+2} + \left( \frac{i_{t+1}}{1+\tau_{t+2}} \right) m_{t+1} - \tau_{t+2} - y_{t+2}}{(1 + r_t)(1 + r_{t+1})(1 + r_{t+2})} + \ldots \frac{d_{t+i}}{(1 + r_t)(1 + r_{t+1})\ldots(1 + r_{t+j})}, \] (3.6)

the pattern becomes clear as the \( d_t \) goes to \( d_{t+i} \). If we break the equation (3.6) into two components the first \( d_{t-1} \), \( y_{t+i} \) and \( \tau_{t+i} \) and the second \( c_{t+i} \) and \( m_{t+i} \) we will get the following:

\[ d_{t-1} = \left[ \frac{1}{1 + r_t} \left( c_t + \left( \frac{i_{t-1}}{1 + \tau_t} \right) m_{t-1} \right) \right] \left( \frac{1}{1 + r_t} \right) (\tau_t + y_t)
\]
\[ + \left[ \frac{1}{(1 + r_t)(1 + r_{t+1})} \left( c_{t+1} + \left( \frac{i_t}{1 + \tau_{t+1}} \right) m_t \right) \right] \left( \frac{1}{1 + r_t} \right) (\tau_{t+1} + y_{t+1}) + \ldots \]
\[ + \left[ \frac{1}{(1 + r_t)(1 + r_{t+1})\ldots(1 + r_{t+j})} \left( c_{t+i} + \left( \frac{i_{t+i}}{1 + \tau_{t+i}} \right) m_{t+i} \right) \right] - \frac{1}{(1 + r_t)(1 + r_{t+1})\ldots(1 + r_{t+j})} (\tau_{t+i} + y_{t+i}). \] (3.7)

Now we have isolated the desired components, we need to move all of the first components \( y_{t+i} \) and \( \tau_{t+i} \) to the LHS and collect all the terms with \( \sum \) and \( \prod \) on both sides, then we will get the following equality according to previous definition of budget constraint in the following form:

\[ d_{t-1} + \sum_{i=0}^{\infty} \prod_{j=0}^{i} \left( \frac{1}{1 + r_{t+j}} \right) (y_{t+i} + \tau_{t+i}) \geq \sum_{i=0}^{\infty} \prod_{j=0}^{i} \left( \frac{1}{1 + r_{t+j}} \right) \left[ c_{t+i} + \left( \frac{i_{t+i-1}}{1 + \tau_{t+i}} \right) m_{t+i-1} \right]. \] (3.8)
The derived budget constraint above, both on the LHS and RHS, represent the part of the equation \( \left( \frac{1}{1+r_{t+j}} \right) \) which is the present discounted factor. In summary, the equation requires that the households transfers value from previous period \( t - 1 \) of bonds and money \( d_{t-1} \), discounted future endowments \( y_{t+i} \) and discounted future lump sum transfers \( \tau_{t+i} \), are bigger or equal to present discounted value of consumption, future consumption and cost of holding money.

However, what if the \( \iota_{t+i-1} < 0 \), meaning that the cost of holding money becomes negative and that the agent can increase both the consumption and the money holdings and still be less or equal to LHS. In the same way, an agent can increase both consumption and money holdings to infinity without violating the budget constraint because mathematically, the consumption and the cost of holding cash would cancel each other out. Then the agent can increase utility by increasing the two variables and the agent would have an unbounded demand for money and hence the problem of maximizing utility does not have a solution. Since negative rates, the agent is paid to hold money and the demand for money increases exponentially.

### 3.2 Cost of Holding Money

Walsh, (2017,) argues that problems that arise from negative interest rates can be illustrated by the money in the utility function (MIU). Hence, the derivation of a basic MIU model to illustrate the steps and assumptions taken in the cost of money model. Therefore, the first order conditions for money holding and bond becomes important and look as follows (for derivations and detailed explanation see Appendix):

\[
U_c(c_t, m_t) = \beta E_t \left( \frac{1 + \iota_t}{1 + \pi_{t+1}} \right) U_c(c_{t+1}, m_{t+1}),
\]

\[
U_c(c_t, m_t) \geq U_m(c_t, m_t) + \beta E_t \left( \frac{1}{1 + \pi_{t+1}} \right) U_c(c_{t+1}, m_{t+1}).
\]

\( U_m \) being the marginal utility of money and \( U_c \) being the marginal utility of consumption, \( \beta \) being the sensitivity of future change, and \( E_t \left( \frac{1}{1 + \pi_{t+1}} \right) U_c(c_{t+1}, m_{t+1}) \) part of the equation being expected future consumption discounted by future inflation. In equation (3.10) the assumption is made that the agent holds positive money, therefore the
inequality sign appears since the private agents cannot issue money meaning \( m_t \geq 0 \). Walsh, (2017) goes on to express that if equation (3.10) were strict that the agents would be holding zero money. Additionally, equation (3.9) and (3.10) can be combined as following: first substituting (3.10) into the LHS of (3.9)

\[
\beta E_t \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) U_c(c_{t+1}, m_{t+1}) \geq U_m(c_t, m_t) + \beta E_t \left( \frac{1}{1 + \pi_{t+1}} \right) U_c(c_{t+1}, m_{t+1}) \quad (3.11)
\]

thereby subtracting \( \beta E_t \left( \frac{1}{1 + \pi_{t+1}} \right) U_c(c_{t+1}, m_{t+1}) \) from both sides and simple algebraic manipulation gives following form:

\[
\left( \frac{i_t}{1 + i_t} \right) U_c(c_t, m_t) \geq U_m(c_t, m_t) \quad (3.12)
\]

Now, under the following constraints assume that \( i_t = 0 \) then the condition requires \( U_m = 0 \) at least in order to satisfy the condition. Furthermore, if there exists \( \bar{m} > 0 \) so that \( U_m(c, m) = 0 \) for all \( m \geq \bar{m} \) meaning that condition would be satisfied under \( i = 0 \) for any and all \( m \geq \bar{m} \),\(^2\) but that condition can never be satisfied if \( i < 0 \).

