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The impact of immigration on the Swedish Economy

Samia Nasir & Elina Larsson
Supervisor: Clas Eriksson
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Authors:
Samia Nasir & Elina Larsson

Supervisor:
Clas Eriksson

Examiner:
Christos Papahristodoulou

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ABSTRACT

The flow of immigrants between Europe and many other countries is often brought up in the discussion of the labor market. It is also questioned whether an increased immigration is beneficial for the society and the economy of the receiving country.

This paper is an investigation of the above-mentioned problem “does immigration benefit a country’s economy and what are the effects of immigration on the economy?”. Furthermore, we will especially look at how immigration benefits the natives in the country with respect to labor and capital income. We will also investigate if there is a difference in education and skills between immigrants and natives and we will also examine the consequences of this. The discussion about benefits and liabilities to the society that has arisen, due to the welfare effects in the economy, will also be brought up and analyzed.

The thesis work will mostly be a study of theoretical findings, where we will discuss the previous researches and important models, for instance Borjas (1999), who uses a model that gives a solid result of how much natives benefit from immigration in term of labor and capital earnings. To support the theoretical part, we have gathered the data from Eurostat and Swedish migration Authority, which will help to find some empirical results as well. This paper also includes a regression model that measures the change in the unemployment rate where the independent variables are GDP growth rate and immigration inflow.

Keywords: Unemployment, immigration, wages, capital income.
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1 INTRODUCTION

The area of study for our thesis is the impact of immigration on the developed economy (with a reasonably flexible labor market). According to Oxford dictionary, the word immigration means the action of coming to live permanently in a foreign country. The study of immigration and its effects is a significant area for both economics and politics. People immigrate due to several reasons like war, trade, work, education, safety etc. People also take decision to move from one country to another to improve their living standard (Swedish Migration Agency).

People who move because of war and political situations are the asylum seekers and some people move for job purpose. According to Statistics Sweden in 2017, 68898 people from over 160 countries became Swedish citizens out of which a large part (8635) of citizenship was granted to Syrian citizens. The immigration increased by 14 percent compared to 2016. Meanwhile, according to Swedish Public Employment Service 2017 has been a good year for the labour market. In 2017, 94000 people got jobs and the employment rates increased to 78.1 percent, which is highest in the EU. Meanwhile, the unemployment rate decreased to 6.7 percent.

With the increase in the immigration the population also increases. In 2004 the population of Sweden exceeded 9million for the first time. In 2017 Sweden crossed the 10million population mark as shown in Figure 1.

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1 The numerical information used in the introduction section is from Statistics Sweden website https://www.scb.se/
1.1 **STATEMENT OF THE PROBLEM**

In recent years, the flow of immigration has increased quickly due to several reasons. The reasons can be political, environmental or economic. The political and environmental reasons include refugees and asylum seekers who migrate due to war, natural disasters and poverty (Swedish Migration Agency). Economic reasons include when people move for the jobs purpose to improve their welfare.

One of the reason is non-stability in the home country, which has arisen a large discussion whether immigration is beneficial for receiving the country or not. Earlier findings and analyses have suggested that it is beneficial for the natives in the country, due to their different qualifications in contrast to the immigrants. However, other findings suggest that immigrants take jobs from natives. While previous research brings up whether immigrants “take jobs” from natives, it also stresses whether immigration lowers the wages or not. With the use of data regarding this, we ask the question, “how is this measured in Sweden”? 

![Figure 1: Population 1800–2017 (SCB)](image-url)
1.2 OVERVIEW OF HISTORY OF IMMIGRATION AND EMIGRATION IN SWEDEN

According to the Swedish Migration Agency, from the mid-1800s to the 1930s people emigrated from Sweden to US, Canada, South America and Australia, which leaves the deepest mark on the development of Sweden. Over 1.3 million Swedes moved because of poverty, religious persecution and lack of political freedom. The peak year of emigration was 1887 when more than 50,000 people left Sweden and most of them moved to America. After World War I, the emigration declined due to US restriction. After World War II, because of German, Nordic and Baltic refugees, Sweden changed from being a net emigration country into a net immigration country. Since 1930, Sweden has larger immigration than emigration².

As immigration increased after World War II, the Swedish Migration Agency was started in 1969 to deal with immigration matters. After World War II over 100,000 refugees from Yugoslavia moved to Sweden. In the 1980s the number of asylum seekers from the countries Iraq, Lebanon, Syria and Eritrea increased significantly in the Western Europe. In 1995 Sweden

² The numerical values and information are from the Swedish Migration Agency website https://www.migrationsverket.se/
The graph is from Statistics Sweden website.
joined the European Union and later in 1999 the European Council set a goal to make asylum and migration policy. In 2001 Sweden joined the Schengen Cooperation which today consists of 26 countries to grant visa free entry in the Schengen area (Swedish Migration Agency).

Figure 3: Residence permits granted 1980-2014 (Swedish Migration Agency)

1.3 AIM

This paper is an investigation of how immigration effects a developed economy. The aim is to understand and analyse benefits and liabilities in the labor market for natives and foreign born. This is achieved by analysing two theoretical models that attempts to describe the change in wage level for natives when there is an inflow of immigration, originally built on Borjas (1999) and Altonji and Card (1991). Moreover, we examine how immigration affects unemployment, using Swedish data.

1.4 LIMITATIONS

The thesis work is both theoretical and empirical. For the theory part, the study of economic articles and relevant theoretical methods will be used, whereas for the empirical part we will use data and statistics from reliable sources. The data are obtained from Statistics Sweden and
the Swedish Migration Agency. The empirical research and analysis of this paper will be restricted to Sweden.

1.5 Literature Review

There have been many previous research studies done by other researchers around the subject of this paper and one of the leading author’s is George G. Borjas who has written a book about immigration economics as well as articles about the benefits from immigration. One of our multifactor models is built on Borjas (1999), who wrote an article that includes a model, trying to measure the change in wage level for natives when there is an inflow of immigrants to the labor market. The article measures the immigration surplus when there is a difference in skill level for the two groups, and when there is no difference. It distinguishes between the groups that is getting beneficial respectively the contradiction.

