MÄLARDALEN UNIVERSITY
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MASTER THESIS IN MATHEMATICS/ APPLIED MATHEMATICS

A Quantitative Risk Optimization of Markowitz Model

An Empirical Investigation on Swedish Large Cap List

by

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ABSTRACT

This paper is an empirical study on Harry Markowitz work on Modern Portfolio Theory. The model introduced by him assumes the normality of assets' return. We examined the OMX Large Cap List[1] by mathematical and statistical methods for normality of assets' returns. We studied the effect of the parameters, Skewness and Kurtosis for different time series data. We tried to figure it out which data series is better to construct a portfolio and how these extra parameters can make us better informed in our investments.

[1] We have chosen 42 stocks from this list from different sectors of length 10 years. The complete Large Cap list is available at appendix X.

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Introduction

The aim of this paper is to construct an empirical study on the concept, Modern Portfolio Theory. The method is appreciated by scientists and without any doubt it is the most practical investment model ever introduced. We are about to introduce the model and its components fully, also covering the mathematical development of the model. But we are not going to give just an introduction to this model.

The first part of this paper, “Theory of the Modern Portfolio Theory” gives a broad view on the theory to the reader. Almost all the parameters and components of the basic model defined in this part. We tried to be careful with references and choose the best literature in order to give this opportunity to the reader to deepen his/her knowledge by referring to these sources. Some historical facts, the risk and reward analysis, mathematical development of the model, diversification and finally some other concepts introduced fully in this section of the paper.

The second part of this paper under title “Construction of the Model on Excel” shows how we established the model on excel for further investigations. This part is brief and references introduced can help the reader to get a better understanding of the process while referring to the excel file provided by this study can also help the reader for these calculations.

The last section under title “Empirical investigation” is the main pat of this research. In the first part we question the validity of one of the critical assumptions of the model and by some statistical test we support our claim, then we introduce a new ratio to handle this inefficiency regarding the model and finally we test these two ratios against each other by different combination of some extra parameters introduced during the process.

In the following part, “Data and Methodology” we introduce the type of the data under use for this study and some practical information about the data.

Data and Methodology

The data to investigate consists of 42 risky stocks listed on the Swedish stock exchange, Large Cap list, one Index and a risk-free government bond. The data is chosen for a period of 10 years, which is aimed to cover large events on the stock market. The aim to choose this period, 1997-2007, is to consider the extreme market events of August of 1998 as well as September 11, 2001 incident. Using this data set, we separate it into two parts, and we define the first period of the data set as Historical Data and the latter as Future Data. Throughout this paper they are referred as historical and future data. The data is analyzed in 5 different time scales, daily, weekly, monthly, quarterly and yearly. For each time period of 5 years, 4 different types of portfolios and each in different time scales constructed and studied.

Practically in analysis of the data there are always some missing cells due to discrepancies or simply the fact that no trade took place under those dates, to solve this issue we considered no changes in the prices occurred during those dates and consequently the assets' return was zero on those dates.

The risk-free interest rate is extracted from the data on SSVX30 (which is the government bond) for both periods and the volatility of the market is studied from the SIXRX (RT) (which is the Index or Benchmark). The portfolio is constructed by Markowitz Model, where we emphasized it as the traditional model compared with what we did adjustments to the parameters of this model.

Theory on Modern Portfolio Theory

Modern Portfolio Theory (MPT)

Modern Portfolio Theory (MPT) is not as modern as it implies in first glance. Like other theorems and models, which went through mysteries, MPT has its own story too. But it is always so that one gets lucky and wins whole the pot. The insight for which Harry Markowitz (born August 24, 1927) received the Nobel Prize was first published 1952 in an article entitled "Portfolio Selection". The article later expanded to a book by Markowitz at 1959, "Portfolio Selection: Efficient Diversification of Investments". The quantitative approach of the model existed far back in time, and they were modeled on the investment trusts of the England and Scotland, which began in the middle of the nineteenth century. Where Gallati [1] cites a quote about diversification which showed that it happened also earlier in time, where in Merchant of Venice, Shakespeare put the words on merchant Antonio who says;

Figure 1 - Harry M. Markowitz[2]

My ventures are not in one bottom trusted
Nor to one place; nor is my whole estate
Upon the fortune of this present year;
Therefore, my merchandise makes me not sad.