One way to fix the model and allow for the accommodation of negative rate, i.e \( i < 0 \), is to assume that there exists a cost for holding large amount of cash, like safe transport or safe storage of the cash. Hence, the introduction of a money cost function denoted as \( \tau(m) \), which is non-decreasing and convex under \( m; \tau' \geq 0 \) and \( \tau'' > 0 \). In the beginning holding small amount so cash might not be that costly hence, \( \tau(m) \) approaches zero but when money amount increases so does the cost of holding that money at an exponential rate. Furthermore, the initial equation (3.11) gets replaced under the assumption that money is held:

\[
\left[ \left( \frac{i_t}{1 + i_t} \right) + \tau'(m_t) \right] U_c(c_t, m_t) = U_m(c_t, m_t) \quad (3.13)
\]

The LHS representing marginal utility cost of holding money while the RHS is the marginal utility of additional money holdings. Additionally, consider if the nominal

\(^2\)If \( \lim_{m \to \infty} U_m(c, m) > 0 \) for all \( c \), no finate level of money holdings satisfies the equilibrium condition when \( i_t = 0 \), which is why the presence of a saturation level of money balances \( \bar{m} \) is often assumed" (Walsh, 2017).
interest rate is negative, demand for money being well defined even if \( m > \bar{m} \) making \( U_m = 0 \) bringing the following condition for \( \tau'(m_t) \):

\[
\tau'(m_t) = -\left( \frac{i_t}{1 + i_t} \right) \geq 0
\]  

(3.14)

The \( \tau'(m_t) \) function is always bigger then or equal to zero and \( m \) increasing, there can be a well characterized finite solution to agents money demand. This explains why people do not take out all their money even though the interest rate is negative. The most important scenario of the model being even with negative interest rate i.e. \( i_t < 0 \), the cost of money will still be positive, thus proving the conventional models belief wrong that nominal interest has to be positive. The model does so under the assumption of storage cost which makes the \( \tau'(m_t) \) non-negative in a negative interest rate world.

### 3.3 Liquidity Traps

Liquidity trap is a situation that occurs when nominal interest rate equals the ELB, a situation that is usually regarded as low interest rate and high saving. In regards to equation (3.10), which illustrates that money is separated into two sides of the inequality signs. First being money that does not give a periodic principal on holding money and bonds which pay a nominal return. That separation between money and bond makes them opposites and complementary to each other, but if \( i = 0 \) and \( U_m = 0 \), then there is very little in terms of distinguishing the two which in fact become perfect substitutes. Walsh, (2017) goes on arguing that monetary policy rules such as the Taylor principle may in fact lead to economic instability and force the economy into a liquidity trap. To explain this, consider equation above (3.12), where Walsh, (2017) argues that there is a possibility of accelerating hyperinflation even if the nominal quantity of money was kept constant. However, an explosive deflation could cause the nominal quantity of money to explode exponentially. But the nominal interest rate would be persistently falling with the deflationary path until reaching the ELB where the rate of expanding deflation would be halted. Consider the following equation:

\[
i_t = r + \pi_{t+1},
\]  

(3.15)
where the $r$ is the real interest rate, $i_t$ interest rate and $\pi_t$ inflation. Then assume that the central banks follow a rule like:

$$i_t = r + \pi^* + \delta(\pi_t - \pi^*),$$  \hfill (3.16)

where $\pi^*$ is the target rate of inflation set by the central bank and $\delta > 1$ so to satisfy the Taylor rule\(^3\). But Mankiw, (2009) considers $\delta = 0, 5$ and when $\delta < 0, 5$, which creates different slopes and therefore leads to alternative equilibrium, that becomes stable as opposite to the case with $\delta > 1$.

Consider combining the two equations above (3.16) and (3.17), then the new dynamics for inflation becomes as follows:

$$\pi_{t+1} = \pi^*(1 - \delta) + \delta \pi_t,$$  \hfill (3.17)

making it unstable with $\delta > 1$ and unfeasible to following the Taylor rule. Moreover, Walsh, (2017) suggests that a stationary equilibrium exists with inflation being equal to $\pi^*$, but for inflation rates that starts below $\pi^*$ meaning that the $\pi$ declines. If inflation declines so will the nominal interest rate eventually.

\(^3\)Taylor rule: A rule for monetary policy according to which the central bank sets the interest rate as a function of inflation and the deviation of output from its natural level (Mankiw, 2009, p.583).
Furthermore, suggesting that the rate of deflation is bounded from below by \( i_L - r \) where \( i_L \) is representing the lower bound of nominal interest rate. Now considering that the deflation is bounded from below which will ultimately lead the economy into a liquidity trap and a new equilibrium inflation rate \( \pi^{**} \) as shown in the figure (3.1).

Seeing that in the proximity of the deflationary equilibrium \( \pi^{**} \) there exists many equilibrium points from which there exist possibility of having perfect-foresight equilibrium. Another essential point assuming that inflation starts at \( \pi^* \) and naturally the central bank cuts the nominal interest rate to stimulate the economy and lower the real rate. However, the expectation of future inflation drops and leading to a decrease in the actual inflation even further, could push prices down even further and drive the economy deeper into deflation equilibrium.