Altonji and Card (1991) wrote the article “The effects of immigration on the labor market outcomes of less-skilled natives”. The article includes a model that measures the wage level for natives when there is an inflow of immigrants but differs from Borjas model in the sense that it uses heterogeneous inputs instead of homogeneous. Their model is not totally different from other economist’s work, in the sense where they all basically are trying to measure the same thing; changes for the natives related to an immigration inflow. However, the model by Altonji and Card differ in two different key aspects, for instance, the inputs of their model consider both unskilled and skilled workers, and it also uses a locally produced good (call it $Y$) to be measured. This is a heterogeneous model, where the author’s build some of the equations and derivations on earlier work by other economists, such as Johnsen (1980a). Although the model to some extent builds on earlier economist’s work, Altonji and Card (1991) clearly emphasize how their work differ from earlier studies.
Jennifer Hunt, Professor of economics has done a lot of previous research regarding immigration and the benefits of it, where she, for instance, stresses the controversial thought that immigration benefits the whole society, especially if immigrants are low skilled or possess different skills than the natives does (DN Fokus 2016).

Marjolaine Gauthier-Loiselle and Jennifer Hunt (2008) in the paper “How much does immigration boost innovation” have done research about immigrants, where they discuss to which extent skilled, highly educated immigrants increase innovation. This paper is especially dedicated to US but can to an extent be applied to Sweden as well.

The empirical part of this study will consist of running five sample regression equations to see how the change in unemployment corresponds to changes in the GDP growth rate, and immigration inflow (refugees and labor force). This study is dedicated to Sweden only, because of the limited time. Although, we realize that other factors than those named above, also have an impact on the change in unemployment. For the five regression equations, two data sets will be encountered in the study; the first one is from the year 1990-2017 and considers immigration as one group. The other data set includes immigration as both refugees and work permits but is only measured from the year 2000-2017. The reason for the measure of two data set is as previously stated, that the second data set includes immigrants as two different groups, so it is interesting to see if this dataset has a different impact on the dependent variable that set one. Furthermore, the regression equations will start with Okun’s law. Where we look at the change in the dependent variable as a change in unemployment, caused by the independent variable as growth in GDP. Okun’s law (studied by Arthur M. Okun) shows a negative relationship between the unemployment and GDP.
2 THEORETICAL BACKGROUND

In the theoretical part, we have analyzed the problem statement by using models and explaining figures. It will give a picture of the effects of immigration for the natives on the labor market given that the immigrants are either low skilled or highly educated.

According to Borjas (2014) “immigration has consequences, and these consequences generally imply that some people will lose while others benefit” ⁴. This theoretical part is a brief introductory description of why people gain and lose in the case of immigration. It explains how immigrants affect the natives when they act as perfect substitutes or complements. This part also includes Cobb-Douglas production function, which explains that the increase in the labor due to immigration increases the rate of return to capital and decrease the real wage of natives. The natives always benefit from immigration either in the form of higher wages or in the form of an increase in the return of capital in the long run.

2.1 LABOR MARKET EQUILIBRIUM

In the labor market, firms want to keep salaries low whereas workers prefer jobs with high salaries. Labor market equilibrium helps to create a balanced situation between workers and firms.

In the labor market, firms constitute the demand side of equilibrium whereas the workers determine the supply side. Equilibrium is the point where the supply of workers is equal to the demand of workers, which means anyone who wants to work can find a job.

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In the above figure $w^*$ is the equilibrium wage, $E^*$ is the equilibrium employment and P is the firm/producer surplus whereas Q is the worker surplus\(^4\).

### 2.2 Labor Market Impact Of Immigration

Let us now add the assumption that the workers are different according to their skill level. Workers can be skilled or unskilled and it applies to both natives and the immigrants. The immigrant workers can be perfect substitutes i.e. have the same skill set and occupation or they can be complements i.e. low skilled workers. If skilled immigrants are substitutes they will compete against each other for similar jobs. Therefore, in the short run the supply of skilled labour will increase due to which the supply curve will shift downwards, which shows a reduction in wages from $w$ to $w'$ and the total employment increases from N to L in the left panel of Figure 5. With the lower wage the number of native workers declines from $N$ to $N'$, due to the positive slope of labor supply curve.

If the immigrants are complements, on the other hand, they do not compete with natives and belong to different markets. In this case, the native skilled workers can use their skills for

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\(^4\) This entire section of the theoretical background (Page 7–10) including figures builds on chapter 4 from the book Labor Economics written by Borjas published in 2016.
something better and the unskilled labor can do the work that does not require a high skill level. As immigrants and natives complement each other, the increase in the immigrants will increase the marginal product of natives, which will shift the demand curve to the right. This upward shift in the demand curve increases the demand for the high skilled natives. The increase in the productivity of natives will increase the wages of natives from $w'$ to $w$ and native employment will also increase from $N'$ to $N$, as can be seen in the panel to the right.

![Diagram showing perfect substitutes and complements](image)

**Figure 5:** Immigration in short run perfect substitute & complements (Borjas, 2016)

If immigrants and natives are perfect substitutes and capital is fixed, the wages of natives will be lower in the short run if the supply of immigrants is sufficient. However, it will raise the return to capital. This leads to more investments and thus higher marginal products of capital, which in turn pull wages upward (this is explained in the next section). If immigrants and natives are complements, then the wages of natives will be higher.

### 2.3 Cobb-Douglas Production Function

To determine the production level, we will use a Cobb-Douglas production function. The production function shows how much output we can produce from given labor, technology and
capital. This Cobb-Douglas production function exhibits constant returns to scale, which means that an increase in both inputs by an equal amount will result in an equal increase in the output.

\[ Q = AK^aL^{1-a} \]

Where \( Q \) is production, \( K \) is capital, \( L \) is labor, \( A \) is a constant and \( a \) is a parameter that lies between 0 and 1.