Prior to Markowitz article, 1952, Hicks mentioned the necessity of improvements on theory of money in 1935. He introduced risk in his analysis, and he stated "The risk-factor comes into our problem in two ways: First, as affecting the expected period of investment, and second, as affecting the expected net yield of investment." Gallati [1] also mentioned in his book that he could not demonstrate a formula relating risk of individual assets to risk of the portfolio as whole.

Since this work is based on MPT we will consider the model developed by Markowitz and his work on mean-variance analysis. He states that the expected return (mean) and variance of returns of a portfolio are the whole criteria for portfolio selection. These two parameters can be used as a possible hypothesis about actual behaviour and a maxim for how investors ought to act.

It is essential to understand the intimates of Markowitz model. It is not all about offering a good model for investing in high return assets. It might be interesting to know that whole the model is based on an economic fact, "Expected Utility". In economic term the concept of utility is based on the fact that different consumers have different desires and they can be satisfied in different ways. Do not forget that we mentioned two parameters, risk and return. It will make more sense to you when we go in the explaining diversification of a portfolio. In behavioural finance we can explain it so; Investors are seeking to maximize utility.

[2] Internet reference: http://www.depotanalyse24.de/portfoliotheorie.html

Consequently if all investors are seeking to maximize the utility, so all of them must behave in essentially the same way! Which this consistency in behaviour can suggest a very specific statement about their aggregate behaviour. It helps us to reach some description for future actions. We will talk more about this in next sections.

Every model or theory is based on some assumption, basically some simplification tools. Markowitz model relies on the following assumptions[3];

- Investors seek to maximize the expected return of total wealth.
- All investors have the same expected single period investment horizon.
- All investors are risk-adverse, that is they will only accept a higher risk if they are compensate with a higher expected return.
- Investors base their investment decisions on the expected return and risk.
- All markets are perfectly efficient.

By having these assumptions in mind, we will go through some concepts and terminologies that will make us understand the model constructed in further part of this paper.

Risk and Reward (Mean and Variance Analysis)

As mentioned above Markowitz model relies on balancing risk and return, and it is important to understand the role of consumer's preferences in this balance. There are different methods to calculate risk and return and the choice of these methods can change the result of our calculations dramatically. The following sections describe these methods in brief and we motivate our choice by mathematical proof.

By assumption for the Markowitz model, investors are risk averse. Assuming equal returns, the investor prefers the one with less risk, which implies that an investor who seeks higher return must also accept the higher risk. There is no exact formula or definition for this and it is totally dependent on individual risk aversion characteristics of the investor.

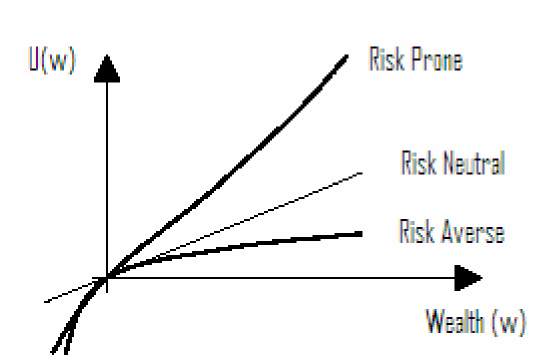


Figure 2 - Utility Curve for Investors with Different Risk Preferences[4]

[3] The assumptions are cited from the WebCab Components home page, the PDF file is available at internet reference [2].
[4] Internet Reference: http://facweb.furman.edu/~dstanford/mecon/b1.htm

A further assumption is that risk and return preferences of an investor can be described via a quadratic utility function. This means when plotted on a graph, your utility function is a curve with decreasing slope, for larger risk. Where *w* is an indicator for wealth and *U* is a quadratic utility function. We have,

$$U(w) = w - w^2$$

A consumer's utility is hard to measure. However, we can determine it indirectly with consumer behaviour theories, which assume that consumers will strive to maximize their utility. Utility is a concept that was introduced by Daniel Bernoulli. He believed that for the usual person, utility increased with wealth but at a decreasing rate. Figure 2 shows the utility curve for investors with different risk preferences.