Walsh, (2017) argues there is a way by combining monetary policy and fiscal policy to get out of the liquidity trap. Assuming that the government have the political power to promise large government deficits every time the inflation becomes low in comparison to the desired target. Thus, the increase in governments total of nominal debt, would lead to higher inflation expectation and would increase the price level equilibrium. Additionally, one of the solution might be monetary policy, Walsh, (2017) suggest that central bank can conduct open-market operations in assets that are complementary to money, meaning monetary policy can affect inflation even when within a liquidity trap. Second suggestion is that central bank devalues the currency and thereby create inflation.
Chapter 4

Actual Consequences of NIRP

"Of course, part of economists fascination with the zero-lower bound is precisely that it forces a rethinking of conventional dogma. Just as the laws of physics imply strange and surprising consequences as an object approaches a black hole, the laws of economics can yield some strange and surprising results as an economy gets too near the zero-lower bound on interest rates." — Kenneth Rogoff, (2017)

This part of the thesis is broadly based on research, data and conclusions of Jurksas, (2017) and Storz et al, (2017). Firstly, structured to present relevant data based on impact of NIRP on various sectors. Secondly, we try to interpret the data, analyze and present gathered data (from Jurksas, (2017) and Storz et al., (2017)) in such a way that makes it easy to follow and easy to understand the content and the connection between various sections.

4.1 Money Market and Longer Term Rates

The logical starting point when analyzing the transmission of the policy rate onto the real economy is by determining the pass-through effect to the money market and short-term rates. As seen in Figures 4.1 and 4.2 the transmission from NIRP to money market rates appear in all the countries that decided to implement NIRP and the short-term money markets have almost instantaneously priced the negative interest rates, as to reflect the new monetary policy.

Jurksas, (2017) argues, that short-term money market rates remained close to the policy rate in Switzerland and Sweden but are somewhat higher in the euro area, Japan and Denmark. Furthermore, arguing that the development might be due to the exceptions that they applied to charges on excess reserves. Jurksas, (2017) goes on to conclude and to add that the negative rates have in fact been transmitted to the short-term money market rates in the same way as positive rates have.
The second big factor and opposite to the short-term money market are long term rates, meaning the yield at which the government borrows. Jurksas, (2017) argues that the belief of investors that central banks would not go below zero, creates a bias in believing in future monetary tightening but that belief gets erupted when central banks crossed the ZLB. Additionally, creating a divergence between actual market rates and the interest rates that investors considered most likely.
Jurksas, (2017) argues that the yield curves become flatter\footnote{"The flat yield curve is a yield curve in which there is little difference between short-term and long-term rates for bonds of the same credit quality. This type of yield curve is often seen during transitions between normal and inverted curves. The difference between a flat yield curve and a normal yield curve is a normal yield curve slopes upward", (Investopedia).}, the issue might become troubling if the sovereign bonds get fundamentally overvalued, therefore increasing a risk of market reversal that could have a negative effect over other markets. Additionally, the low long-term rate can lead to government overspending since the interest rate on the borrowed money would be low, leading in a short-term spike in the GDP and lower unemployment. However, long term can cause problems if the long-term interest rates start rising as seen in Figure (4.2), where the long-term interest has been increasing in all five countries since 2016, especially in Denmark, Sweden and euro area. The biggest problem being euro area since they have countries like Greece that are overly indebted and cannot afford higher rate and might risk default or debt restructuring if allowed by creditors.

### 4.2 Households

According to Jurksas, (2017), since the implementation of NIRP the purchasing power has gone from net savers to net borrowers. There are more people borrowing since low interest than those willing to save at a negative interest rate. Decreasing interest rate only boost banks incentive to lower lending rate on new loans for consumption and home purchases (Figure 4.3). Since the NIRP put pressure on the banks net interest margin and forced them to compete for consumers with lower rates.

Taking into account that consumption has already been in a declining trend, thus isolating effect of NIRP on consumption is difficult. Looking at Figure (4.3) it is apparent that Denmark NIRP had the biggest short-term effect on consumption. After a short period, the consumption leveled off and shortly thereafter dropped even further, crossing the previous low, where the NIRP were implemented. Furthermore, Jurksas, (2017), argues that mortgage rates declined marginally in Switzerland, while in Sweden the decline in mortgage rates has been far less than the policy rate. The author goes on to conclude based on ECB (2015), that household payment has declined substantially. From previous 4 to 1.1 percent of disposable income.
The data shows that Walsh (2017) budget constrain equation (3.8) is half correct only in the short term in the case of Denmark, where the consumption increases in negative interest environment but subsequently the consumption fell right after the short-term boost. Walsh, (2017) assumes that if cost of holding money becomes negative, the agent can increase both consumption and money holdings and still be less or equal to LHS. Therefore, the agents propensity to consume becomes exponentially large.

Run to hard currency is one of the issues discussed by many such as Bernanke, (2017) and Rogoff, (2017). The run into hard currency does not appear to have happened yet and the data (see Figure 4.4) shows a steady increase and there was no notable or exponential increase in circulation since the introduction of the NIRP. However, Sweden has been in a downtrend and the downtrend does not seem to be changed since the introduction of NIRP. One of the explanations for Sweden’s negative downtrends can be caused by Sweden’s increasingly push for a cashless society. In summary, the low mark in NIRP where it would be more sensible to hold hard currency does not seem to have been reached.
4.3 Non-Financial Corporations (NFC)

The decrease in interest rates followed by NIRP lead to a reduction of funding cost for non-financial corporations (NFC) (see Figure 4.5). The biggest reduction can be seen in the euro area followed by Sweden. Sweden has been in a downtrend before the introduction of NIRP and continued to decline after the introduction of NIRP, meaning that Swedish banks might have been deleveraging even before the introduction of NIRP. As a result of NIRP, the corporations debt became “less expensive” in terms of interest payment on the issued debt, hence, the NFC balance sheet became better with higher profitability and lower debt-to-equity ratio.

Additionally, banks also lowered the deposit rate for NFC’s like they did for the households. Taking it one step further, the banks have introduced negative overnight rates for some institutional investors and big firms.