According to marginal productivity theory, the factor of production is paid by the value of its marginal product. Assuming that, the price of \( Q \) is normalized to 1, this means that the labor is paid the real wage that is equal to the marginal product of labor and the price to capital is equal to the marginal product of capital (Borjas, 2016).

\[ w = MPL = \frac{dQ}{dL} = (1-a)AK^aL^{-a} \]

\[ r = MPK = aA \left(\frac{K}{L}\right)^{a-1} \]

Due to immigration the labor \( L \) increases, which increases the rate of return to capital and decreases the wage. However, in the long run firm invest in more capital which increases the capital \( K \) and thus wages and therefore people have more incentives to work.
3 Methodology

The two models that we have theoretically explained are by Altonji and Card (1991) and Borjas (1999), which includes immigrants and natives that have the same productivity and skill level (homogeneous labor) and also the case when they have different productivity and skill level (heterogeneous labor). In both cases, we will examine how the natives get benefit from immigration. The benefits of natives will be higher if there is more difference in the productivity and competence of immigrants. If immigrants and natives are complements, then natives will gain. If immigrants and natives compete against each other then natives will lose. Both models measure the change in wage level for native workers and the model by Borjas (1999) examines the total surplus, which also is explained in a graph.

3.1 Homogeneous Labor and Inelastic Capital

We will here use the model of homogeneous labor and inelastic capital that Borjas (1999) has explained. This model will more stringently show the income of natives before and after immigration and the gain of native capitalist after immigration.

This analysis shows the effect of immigration on the labor market when immigrants and natives are perfect substitutes, which means that both share the same productivity and skill level. If immigrants and natives compete against each other for similar jobs, then in the short run natives will face a reduction in wages but the natives who own capital will gain.

We have a production function where $K$ is the capital and $L$ is the Labor, as before, but the function is now general.

$$Q = f(K, L)$$

The labor ($L$) consists of natives ($N$) and immigrant workers ($M$). The workers are perfect substitutes and $L = N + M$. 
We start from a situation with no immigrant workers. The rental rate of capital when $M = 0$ is

$$ r_0 = f_k(K, N) $$

where the foot index $k$ indicates the partial derivative with respect to capital. Similarly, the price of labor that is wage is

$$ w_0 = f_L(K, N) $$

The output from the production function is distributed to the native capitalists and the native workers, because the production function exhibits constant returns to scale.

### 3.1.1 Income to Natives Before Immigration

The national income to natives before immigration is $Q_N$.

$$ Q_N = r_0K + w_0L $$

The function $f_L$ is the labor demand function. At given $K$ the surface under this curve is the total income. To see this, note that

$$ \int_0^N f_L(K, L) \, dL = [f(K, N)]_0^N = f(K, N) - f(K, 0) = f(K, N) = Q_N $$

The figure below illustrates the initial equilibrium. The capital is fixed and the area $ABN0$ is the national income $Q_N$ to the natives. With the entry of immigrants, the supply curve shifts to the right and the wages fall from $w_0$ to $w_1$. 


Figure 6: The immigration surplus in a model with homogeneous labor and fixed capital (Borjas, 1999)

The area $ACL0$ is the new national income $Q_1$. The part of the increase in the national income $w_1M$ is distributed to the immigrants. The area $BCD$ is the immigration surplus and gives an increase in the national income. With immigration, the wages fall whereas the capital income $r$ increases (because $f_{LK} > 0$). Immigration surplus is the portion that increases the return of capital and is the result of an increase in the supply of labor. Natives who own more capital will gain from the surplus and those who own less capital will primarily be affected by the lower wage.

### 3.1.2 Income After Immigration

With the entry of immigrants $M$, the rental rate of capital $r_1$ and the price of labor $w_1$ is:

\[ r_1 = f_k(K, L) \]
\[ w_1 = f_l(K, L) \]

As mentioned above, income after immigration is the area $ACL0$ in Figure 6. Now the labor $(L)$ consists of both natives $(N)$ and immigrants $(M)$.

\[ Q_1 = r_1K + w_1L \]
\[ Q_1 = r_1K + w_1N + w_1M \]
Because $w_1 < w_0$, native workers will be worse off because of the fall in wages. The native capital owners will be better off as the marginal product of capital gets higher when $L$ increases.

The net gain is $BCD$ which is positive.

Let us now examine these welfare effects further. We know the wage $w = f_L(K, L) = f_L(K, N + M)$. To find an expression for the net income gain we will take differential of wage with respect to immigration $M$.

$$dw = \frac{\partial f_L}{\partial L} (K, N + M) dM$$

$$dw = \frac{\partial f_L}{\partial L} \frac{L}{f_L} dM$$

Now define $\varepsilon$ as the elasticity of factor price for labor (the percentage change in the wage resulting from a one percent change in the size of labor force), i.e. $\varepsilon = \frac{\partial f_L}{\partial L} \frac{L}{f_L} \frac{\partial w}{\partial w} = \frac{\partial w}{\partial L} \frac{L}{w}$

The differential then is:

$$dw = \varepsilon \frac{f_L}{L} dM$$

### 3.1.3 Net Income Gain

The immigration surplus or the social net income gain $BCD$ in the Figure 6 can be approximated by a triangle

$$\Delta Q_N = \frac{1}{2} (w_0 - w_1) M$$

Note that $M$ is changing from 0 to $M$, so $M = M - 0 = dM$. Whereas $(w_0 - w_1) = -\frac{\partial w}{\partial L} M$

$-dw = -\varepsilon \frac{f_L}{L} dM = -\varepsilon \frac{f_L}{L} M$. This means that

$$\Delta Q_N = -\frac{1}{2} \varepsilon \frac{f_L}{L} MM$$
To simplify this, denote the income share of labor by 
\[ \alpha_L = \frac{w_L}{q} = \frac{f_L L}{q} \] and let \( m = \frac{M}{L} \) be the fraction of foreign-born in the labor force. The alternative expression for social net gain then is:

\[ \Delta \frac{Q_N}{Q} = -\frac{1}{2} \epsilon \frac{f_L L M M}{Q L L} \]

Using the definitions above the equation below shows that immigration surplus is proportional to the elasticity \( \epsilon \).

\[ \Delta \frac{Q_N}{Q} = -\frac{1}{2} \epsilon \alpha_L m^2 \]

The equation above is an alternative expression for the social net income gain and it shows how much the country gains from immigration.