Risk aversion can be determined through defining the risk premium, which by Markowitz defined to be the maximum amount that an individual is prepared to give up to avoid uncertainty. It is calculated as the difference between the utility of the expected wealth and the expected utility of the wealth.

$$U[E(w)] - E[U(w)]$$

This allows us to determine the characteristic of the behaviour of the investor regarding risk;

- If $U[E(w)] > E[U(w)]$, then the utility function is concave and the individual is risk averse;
- If $U[E(w)] = E[U(w)]$, then the utility function is linear and the individual is risk neutral;
- If $U[E(w)] < E[U(w)]$, then the utility function is convex and the individual is risk seeking.

It is what was defined by Markowitz (1959) and cited by Amenc et al [3]. Figure 2 gives a graphical interpretation of what was stated above.

A Short Note on Mean Calculation

Before we move to the main challenge of MPT, the risk, we determine a method to calculate the first parameter in use for constructing the model. It is possible to calculate mean of an investment with several methods, but mainly arithmetic and geometric. We have chosen geometric method and in following sections we motivate our choice by mathematical proofs and examples. Before all these, we introduce them briefly;

Arithmetic Mean

The arithmetic mean of a list of numbers is the sum of all the members of the list divided by the number of the items in the list.

$$\bar{a} = \frac{1}{n}\sum_{i=1}^{n} a_i = \frac{1}{n}(a_1 + \cdots + a_n)$$

Where,

$\bar{a}$ Arithmetic mean

a_i Sample's data where $(i = 1, 2, \cdots, n)$

n Number of data set's memeber

Geometric Mean

The Geometric Mean of a collection of positive data is defined as the *n*th root of the product of all members of the data set, where *n* is the number of members. The Geometric Mean of the data set $\left[a_1, a_2, ..., a_n\right]$ is:

$$\left(\prod_{i=1}^{n} a_i\right)^{1/n} = \sqrt[n]{a_1 \cdot a_2 \cdots a_n}$$

Where,

a_i Sample's data where $(i = 1, 2, \cdots, n)$

n Number of data set's memeber

Geometric Versus Arithmetic Mean

Mathematics makes it easier for us to illustrate a problem more concrete. The comparison between these two average methods is possible by Jensen's Inequality. It states that for any random variable *X*, if *g(x)* is a convex function, then

$$Eg(X) \geq g(EX)$$

Equality holds if and only if, for every line *a+bX* that is tangent to *g(x)* at *x=EX*, $P(g(X) = a + bX) = 1$.

This theorem can be used to prove the inequality between these two methods of averaging. If $a_1, a_2, \ldots, a_n$ are positive numbers, defined as;

$$a_A = \frac{1}{n}(a_1 + a_2 + \cdots + a_n), \qquad \text{(Arithmetic mean)}$$

$$a_G = [a_1 a_2 \cdots a_n]^{1/n}. \qquad \text{(Geometric mean)}$$

Where an inequality relating these means is

$$a_G \leq a_A.$$

In order to apply the Jenson's Inequality, let *X* be a random variable with range $a_1, a_2, \cdots, a_n$ and $P = (X = a_i) = 1/n, i = 1, \cdots, n.$ since *log x* is a concave function, Jensen's Inequality shows that $E(\log X) \leq \log(EX);$ so,

$$\log a_G = \frac{1}{n}\sum_{i=1}^{n} \log a_i = E(\log X) \leq \log(EX) = \log\left(\frac{1}{n}\sum_{i=1}^{n} a_i\right) = \log a_A,$$

So $a_G \leq a_A$.[5]

[5] The proof is taken from Statistical Inference by Casella et al. [4].

When to use Geometric Mean

When it comes to the calculation of the growth rate of a portfolio, Markowitz suggests the use of Geometric Mean method. He argues that this method of calculation will guarantee us a more realistic result in compare with Arithmetic and Compounding Average methods. Since we should consider reinvestment of the original amount invested plus its growth in the following period, we should use a method to cumulate the growth in investment at the end of each period. Arithmetic method can not fulfil this criterion. The second reason becomes more touchable when we consider the average of two extremes. Consider two real numbers as ratios, 100000 and 0.00001. The average calculated by Geometric mean is equal to 1, while the arithmetic method gives us an average of approximately 50000. This example is a result of the above argument.