Jurksas, (2017) argues that one reason might be that interest on new loans for NFC’s are lower than those for households so banks feel more pressure to reduce the deposit rate. Additionally, Jurksas, (2017) adds that lower deposit earnings put pressure on the profitability of firms with large money holdings. This creates a need for investing in risky assets, but the problem occurs when there are less and less low risk investment options. Thus, the investments become riskier and riskier, making way for fundamentally flawed
firms to stay alive. Jurksas, (2017) goes on to conclude that those cash rich NFC that have higher willingness to invest will win over those NFC with high deposits.

One of the effects of prolonged negative interest rates have been researched and presented as creation of “zombie” firms (firms generally continued to go deeper into debt during the 2010-2014 instead of defaulting). Meaning that the only reason for these firms being able to finance day to day operations is depending on the low yield that they have to pay (McGowan, Andrews and Millot, 2017).

4.4 Banks

Banks in countries that have implemented NIRP have had their net interest margins reduced (see Figure 4.6). The decrease is most notable when looking at the amount of NFC loans. For example, the euro area saw the biggest decline, while Sweden and Denmark saw some volatility but was mostly constrained within 1 and 1.5 percent (Figure 4.6). Similarly, despite implementation of NIRP, house purchases remained stable over the period in all countries with exception of Denmark were some volatility was seen during the period. Home purchases in euro area where most stable and even saw an increase in medium term after the implementation of NIRP. Additionally, Jurksas, (2017) argues based on Praet, (2016) research that NIRP stimulated banks to increase lending volumes, so that the shrinking margin would not affect banks stability.
Jurksas, (2017) argues that in the short term the profitability of banks could increase as the positive effects of NIRP overshadow the negative. "Rising lending volumes and non-interest income, higher values of securities portfolios due to lower yields, smaller NPLs\textsuperscript{2} because of the decrease of clients debt service burden and reduced funding costs partly offset the problem of deposit rate stickiness" (Jurksas, 2017). Meaning and shown in Figure (4.7) that banks return did not change within NIRP region. As shown in Figure (4.7), the banks with highest return were in Sweden and Switzerland while the worst preforming one was in Denmark and fairly lower returns in Japan.

The importance of long term bank viability becomes a little more complicated, because the short term positive effects of NIRP will fade and the negative effects become somewhat bigger and outweigh the positive. The banks will start feeling the pressure on net interest margins, and as a result they will need to alter the business model (Jurksas, 2017). Additionally, Jurksas, (2017) argues that the longer the NIRP is in effect, the bigger concerns it will create for the banks, in form of a flatter yield curve which implies lower profit from maturity and deposit stickiness. Banks will need to replace matured positive yielding bonds with negative yielding ones. Jurksas, (2017) goes on to conclude,

\textsuperscript{2} A nonperforming loan (NPL) is the sum of borrowed money upon which the debtor has not made his scheduled payments for at least 90 days. A nonperforming loan is either in default or close to being in default (Investopedia).
based on research of Borio et al. (2015) and The Bank of Lithuania, (2016), that long term NIRP policy investors will have to reassess how low the interest rates can actually go. Secondly, banks will have to change their business model even further, re-balance their portfolios into risk carrying strategies and focus more on the borrowers.

### 4.5 Rise of ”Zombie” Firms

Storz et al., (2017), argues, based on Cecchetti et al., 2011; Chen et al., 2015 that excessive levels of debt are a major obstacle to economic growth, where the question remains what are ”excessive” debt levels. However, there exist historically higher level of corporate indebtedness in many euro area countries (see Table 4.8), which might have been an obstacle for economic recovery. Storz et al. (2017), defines zombie firms as those firms that have negative return on investment and low debt servicing, and those two criteria need to be met for two consecutive years. Firstly, Storz et al., (2017), sets parameters for the ”zombie” firms so to limit the sample space, in the following manners: firms with negative returns, firms with negative debt, debt servicing capacity (EBITDA/financial debt) below 5 percent, for at least two consecutive years and firms that employ more than 250 and either have more than 43 million in assets or more than 50 million in turnover. Secondly, Storz et al., (2017), exclude firms that are classified as large according to the European Commission, (2015) and publicly listed companies. The second point can be argued against since that assumption leaves all the publicly
traded firms and large firms as unquestionable and perfectly solvent even if the history has proven otherwise.

The Storz et al., (2017) article identified a sample of 423,000 firms and 900 banks linked to those companies, within five stressed euro area countries (Spain, Greece, Ireland, Portugal, and Slovenia) and two strong and non-stressed countries (Germany and France). With highest number of zombie firms being in Portugal (22%) and Spain (17.7%) while Germany (3%) and Slovenia (7.8%) lowest number of zombie firms.

Using a difference-in-difference framework, their main conclusion is that corporate debt overhang problems paired with rollover risk implied less investment among European cooperates. Under-investment was aggravated for firms that were tied to banks with high sovereign exposures that lost in value after the sovereign debt crisis took off in early 2010. Consequently, especially NFCs in the periphery of the euro area, exhibited sluggish investment and thereby slowing down economic recovery (Storz et al. 2017).

Furthermore, Storz et al., argues based on Caballero et al. (2008), that capital allocation in zombie firms reduce the capital investment into healthy firms and thus creating a long-term problem. Additionally, Storz et al., found that on average zombie firms, in non-stressed countries, increase its leverage by 2.4 percent annually in comparison to the non-zombie firms.

In conclusion, the only reason that zombie companies are artificially alive today is because of low cost of borrowing (i.e. negative interest rate or extremely low interest rate in other non-euro countries). Furthermore, in Figure (4.8) the sample of zombie firms is only euro area but if one would look world-wide like Japan (where the characteristics of its economy are weak and a weak financial sector with high level of zombie firms at
a zero-interest environment (Storz et al. 2017)), Sweden, Denmark, US and so on the number of these zombie firms would increases and the sample would also increase.