Borjas (1999) has used the numerical value of the share of labour income as 70%, the fraction of immigrants in the workforce slightly below 10% and the elasticity of factor price for labor around -0.3. Using these values in the above equation we get 0.1%. Which means the immigration surplus of US is 0.1% of GDP.

3.1.4 Native Labor Earnings

After the immigration, income is redistributed from labor to capital. Due to immigration the area \( w_0 BDw_1 \) is lost by natives, as shown in the Figure 6.

The native labor earnings are

\[ wN = f_L (K, N + M)N. \]

We will take the differential with respect to \( M \):

\[ d(wN) = \frac{\partial f_L}{\partial L} (K, N + M)NdM \]

As \( M \) is changing from 0 to \( M \) so we have \( M = M - 0 = dM = M \) and defining \( \epsilon = \frac{\partial f_L}{\partial L} f_L \) as the elasticity of factor price for labor.
\[ d(wN) = \frac{f_L}{L} \frac{\partial f_L}{\partial L} L N \, dM \]

\[ d(wN) = \frac{f_L}{L} \varepsilon NM \]

\[ d(wN) = \varepsilon \frac{f_L L N M}{1} \frac{1}{L} \]

As we know both natives and immigrants are in the labor market so \( L = N + M \), we will solve for \( N, N = L - M \). We will use \( N \) in the above equation and consider \( \frac{M}{L} = m \) to solve the equation further.

\[ \frac{d(wN)}{Q} = \varepsilon \frac{f_L L}{Q} (1 - m)m \]

\[ \frac{d(wN)}{Q} = \varepsilon \alpha_L (1 - m)m \]

The equation shows the net change in the labor income of natives as a fraction of GDP \( Q \).

Using the same numerical values that Borjas (1999) has used that is \( \alpha_L = 0.7, \varepsilon = -0.3 \) and \( m = 0.1 \) in the equation above we get -1.9%. The minus sign shows a reduction in the wages which means that with the increase in the immigration the native labor earnings decreases by 1.9% of GDP.

### 3.1.5 Capitalists Earnings

The relative change of income to capital is obtained by subtracting the native labor earnings from the social net gain.

\[ \frac{d(rK)}{Q} = \frac{\Delta Q_N}{Q} - \frac{d(wN)}{Q} \]

\[ = \frac{1}{2} \varepsilon \alpha_L m^2 - \varepsilon \alpha_L (1 - m)m \]

\[ \frac{d(rK)}{Q} = -\varepsilon \alpha_L m \left(1 - \frac{m}{2}\right) \]
The equation above shows the net change in the income of capitalist as a fraction of GDP $Q$. The rectangle area $w_0 B D w_1$ and the immigration surplus $BCD$ in Figure 6 goes to the native capitalists. Using the same values as above ($\alpha_L = 0.7, \epsilon = -0.3$ and $m = 0.1$) from the Borjas (1999) in the equation above we get 2% which means that the capital earnings of natives are about 2% of GDP. To sum up: While the net social gain is quite small, the redistribution is considerably larger.

For the implications of immigration in the case of heterogeneous labor we will use the next model of Altonji and Card (1991) because we think it is more suitable. The homogeneous model by Borjas (1999) looks at the economy in a big perspective, whereas the model by Altonji and Card (1991) looks at the economy at a smaller perspective which could be for example Stockholm or Västerås. We think it is interesting to look at both perspectives, hence we include a heterogeneous model into our paper as well.

### 3.2 Heterogeneous Labor

This model is built on Altonji and Card (1991) where a municipal economy is distinguished locally. The economy produces a local good $Y$ in a competitive industry, which is assumed to have CRS$^5$. Inputs are unskilled and skilled labour, as well as other materials, such as capital and raw material. The output $Y$ is then consumed locally within the economy and exported to other cities.

The total industry cost in the model is:

$$C(w_s, w_u, Y) = Y \cdot c(w_s, w_u)$$

Where the above equation measures the total industry cost as a function of the wages for unskilled and unskilled labor multiplied by the quantity of $Y$. Let $w_s$ and $w_u$ denote the wage

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$^5$ An increase in inputs (capital and labor) cause the same proportional increase in output.
levels for skilled workers and unskilled works respectively. By CRS, it is possible to pull out $Y$, and thus $c$ is the unit cost function.

### 3.2.1 Product Market Equilibrium

Demand for the good $Y$, arises from local demand from skilled workers, denoted as $Y_s$, local demand from unskilled workers, denoted as $Y_u$, and export demand from the rest of the economy, denoted $Y_x$. From this, we can form 3 demand functions, specified as per person demand, $D_s(q, w_s)$, $D_u(q, w_u)$ and $D_x(q)$. Which are demand for a skilled, unskilled and export respectively.

These are in turn used to form the following equation for the product market equilibrium:

$$ Y = P_s \cdot D_s(q, w_s) + P_u \cdot D_u(q, w_u) + D_x(q) $$

Let $P_s$ and $P_u$ denote the parts of the population in the economy that consist of skilled workers and unskilled workers, respectively. The last expression determining the market equilibrium is the demand for exportation.

For equilibrium in the labour market, the following requirement must be satisfied:

$$ (2a) \; P_s \cdot L_s(w_s, q) = Y \cdot c_1(w_s, w_u) $$

$$ (2b) \; P_u \cdot L_u(w_s, q) = Y \cdot c_2(w_s, w_u) $$

The functions $c_1$ and $c_2$ denotes the partial derivative of the unit cost function with respect to unskilled and skilled wage rates respectively. Note that the RHS is the demand functions, which can be derived from the unit cost function.

### 3.2.2 Inflow Of Immigrants

Assume that we wish to analyse an inflow from immigrants and its effects. One of the elasticities used to analyse the model is:
\[ \eta_{ij} = \frac{\partial c_i}{\partial w_j} w_j c_i, \quad i = 1, 2, \quad j = s, u. \]

This is the elasticity of labour demand for skill group \( i \) with respect to the wage of group \( j \).

The elasticity of labor supply, \( \varepsilon_i \) of group \( i \) is given by:

\[ \varepsilon_i = \frac{\partial L_i}{\partial w_i} L_i, \quad i = s, u. \]

Let \( \beta = \frac{p_u}{p} \) and \( \alpha = \frac{p_s}{p} \), where \( \beta \) and \( \alpha \) denote the fractions of unskilled and skilled workers in the population respectively.

Let \( \lambda_s, \lambda_u \) be numbers between 0 and 1 for the skilled and unskilled workers respectively, which can be determined with the following equations:

\[ \lambda_u = \frac{Y - Y_u - k_1 \cdot Y_s}{Y}, \quad k_1 = \beta (1 - \alpha)/[(\alpha (1 - \beta))], \]

\[ \lambda_s = \frac{Y - k_2 \cdot Y_u - Y_s}{Y}, \quad k_2 = \beta (1 - \beta)/[\beta (1 - \alpha)]. \]

Finally, the stock of immigrants is denoted as \( I \), so that the immigrant flow is given by \( \Delta I \).

Thus, the model above can be differentiated (see Appendix) to analyse the result from an inflow of immigrants. The differentiated system is formed as following:

\[ (3a) \lambda_u (\alpha / \beta) \Delta I / P = (\eta_{uu} - \varepsilon_u) \Delta \log w_u + \eta_{us} \Delta \log w_s \]

\[ (3b) \lambda_s [(1 - \alpha) (1 - \beta)] \Delta I / P = \eta_{us} \Delta \log w_u + (\eta_{ss} - \varepsilon_s) \Delta \log w_s \]

The left-hand sides of the above equations measure the percentage increase in unskilled workers and skilled workers that results from an inflow of immigrants by \( \Delta I \). The right-hand sides describe the changes in the wages of skilled and unskilled workers. To further understand these equations, note that \( \alpha \Delta I \) and \( (1 - \alpha) \Delta I \) are the changes in the portion of unskilled workers and skilled workers respectively. Thus, the proportional increases in the populations
of unskilled and skilled workers are given by, \((\alpha/\beta)\Delta I/P\) and \([(1 - \alpha)(1 - \beta)]\Delta I/P\), respectively.

The model uses \(\lambda\), as a factor to adjust for the gross increase in labor supply, for net increases in the demand, caused by immigrants. Furthermore, Altonji and Card (1991) gives a brief analyse that is dedicated especially to when the factor \(\lambda\) attains a specific value. The first case yields that, if immigrants possess the same skills as the natives, the following relationship will be true:

\[
\lambda_u = \lambda_s = \frac{Y_x}{Y},
\]

Where \(\frac{Y_x}{Y}\), is the fraction of the good \(Y\) that is exported. This seems to be an accurate result, because this would mean that all labor force is equal and probably then give rise to some competition on the actual labor market.

On the other hand, if immigrants are less skilled than the natives, we have that,

\[
\lambda_u > \frac{Y_x}{Y} > \lambda_s.
\]

This inequality is trustworthy since it shows exactly the relationship that is mentioned above and makes sense. It is also worth mentioning due to the previous statement, that Altonji and Card (1991), tells us that this relationship accentuates the effective increase in labor supply (newly arriving immigrants are considered as less skilled than natives).

Further investigation of the model determines that if the coefficient \(\lambda = 1\), meaning that a 1 percentage increase of immigrants in the population (of the city) add nothing to the demand locally for produced output and decreases native wages by approximately 0.3-0.5 percent. Note that this is a finding by Altonji and Card (1991).
Let $f$ be the fraction of immigrants in the local population. If equations (3a) and (3b) are used, the model shows a relationship between the change in the wage rates, and the change in the fraction of immigrants:

$$\Delta f = \Delta (I/P) = (1 - f)\Delta I/P.$$  

We note that when the demand for unskilled labor is not dependent of the wage rate for skilled labor, we have that:

$$\eta_{us} = 0.$$  

Equation (3a) can be simplified to:

$$\Delta \log w_u = \frac{-\lambda_u (\alpha/\beta)}{\epsilon_u - \eta_{uu}} \Delta I/P$$

$$= \frac{-\lambda_u}{(1 - f)(\epsilon - \eta)} (\alpha/\beta) \Delta f$$

Equation (4) is originally built on Johnson (1980a), a special case where the notation $\lambda_u$ here, is exactly equal to one and $\alpha = \beta$. Although, the above equation is built on the previously stated economist, this equation differs in two perspectives; namely, that it considers both unskilled and skilled workers, and the effect from demand of the good $Y$.

However, if $\eta_{us} \neq 0$, the change in unskilled wage is:

$$\Delta \log w_u = B_u \Delta I/P,$$

where

$$B_u = \frac{-\lambda_u (\alpha/\beta) - \lambda_s (1 - \alpha) \eta_{us}/(\epsilon_s - \eta_{ss})}{(\epsilon_u - \eta_{uu}) - \eta_{us}\eta_{su}/(\epsilon_s - \eta_{ss})}.$$  

Recall that $\beta$ and $\alpha$ denote the fraction of unskilled and skilled workers in the population respectively. The above expression is a function of the supply and demand elasticities for
unskilled and skilled labor, respectively. The $\lambda_s$ denotes the fraction of $Y$ that is exported to other cities.