4.6 Governments

The NIRP has lowered the cost of borrowing for governments in all five countries that implemented NIRP, as the yield of the short-term bond have been decreasing significantly, some even negative. But the long-term bond yield has also been decreasing as seen in Figure (4.2), while Japan and Switzerland (briefly) went negative but as of now all long-term yield are positive and slightly increasing since middle of 2016. "While policy rates have a more direct effect on the short-end of the yield curve (the signalling channel), its impact on market participants expectations about the path of future policy rate had an impact on longer-end yields too, mostly through the balance sheet channel . The extra savings of refinancing led to lower deficits and, to some extent, higher spending” quoted by Jurksas, (2017).

The negative consequences (foreseen or unforeseen) of NIRP are likely to become a concern in the long run, since most of the countries that have NIRP have already spent the extra savings on short-term political gain (Jurksas, 2017). Spending the excess saving, no necessary structural reforms and no rebuilding of spent reserves have been made. Jurksas, (2017) argues adding on research by Borio et al. (2016), that governments need to break their reliance on "cheap money” at the exceptionally low rates, since it does not have a long-lasting effect on the economy. Politicians have much lower time span than an economic cycle, thus they can enact and spend excess saving in an attempt of gaining popularity and winning votes. However, when interest rates rise the service of bonds becomes more expensive and politicians that enacted on those decisions in 2012 are out of office, thus creating a problem for the current politician. Who most probably will blame the problem on an unrelated variable and enact policy reform that in mostly suited for him/her. Ultimately, financially unprepared to deal with the problem of positive/rising interest rates, with higher debt servicing cost and even not being able to pay existing debt on the agreed terms. This would create a shock in the financial markets, increased vulnerability, reduced bondholders willingness to invest and ultimately economic slowdown (Jurksas, 2017). Jurksas, (2017) goes on to conclude that the situation could get much worse with longer period of NIRP and the faster they are normalized.
The short-term benefits of NIRP to the government are short term boost to the consumption and production, because of all the government spending. The long-term benefits do only appear if a country borrows at the low rates and invest in the infrastructure and enact structural change so that the country becomes stronger and more long term sustainable in the long-run. Those countries that do that will have become stronger and increased its competitiveness in the long-run. Otherwise, the country would have spent large amounts of money and be in a deeper debt than before.

4.7 Asset Markets

The effect on currency exchange rate in the countries that implemented NIRP contributed to a depreciating currency (or at least stable) such as intended by central banks. The portfolio rebalancing being the main channel which led to capital outflow and thus decreasing the demand for currency, leading to the depreciation of the currency. This can be seen on a short-term basis across the countries that implemented NIRP (see Figure 4.9). Jurksas, (2017) suggests that currency depreciation can lead to "currency wars" (as mention in previous chapter 2.1), leading to a cycle of rate-cutting across competing countries.

The main reason for wanting to devalue one's currency is the short-term benefit in net export, firstly it becomes less expensive for foreigners to buy homemade products, therefore boosting short-term production and increasing the demand for home made goods. Secondly, foreign goods become more expensive when priced in the depreciating currency, therefore lowering the imports of foreign goods and those goods need then to be produced in the home country, thus increasing the production and the net export of the country that devalues first. Rickards (2012), makes the argument that in the long-run the country that initiates currency "manipulation" might be worse off assuming that trading partners don’t respond with currency devaluation. Rickards (2012), argues that even if successfull with being the only one to depreciate the currency, the foreign consumers will consume less of their home made goods leading to their productivity worsening leading to less income and less savings. So thereby in the long-run they would import less and thereby slowing the productivity of the country that initiated the currency devaluation. The problem becomes when several countries attempt to devalue their currency’s leading
to a "currency war" and hard to reverse. Additionally, Rickards (2012) makes the argument that there is a possibility for further escalation if "currency wars" were to initiate and that it could lead to tariffs, sanctions and possibly even war, making it extremely dangerous.

![Figure 4.9: Nominal effective exchange rate, normalized to 100 at 01/01/2014, %](image)

NIRP effect on the stock market was positive in the short term (see Figure 4.10), but somewhat muted in the medium-term with the exception of Denmark, which outperformed all other stock market indices measured in performance after implementation of NIRP. Meanwhile medium-term after the implementation of NIRP all indices started under-performing with exception of Denmark. Jurksas, (2017) suggests that this might be concern for long-term banking sector and the pressure on net interest margins.

NIRP have had same effect on corporate bond yield as they did on government bond yield. Accounting for $500 in corporate negative yielding bonds out of $10 trillion in negative yielding bonds (Jurksas, 2017). Furthermore, Jurksas, (2017) argues that this might be a consequence of expectation for negative yielding bond to remain negative in the long-term.
In conclusion, the long-term implications of NIRP on asset market could eventually outweigh the positive short-term effects. Additionally, if NIRP is kept for a long period of time, which will lead to portfolio rebalancing and could lead to overvalued asset prices. But eventually central banks will need to start signalling for monetary policy normalization and the new monetary policy accommodations get priced in by the investors, the risk for investors pulling their money back and creating a stock market crash, which historically has led to an economic downturn (Jurksas, 2017).
Chapter 5

Conclusion

The main reason for the world’s current unconventional monetary policy is the actions of central banks and the seriousness of the financial crises of 2008. The crises left policy makers with little choice. Because, as Yellen, (2016) argued, policy makers usually on average need 5.5 percent interest rate cut in effort to fight economic recession. But in the wake of 2008 the central banks had very little room to cut interest rates (see Figure 4.1). They needed something that would restore confidence and get the economy going, hence the introduction of NIRP and QE as a short-term solution to the problem.

The impact of negative interest rates on the economic development can be seen through its effect on long-term rates, household spending, asset markets, NFC reduction in lending cost, banks short term profitability but long term squeezing net interest margin and the zombie firms that arise in the environment.