Relating to the wage changes to immigrant inflows; suppose that $\alpha = \beta$, so that $\lambda_u = \lambda_s$.

Then, equation (5) can be rewritten as:

$$\Delta \log w_u = \lambda b_u \Delta I/P$$

Where the coefficient $b_u$ for this case is negative. The value of $b_u$ depends on how large in size the immigration inflow to a city is and also on the demand for the output of the good $Y$.

Furthermore, Altonji and Card (1991) suggest a result that; even if the parameters of the demand and supply is chosen widely, differently, the coefficient $b_u$ will still be in the same interval between -0.49 up to -0.27, which suggests a relatively small interval difference.

Moving to a most extreme case where the whole immigration inflow to the city will consist of unskilled labor, and the proportion of already existing skilled labor is equal to 0.5, the value of $b_u$ then will be -2 up to -1. This means that the effect on the unskilled (already existing) labor’s wages will be as much as two to three times larger.

Also, worth mentioning is that another finding by Altonji and Card suggests that if labor supply is between zero and one, the reduction in per capita labor supply for natives will be between 0-0.5 percentages.
4 Empirical Model

This section includes a regression analysis, which will examine how unemployment is affected by immigration. The data that is used to perform the regression is mainly from Eurostat and the Swedish Migration Agency. The model includes two sets of data, where the first one (set 1) is from the period 1990-2017 and the second is from 2000-2017. Two sets of data are used, because in the second one immigration is divided into two categories, refugees and labor force, whereas in set 1 there is a lump of both, and the essence behind this is to see if the result will differ when the analysis includes both categories.

We have analyzed the data using excel and then discuss the results using some tables and explanation. The regression equation is an estimate of the change in unemployment explained by an inflow of immigrants and the growth rate of GDP in Sweden.

If you compare the growth in GDP between many countries, one can expect that it can be described by the change in unemployment $\Delta u$ over a time period, and the inflow of immigrants ($IM$) in a country. We would expect that unemployment will have a negative effect on the growth in GDP ($Y$), due to decreased spending and immigration to have a positive effect as a contradiction. To see this, we begin by looking at Okun’s law.

4.1 Okun’s Law

Okun’s law is an empirical phenomenon, which was firstly studied by Arthur M. Okun. The Okun’s law shows a negative relationship between the unemployment and GDP\(^6\). It describes how unemployment is related to growth in $Y$, using the following regression equation:

$$\Delta u_i = \beta_0 + \beta_1 Y_i$$  \hspace{1cm} (1)

We would expect the signs to be $\beta_0 > 0$ and $\beta_1 < 0$. This is confirmed by the estimation.

4.2 Estimation

Adding immigration to equation to (1), the equation becomes:

\[ \Delta u_i = \beta_0 + \beta_1 Y_i + \beta_2 IM \]  

(2)

Now, we turn to looking at the regression equation from data set 2, where immigration is consisting of both refugees and work permits. Again, we start with Okun’s law, where change in unemployment is described by the growth in \( Y \) and refugees (\( REF \)):

\[ \Delta u_i = \beta_0 + \beta_1 Y_i + \beta_2 REF_i \]  

(3)

Then we look at how the unemployment is explained by work permits (\( L \)), and the equation instead becomes:

\[ \Delta u_i = \beta_0 + \beta_1 Y_i + \beta_2 L_i \]  

(4)

Finally, we include both refugees and labor force into the regression equation:

\[ \Delta u_i = \beta_0 + \beta_1 Y_i + \beta_2 L_i + \beta_3 REF_i \]  

(5)

When change in unemployment is the dependent variable, the expected signs are: \( \beta_0 > 0, \beta_1 < 0, \beta_2 < 0 \) and \( \beta_3 < 0 \). The constant is expected to be greater than zero because if all other independent variables are zero, the change in unemployment will be positive. The negative sign of GDP is because unemployment and GDP are expected to have a negative correction to each other. If unemployment increases with some number, GDP is expected to decrease due to a fall in the purchasing power. The labor force (immigrants) consisting of refugees and work permits is expected to have a negative sign because the inflow of immigrants will increase the labor market and hence reduce unemployment.
### 4.3 Results

<table>
<thead>
<tr>
<th>Regression equations</th>
<th>Equation (1)</th>
<th>Equation (2)</th>
<th>Equation (3)</th>
<th>Equation (4)</th>
<th>Equation (5)</th>
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T-values in parentheses
**Regression Results From Equation (1)**

\[ \Delta u_i = \beta_0 + \beta_1 Y_i \]

We have started with equation (1) using Okun’s law where the dependent variable is change in unemployment rate and the independent variable is the growth rate of GDP. The variables in equation (1) are in the expected direction. The negative sign of the GDP growth rate \( Y \), indicates that when there is a high growth in \( Y \) the unemployment in Sweden decreases. This is because if unemployment \( (u) \) is low, people can spend more money on buying which would instead raise \( Y \). The intercept is quite close to zero, which means that if \( Y \) is equal to zero, the corresponding change in \( u \) is almost zero as well. The t-values of the coefficients are high and significant and looking at \( R^2 \), we can tell that the fit of the data to the equation is quite okay. The t-values are quite high, so both the coefficients are significant here.

**Regression Results From Equation (2)**

\[ \Delta u_i = \beta_0 + \beta_1 Y_i + \beta_2 IM \]

In equation (2) we have added the immigration inflow \( (IM) \) in Okun’s law and it does not change the other coefficients substantially. Looking at \( R^2 \), we note that adding immigration raises the value significantly, which means that this variable has an explanation to the change in unemployment. Since the sign is negative, it means that unemployment is reduced by an immigration inflow to the labor market. The t-values are quite high, with 16 degrees of freedom. We can conclude that all the coefficients are significant.

**Regression Results From Equation (3)**

\[ \Delta u_i = \beta_0 + \beta_1 Y_i + \beta_2 REF_i \]

In equation (3) we have added the refugees to the regression equation. This raises \( R^2 \) considerably, so the variable explains the unemployment well. Refugees seem to have a
negative effect on unemployment, which could be because refugees might join the labor force with the natives. As previously discussed, if immigrants that join the labor market have different skills than natives, it will be beneficial for everyone.

**Regression Results From Equation (4)**

\[ \Delta u_i = \beta_0 + \beta_1 Y_i + \beta_2 L_i \]

In the equation (4) we have added the immigration inflow as labor force in the Okun’s law to see the how it effects the change in unemployment rate. The group of labor force seems to have an even stronger negative effect on the unemployment than refugees, which could be expected. Although the adding work permits instead of refugees seems to lower the \( R^2 \), so the unemployment variable seems to be better explained if we only include refugees as a group.

One would have expected that the inflow of scarce competence should increase the marginal productivity of natives, and thus reduce unemployment. The t-values here are quite high as well, except the one for the labor force which is not significant.

**Regression Results From Equation (5)**

\[ \Delta u_i = \beta_0 + \beta_1 Y_i + \beta_2 L_i + \beta_3 REF_i \]

In the equation (5) we have refugees, labor force and the GDP growth rate as independent variables, describing the unemployment. We notice that the t-values gets too low to be significant. All the coefficients are still in the expected direction and the raise in \( \bar{R}^2 \) suggest that both the labor force and refugees describe unemployment in an efficient way (if we don’t consider the insignificance of the t-values). The problem with the low t-values suggests that there could be multicollinearity, which means that one or more of the independent variables is an imperfect linear combination of another independent variable.
5 SUMMARY AND CONCLUSION

This study shows that immigration can be beneficial for the economy in a country. If we recall the model by Borjas (1999) we can determine the wage level for natives after an inflow of immigrants into the labor market. Recall that this model assumes the natives and immigrants will possess the same skill level and thus compete with the same jobs. In the short run, natives will gain from capital they own as shown in figure 6, page 16, while native workers will lose.

To understand the results from the model by Altonji and Card (1991), we can look at the coefficient lambda, a percentage of how much immigrants add to the economy of a city, producing a local good. The model uses factors that will adjust the gross increases in the labor supply for net increases in the demand, caused by immigrants. If immigrants possess the same skills as the natives, the model tells us that the signs of lambda for unskilled workers, skilled workers and the fraction of the good $Y$ that is exported all are equal to each other (i.e. have the same numerical value). If the lambda coefficient equals to the value of 1, this specifically means that a 1 percentage increase in the population of immigrants (for a city) adds nothing to the economy and we can see that in this case that immigrants reduce native wages, due to competition in the labor market. Also, this reduction of per capita labor supply is proportional to the reduction in wages (for natives and existing immigrants), multiplied by the elasticity for the labor supply. If the elasticity of the labor supply is between zero and one, it means that the decrease in native labor supply is between 0-0.5 percentages.

The empirical section of this study includes a regression analysis, where several regressions are run to see how change in unemployment rate can be explained by both immigration inflow and growth in the gross domestic product in Sweden. The result shows a negative correlation between both change in unemployment rate and gross domestic product, and with immigrants (refugees and work permits), so we can conclude that immigrants increase the labor force in Sweden into some extent. One would think that immigrants increase the unemployment due to
an increased competition on the labor market, but the empirical study suggests the contradiction.

The negative correlation between unemployment and gross domestic product in all five regression equations, suggests that immigration increase the economic growth in Sweden, due to an increase in purchasing power of products on the market.

Regression equation (5) includes immigrants divided into two groups (refugees and work permits). The result shows in-significance, looking at the t-values and comparing them with a critical value. It is hard to have significance in a multifactor regression model when the dependent is explained by many independent variable, since it can give arise for multicollinearity. But if we do not consider the low t-values, all the variables are in the expected direction as before and the raise in the value for $\bar{R}^2$, comparing with equation (5) tells us that all the independent variables explain the dependent variable in an efficient way.

This study suggests that immigration benefits both the economic growth and the labor market. However, it does not tell you how much it benefits the natives. An extended analysis could be done by investigating further to what extent and magnitude this affects a country.
REFERENCES


## APPENDIX

### Table 1

<table>
<thead>
<tr>
<th>Years</th>
<th>Unemployment Rate</th>
<th>GDP growth</th>
<th>Immigrants</th>
<th>Immigrants as Labor force</th>
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Table 2

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MATHEMATICAL APPENDIX

Model by Card and Altonji (1991)

1. Product market equilibrium:

\[ Y = P_s D_s(q, w_s) + P_u D_u(q, w_u) + D_x(q) \]

2. Equilibrium in the local labor market:

\[ (2a) L_s P_s(w_s, q) = Y c_1(w_s, w_u) \]
\[ (2b) L_u P_u(w_u, q) = Y c_2(w_s, w_u) \]

Assumptions:

Elasticity of labor demand for skill group \( i \) with respect to the wage of group \( j \)
\[ \eta_{ij} = \left( \frac{\partial c_i}{\partial w_j} \right) \frac{w_j}{c_i}, \quad i = 1, 2 \quad j = s, u \]

Elasticity of labor supply, \( \varepsilon_i \) of group \( i \)
\[ \varepsilon_i = \left( \frac{\partial L_i}{\partial w_i} \right) \frac{w_i}{L_i}, \quad i = s, u \]

The differential of (1):

\[ dY = D_s dP_s + P_s \frac{\partial D_s}{\partial q} dq + D_u dP_u + P_u \frac{\partial D_u}{\partial q} dq + \frac{\partial D_x}{\partial q} dq \]