As argued by Vinals et al. (2016); Blanchard and Summers, (2017) and Rachel and D. Smith, (2017), long-term interest rates have been falling for the past three decades even before the implementation of NIRP. The only long-term bonds that went negative for a short while as a result of the NIRP were the Japanese and Switzerland’s. The problem created by that scenario is that governments take advantage of the low funding cost and borrow at low rates without enacting structural change or investing that money in viable future sustainable development. Therefore, the low interest rates are creating an indebted country and possibly fundamentally overvalued bond market and creating risk of market reversals that might lead to a financial crisis. This phenomenon is something that has been going on for a while and is not something new that appeared because of NIRP. The only thing that changed in the new NIRP environment is that the decline might have accelerated downward and ultimately reaching negative for some countries like Japan and Switzerland. Finally, on August 1, 2017 the famous ex FED chairman Alan Greenspan stated ”By any measure, real long-term interest rates are much too low and therefore unsustainable”.

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Walsh, (2017) definition of a liquidity trap and our current situation are not too far apart. The first condition for liquidity trap is a low interest rate environment and the second is high savings or cash hoarding. The current situation of low interest rate environment and in some countries negative interest rate have already fulfilled the first condition for liquidity trap. The second condition of high savings is not fulfilled yet but there is risk that needs to be considered such as the possibility for a major crisis (financial crash, war, natural disaster and so forth) that might lead to a run into hard currency as seen in 2008. But the current situation with low interest rates creates problems and the idea of cutting deeper into negative territory might result in a central bank crisis of confidence. Otherwise positive thing is that the implementation of NIRP did not cause a run into hard currency (see Figure 4.4) and according to the Walsh (2017) (see Section 3.2), the cost of money has yet to reach that boundary where it’s less expensive to hold cash than pay negative interest rate. The possibility of going deeper into negative territory might create problems around minus 0,75 percent to minus 2 percent (Vinals et al. 2016) and breach the point where the cost of holding money is equal to the interest payment, at which point there might be a run into hard currency and pushing the economy into a liquidity trap. Therefore, as mention by Bernanke, (2017) and Rogoff, (2017), the need is grave for additional monetary policy tools to be considered. As suggested by Rogoff, (2017) there are measures to deal with the deeper NIRP if there would ever be a need.

The stock markets have performed well in all countries that introduced NIRP. Denmark index was one that outperformed all the others (see Figure 4.10). The gains in the stock market were short-term because all the indices turned downward after the initial short-term out-performance. The reason for the downturn being concerns for the banks long-term profitability and stability. Some of the short-term gains in the stock markets can be attributed to the exchange market, which was depreciating in all the countries that implemented NIRP (see Figure 4.9). A depreciating currency makes it cheaper and therefore more attractive for foreigners to invest in the stock market and import homemade products. Therefore, creating an environment where other countries interpret the implementation of NIRP as an attempt to cheapen the currency and thereby boosting the exports. But the depreciation in the exchange market were short-term which caused the currencies of the countries that implemented NIRP to return slightly to where they started out. Looking at the Figure 4.9 its clear that the only success in the currency "stabilization" (depreciation) was the euro area which saw its currency depreciate minus
5 percent, while Denmark and Sweden were mostly unchanged in the long-run whereas Japan and Switzerland saw an appreciation in their respective currencies with an increase of 5 percent in Switzerland and 7 percent in Japan. In conclusion, the purpose of lowering exchange rates by introduction of NIRP has widely failed with exception of euro area.

Jurksas, (2017) argues that when it comes to household consumption the implementation of NIRP shifted the purchasing power from net saver to net borrower. The household consumption has been in a declining trend long before the implementation of NIRP and there was little to none change in the declining trend with introduction of NIRP (see Figure 4.3). Additionally, as mentioned in section 3.1 Walsh, (2017) argues that the budget constraint of a household under specific assumptions (see equation 3.8), would lead to significant demand for money holdings (in form of loans since they pay negative interest) and consumption. The model argues that if the agent where to increase both consumption and money holding at a similar pace there would be no limit for money holdings and consumption, which could theoretically go to infinity. The practical limitation of equation 3.8 is that borrowing at the negative interest rates is only possible for banks and not private individuals. In summary, based on the data gathered from the negative interest rate countries, it shows that the effect of NIRP have not delivered the expected boost to consumption, not even in the short-term (see Figure 4.3).

One of the results of NIRP is some decrease in funding cost for the NFC, eventually going negative and creating an environment where fundamentally flawed firms can fund day to day operations. NIRP has led banks to change their business model and re-balance their portfolio into risk carrying strategies so to stay profitable in the long-run. The short-run profitability of the banks was somewhat unchanged, but the long-term profitability becomes much more complicated. Because of the new model of lending at low rates to riskier costumers and eventually funding/creating zombie firms that would not exist in higher interest rate world. Storz et al., (2017), found a sample of 423,000 firms and 900 banks linked to those companies only in the euro area. Meanwhile, 60,688 of those were zombie companies making up 14.3 percent of the sample space. One of the criteria is that the firm must have 250 employees or more. Calculating 250 times 60,688 firms and the minimum number of employees (if all firms has only 250 employees) in the euro area that is employed by a fundamentally flawed firm is 15.8 million people, that can lose their jobs if the interest rate starts rising. The concerning thing is that the study was done only in the euro area, but if one would to study all the other countries that
employed NIRP the number of zombie firms would increase and so would the number of people that are employed by those firms. Second part of the equation are the banks that finance the zombie firms, which would take on heavy losses even if a small portion of the zombie firms were to default on their obligations. Thereby creating a financial crisis scenario and the possibility of worldwide financial crisis, because the interdependence and connectives of the worldwide financial sector.

To summarize the long term-effect of NIRP on the economic stability; Firstly, the households will be discouraged from saving and encouraged to borrow, which will lead to debt per household to rise and become addicted to easy money. Similarly, the government borrowers in the short term for political gain and become addicted and dependent on abnormal low rates; Secondly the lending terms for the zombie companies will continue to be favorable and the capital will be averted from sound companies, which will create long term problems; Finally, the current expansion is the second longest on the record, meaning that the probability of a recession increases by every passing day, week and month. The first sign of stress will appear in the bond market as the short-term rates and long term rates become higher. Investors will see more and more unsustainable debt in government and companies and therefore will not want to invest at the current yield. The yield will rise and the conditions will become tougher for government bonds and cooperate bonds, while the banks and financial institutions hold most of those bond the investors will start re-balancing the portfolios. Therefore, this will likely lead to a stock market crash with banks and zombie firms leading the charge. Most likely this will start in countries like Portugal, Spain and Greece that have the highest rate of zombie firms and some other countries like Italy that have large amounts of government debt. Therefore, the central banks would need to act in the new crisis that was unforeseen (Fligstein, et, 2014). The solution would most probably will be something in the lines that Bernanke, (2017) and Rogoff, (2017) suggested and then the question becomes of the peoples limit where they lose some or total confidence in central banks.

Lastly, the issue of getting out of negative interest rates, it is very possible in theory and somewhat in reality. In theory getting out of NIRP and rising them somewhat towards the historical mean, would cause a lot of defaults and create a depression with low growth and low inflation. Additionally, defaults in some countries on the bond interest payments would allow for restructuring and realization that past decisions were not long-term sustainable. But on the contrary, there is little chance of that happening
since someone needs to take responsibility for the past mistakes and acknowledge the mistakes made. Most probably the world will continue to be dependent on abnormal low rates and continue to search for additional unconventional monetary policy tools. The problem becomes between a short term “feel good” solution and by smart and long term healthy solution for the economy. Unfortunately, the probability is high that in the end it will be the short-term solution.

Ultimately the macro economy might be too unpredictable as Friedrich Hayek, (1945) and Milton Friedman famously suggested and should be left to the free market to determine the interest rates and so forth.
Chapter 6

Further Research

It would be very interesting if someone would continue on this research and expand on the idea of the theoretical model, such as building a simulation in MATLAB or some other program. Additionally, it would also be really interesting to read if someone were to continue based on this research and find data that this thesis had in the empirical section but instead of the data being from 2012-2016 instead looking for data from 2012-2018 or 2006-2018. The data presented by Jurksas (2017) is quite short around the time of introduction of the NIRP, it would be interesting to get more data before the introduction of the NIRP to see the trends before so to be able to make a more detailed assessment of the NIRP. That would give the researcher much larger time period to analyze NIRP on. Then again it would be really interesting to read if someone were to analyze the next recession and see what role the NIRP played in that recession and how the policy makers responded.


**Quote references:**


Appendix

Derivation and explanation of MIU function:

Walsh, (2017,) argues that problems that arise from negative interest rates can be illustrated by the money in the utility function (MIU). Hence, the derivation of a basic MIU model to illustrate the steps and assumptions taken in the cost of money model. Therefore, the first order condition for money holding and bond becomes important. Walsh, (2017) ignores uncertainty and any labour-leisure choice and focuses strictly on the implications of the model for money demand, the value of money and cost of inflation. Hence the utility function of households takes the following form

\[ U_t = u(c_t, z_t), \]

the utility function at time \( t \) is described by two variables. Firstly, \( c_t \) representing consumption per capita at time \( t \) and secondly, \( z_t \) which represents flow of services yielded by money holdings. Walsh, (2017) goes on to assume that the utility function is increasing in both variables, strictly concave, and continuously differentiable. Meaning that the demand for monetary services is always positive if and only if \( \lim_{m \to 0} u_z(c, z) = \infty \) for all \( c \), where \( u_m = u(c, z) \frac{\partial}{\partial z} \). Note that \( z_t \) is not only representation of dollars (or some other currency) that some individual agent holds, but \( z_t \) is a representation of the goods that those dollars hold. Meaning some measure of transaction services that are expressed in terms of dollars (paper currency). As a result the \( z_t \) term is represented by number of dollars (\( M \)), times their price \((1/P)\), in terms of goods: \( M(1/P) = M/P \). Therefore, if the flow of services is proportional to the real value of the money holdings and \( N_t \) is the population, then \( z_t \) becomes equal to real per capita money holdings (Walsh, 2017):

\[ z_t = \frac{M_t}{PN_t} \equiv m_t, \]

To ensure that the monetary equilibrium exists, it can be assumed that for all \( c \), there exists a finite \( \bar{m} > 0 \) so that \( u_m(c, m) \leq 0 \) for all \( m > \bar{m} \). Meaning that the marginal utility of money must become negative for a large amount of money holdings.

The household total utility becomes:
\[ W = \sum_{t=0}^{\infty} \beta^t U_t = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t), \] (6.1)

where \( 0 < \beta < 1 \) is a subjective rate of discount. The equation (6.1) suggests that there is a much stronger notation of utility provided when holding money. If the marginal utility of money is positive, then the equation (6.1) suggests that, holding constant the path of real consumption for all \( t \) and that the utility increases by the increase in money holdings. In summary, money-in-the-utility-function is useful as a shortcut for ensuring that there is demand for money (Walsh, 2017).