Collecting terms:

\[ (1') dY = D_s dP_s + D_u dP_u + \left( P_s \frac{\partial D_s}{\partial q} + P_u \frac{\partial D_u}{\partial q} + \frac{\partial D_x}{\partial q} \right) dq \]

Differential of (2a):

\[ L_s dP_s + P_s \frac{\partial L_s}{\partial w_s} dw_s + P_s \frac{\partial L_s}{\partial q} dq = c_1 dY + Y \frac{\partial c_1}{\partial w_s} dw_s + Y \frac{\partial c_1}{\partial w_u} dw_u \]
Using the second part of the assumption (A1)

\[(2a') \] \[ L_s dP_s + P_s \frac{\partial L_s}{\partial w_s} \; dw_s = c_1 dY + Y \frac{\partial c_1}{\partial w_s} \; dw_s + Y \frac{\partial c_1}{\partial w_u} \; dw_u \]

The differential of (2b):

\[ L_u dP_u + P_u \frac{\partial L_u}{\partial w_u} \; dw_u + P_u \frac{\partial L_u}{\partial q} \; dq = c_2 dY + Y \frac{\partial c_2}{\partial w_s} \; dw_s + Y \frac{\partial c_2}{\partial w_s} \; dw_u + Y \frac{\partial c_2}{\partial w_u} \; dw_u \]

Using the second part of the assumption (A1):

\[(2b') \] \[ L_u dP_u + P_u \frac{\partial L_u}{\partial w_u} \; dw_u = c_2 dY + Y \frac{\partial c_2}{\partial w_s} \; dw_s + Y \frac{\partial c_2}{\partial w_u} \; dw_u \]

Assuming CRS and perfect competition, a function \( q \), representing the unit price of the local output can be described as following:

\[ q = c(w_u, w_s) \]

Which implies:

\[ dq = \frac{\partial c}{\partial w_s} \; dw_s + \frac{\partial c}{\partial w_u} \; dw_u \]

\[ dq = \frac{\partial c \; w_s}{c \; w_s} \; dw_s + \frac{\partial c \; w_u}{c \; w_u} \; dw_u \]

\[ dq = \theta_s \frac{dw_s}{w_s} + \theta_u \frac{dw_u}{w_u} \]

Where \( \theta_s \) and \( \theta_u \) is the share of the value of output that is paid as wages to skilled and unskilled workers, respectively.

Recall that \( P_s L_s(w_s, q) = Y c_1(w_s, w_u) \) and thus:

\[ c_1 = \frac{P_s L_s}{Y} \]

Recall also that:
\[ c_1 = \frac{\partial c}{\partial w_s} \]

Substituting \((1')\) into \((2a')\):

\[ L_s dP_s + P_s \frac{\partial L_s}{\partial w_s} \, dw_s = \]

\[ = c_1 \left[ D_s dP_s + D_u dP_u + \left( P_s \frac{\partial D_s}{\partial q} + P_u \frac{\partial D_u}{\partial q} + \frac{\partial D_x}{\partial q} \right) d q \right] + Y \frac{\partial c_1}{\partial w_s} \, dw_s + Y \frac{\partial c_1}{\partial w_u} \, dw_u \]

Eliminating \( \frac{dq}{q} \) (and completing some elasticities):

\[ L_s dP_s + P_s L_s \frac{\partial L_s}{\partial w_s} \frac{dw_s}{w_s} = \]

\[ = c_1 \left[ D_s dP_s + D_u dP_u + \left( P_s \frac{\partial D_s}{\partial q} + P_u \frac{\partial D_u}{\partial q} + \frac{\partial D_x}{\partial q} \right) q \left( \frac{dw_s}{w_s} + \frac{dw_u}{w_u} \right) \right] + Y c_1 \frac{\partial c_1}{\partial w_s} \frac{dw_s}{w_s} + Y c_1 \frac{\partial c_1}{\partial w_u} \frac{dw_u}{w_u} \]

Collecting terms:

\[ L_s dP_s - c_1 D_s dP_s - c_1 D_u dP_u = \]

\[ = Y c_1 \frac{\partial c_1}{\partial w_s} \frac{dw_s}{w_s} + Y c_1 \left( P_s \frac{\partial D_s}{\partial q} + P_u \frac{\partial D_u}{\partial q} + \frac{\partial D_x}{\partial q} \right) q \left( \frac{dw_s}{w_s} + \frac{dw_u}{w_u} \right) \]

Using \( P_s L_s = Y c_1 \) and the definition:

\[ \gamma = \left( P_s \frac{\partial D_s}{\partial q} + P_u \frac{\partial D_u}{\partial q} + \frac{\partial D_x}{\partial q} \right) q \]

Of the elasticity of demand with respect to \( q \), we have:

\[ \frac{L_s dP_s - c_1 D_s dP_s - c_1 D_u dP_u}{Y c_1} = \]
\[ \frac{\partial c_1 w_s}{\partial w_s c_1} \cdot \frac{dw_s}{w_s} + \gamma \left( \theta_s \frac{dw_s}{w_s} \right) - \frac{\partial L_s w_s}{\partial w_s} \cdot \frac{dw_s}{w_s} + \frac{\partial c_1 w_u}{\partial w_u c_1} \cdot \frac{dw_u}{w_u} + \gamma \left( \theta_u \frac{dw_u}{w_u} \right) \]

Rearranging and using the definition of \( \varepsilon_s \)

\[ \frac{L_s dP_s - c_1 D_s dP_s - c_1 D_u dP_u}{Yc_1} = \]

\[ \left( \frac{\partial c_1 w_s}{\partial w_s c_1} + \gamma \theta_s - \varepsilon_s \right) \frac{dw_s}{w_s} + \left( \frac{\partial c_1 w_u}{\partial w_u c_1} + \gamma \theta_u \right) \frac{dw_u}{w_u} \]

Noting that \( \frac{dw_s}{w_s} = \Delta \log w_s \), it appears that \( \gamma \) must possess a negative sign.