Lastly, in completing the specifications for the model, assume that households can hold money, bonds that pay a nominal interest rate \( i_t \) and physical capital\(^1\). According to household’s income, its assets and any transfers from the governments (\( \tau_t \)), the individual household allocates its holdings into consumption (\( C_t \)), gross investment into physical assets and gross accumulation of bonds (\( B_t \)) and real money holdings (\( M_t \)). Therefore, the aggregate economy-wide budget constraint of households becomes (Walsh, 2017):

\[ Y_t + \tau_t N_t + (1 - \delta)K_{t-1} + \frac{(1 + i_{t-1})B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = C_t + K_t + \frac{M_t}{P_t} + \frac{B_t}{P_t}, \] (6.2)

where \( Y_t \) represents aggregate output, \( \delta \) representing rate of depreciation of physical capital, \( K_{t-1} \) represents aggregate stock of capital at the beginning of the period \( t \) and \( \tau_t N_t \) represents aggregated value of lump sum transfers from the government or taxes. The output \( Y_t \) combined with employment (\( N_t \)) and capital stock (\( K_{t-1} \)) available relates to: \( Y_t = F(K_{t-1}, N_t) \), because output is a function of available capital stock and employment. Therefore, under the assumption that the production function is linear homogeneous and with constant return to scale, output per capita \( y \) becomes a function of the per capita capital stock \( k_{t-1} \) (Walsh, 2017).

\[ y_t = f \left( \frac{k_{t-1}}{1 + n} \right)^2, \] (6.3)

\(^1\)“Physical capital produces output according to a standard neoclassical production function” Walsh, (2017).

\(^2\)One could write \( Y_t/N_t \equiv y_t = F(K_{t-1}, N_t)/N_t = F(K_{t-1}/N_{t-1}) \equiv f(k_{t-1}/(1 + n)) \), where \( n \) represents constant labour force growth (\( N_t - N_{t-1} \))/\( N_{t-1} \) (Walsh, 2017).
where \( n \) represents population growth (under the assumption that its constant), additionally the output is produced in period \( t \) while the capital stock is carried over from the previous period \( t-1 \). Furthermore, the function is placed under specific conditions such as, \( f_k \geq 0, f_{kk} \leq 0, \lim_{k \to 0} f_k(k) = \infty, \lim_{k \to \infty} f_k(k) = 0 \). The next step is to divide both sides of the budget constraint (6.2) with population \( N_t \) and the per capita becomes (Walsh, 2017):

\[
\omega_t \equiv \left( \frac{k_{t-1}}{1 + n} \right) + \tau_t + \left( \frac{1 - \delta}{1 + n} \right) k_{t-1} + \frac{(1 + \delta_{t-1})b_{t-1} + m_{t-1}}{(1 + \pi_t)(1 + n)} = c_t + k_t + m_t + b_t, \tag{6.4}
\]

where \( \pi_t \) represents the rate of inflation, \( b_t = B_t/P_t N_t \) and \( m_t = M_c/P_c N_c \). The respective households will face a problem of choosing between \( c_t, k_t, m_t, \) and \( b_t \) so to maximize the total utility (6.1) subject to (6.4). Making the optimization of the different variables in order to reach maximum utility increasingly important, especially when choosing among variety of different selections: consumption, capital holdings, bond holdings and money holdings. The initial level of resources \( \omega_t \) and the value function is defined as follows:

\[
V(\omega) = \max_{c_t, k_t, m_t, b_t} \{ u(c_t, m_t) + \beta V(\omega_{t+1}) \}, \tag{6.5}
\]

but the maximization is subject to budget constraint (6.4) and therefore the equation becomes as follows:

\[
\omega_{t+1} = f \left( \frac{k_t}{1 + n} \right) + \tau_{t+1} + \left( \frac{1 - \delta}{1 + n} \right) k_t + \frac{(1 + \delta_t)b_t + m_t}{(1 + \pi_{t+1})(1 + n)}, \tag{6.6}
\]

Making use of (6.4) to express \( k_t = \omega_t - c_t - m_t - b_t \), also making use of the definition \( \omega_{t+1} \), (6.5) it can therefore be written as:
\[ V(\omega) = \max_{c_t, m_t, b_t} \left\{ u(c_t, m_t) + \beta V \left( f \left( \frac{\omega_t - c_t - m_t - b_t}{1 + n} \right) + \tau_{t+1} + \left( \frac{1 - \delta}{1 + n} \right) (\omega_t - c_t - m_t - b_t) \right) + \frac{(1 + \delta) b_t + m_t}{(1 + \pi_{t+1})(1 + n)} \right\}, \] (6.7)

with the optimization problem being unconstrained for the \( c_t, b_t \) and \( m_t \) additional condition are necessary, hence the following first order conditions:

\[ u_c(c_t, m_t) - \frac{\beta}{1 + n} \left[ f_k \left( \frac{k_t}{1 + n} \right) + 1 - \delta \right] V_\omega(\omega_{t+1}) = 0, \] (6.8)

\[ \frac{1 + \delta}{(1 + \pi_{t+1})(1 + n)} - \frac{1}{1 + n} \left[ f_k \left( \frac{k_t}{1 + n} \right) + 1 - \delta \right] = 0, \] (6.9)

\[ u_c(c_t, m_t) - \frac{\beta}{1 + n} \left[ f_k \left( \frac{k_t}{1 + n} \right) + 1 - \delta - \frac{1}{1 + \pi_{t+1}} \right] V_\omega(\omega_{t+1}) = 0, \] (6.10)

together with the optimization conditions:

\[ \lim_{t \to \infty} \beta^t \lambda_t x_t = 0, \text{ for all } x = k, b, m, \] (6.11)

where \( \lambda_t \) represents the marginal utility for the consumption of period \( t \), therefore the theory asserts that:

\[ V_\omega(\omega_t) = \frac{\beta}{1 + n} \left[ f_k \left( \frac{k_t}{1 + n} \right) + 1 - \delta \right] V_\omega(\omega_{t+1}), \] (6.12)

therefore, when combining together with (6.8) we get:

\[ \lambda_t \equiv u_c(c_t, m_t) = V_\omega(\omega_t). \] (6.13)

The first order condition is straight forward and simple to grasp, since initial resources \( \omega_t \) has to be divided among consumption, capital, bonds, and money, which must lead to
some marginal utility when optimized in a correct manner. Therefore, the maximization of the utility function become increasingly important.