Variance and Standard Deviation

According to Wackerly [5], variance of a sample of measurements $a_1, a_2, \ldots, a_n$ is the sum of the square of the differences between the measurements and their mean, divided by $n-1$, where n denoted the sample size. The sample variance is denoted as:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(a_i - \bar{a}\right)^2$$

Where,

s The variance

n Number of the sample's members

a_i The corresponding member of the data set where $i = 1, 2, \cdots, n$

$\bar{a}$ The mean of the sample

When referring to the population variance, we denote it by the following symbol σ^2. A complete proof for the mean and variance just presented above is available in Appendix 1 and 2 respectively. This formula is an unbiased estimator of the population variance.

The standard deviation of a sample of measurements is the positive square root of the variance, which can be denoted as:

$$s = \sqrt{s^2}$$

The population standard deviation is denoted by $\sigma = \sqrt{\sigma^2}$. The proof of the standard deviation is similar to the variance, but it is squared.

For both the variance and standard deviations in these cases, they are assumed to be unbiased estimators for σ , meaning that the random variables $a_1, a_2, \ldots, a_n$ are assumed to be normally distributed.

Description of Standard Deviation in Portfolio theory

In portfolio theory the standard deviation measures how much the return of a portfolio or the stock moves around the average return. The standard deviation grows as returns move further above or below the average. This is considered as a measure of risk, where most investors only care about the standard deviation of a stock in one direction, above or below the mean. For investors who are long stocks do not want returns to dip below mean, but would be happy with returns that exceed it. If the returns on a portfolio or stock are normally distributed, then the standard deviation is a valid measure of the returns that are below the mean Markowitz [6]. If returns are not normal but skewed, then the standard deviation is less meaningful. This will be explained more later on.

Annualizing Returns and Standard deviation

To annualize returns and standard deviations from sets of periodic data, it is important to realize what type of mean calculations you are using and how it works. Since there are two different methods of annualizing returns and standard deviations, in the case of either arithmetic mean or geometric mean calculations.

According to Chincarini [7], in the case of the arithmetic mean or average mean as it is also called, we should have in mind that this method assumes no compounding and the set of equations for annualizing the return and standard deviations is:

$$\overline{R}_{annual} = m * \left(\overline{R}_{Periodic}\right)$$
$$\sigma_{annual} = \sqrt{m} * \sigma_{Periodic}$$

Where,

$\overline{R}_{annual}$ = Annualized return

m = The number of periods per year

$\overline{R}_{Periodic}$ = The Periodic return

σ_{annual} = Annualized standard deviation

In the case of the geometric mean and standard deviation we should have in mind that it assumes compounding. We then have the following set of equations for annualizing compounding returns and standard deviations:

$$\overline{R}_{annual} = \left(1 + \overline{R}_{Periodic}\right)^m - 1$$
$$\sigma_{annual} = \sqrt{\left[\sigma^2_{Periodic} + \left(1 + \overline{R}_{Periodic}\right)^2\right]^m - \left(1 + \overline{R}_{Periodic}\right)^{2m}}$$

Where,

$\overline{R}_{annual}$ = Annualized return

m = The number of periods per year

$\overline{R}_{Periodic}$ = The Periodic return

σ_{annual} = Annualized standard deviation

Mathematics of the Markowitz Model

Markowitz model involves some mathematics. Before implementing the model in Excel it might be better to develop the model mathematically to get a better understanding. In the previous sections of the theory part we introduced some basic definitions and building blocks of Markowitz model, risk and return. Markowitz model makes it possible to construct a portfolio with different combinations where short sales and lending or borrowing might be allowed, or not. The case might be the best alternative to consider for the purpose of our paper, which is clarifying the construction of a portfolio when short sales are allowed and riskless borrowing and lending is possible. The Markowitz model is all about maximizing return and minimizing risk, but simultaneously.

The investor preferences are the most important parameter which is hidden in the balancing of the two parameters of Markowitz model. We should be able to reach a single portfolio of risky assets with the least possible risk that is preferred to all other portfolios with the same level of return. Let's consider the following coordinate system of expected return and standard deviation of return. It will help us to plot all combinations of investments available to us. Some investments are riskless and some are risky. Our optimal portfolio will be somewhere on the ray connecting risk free investments R_F to our risky portfolio and where the ray becomes tangent to our set of risky portfolios or efficient set it has the highest possible slope, in Figure 3 this point is showed by *B*. Different points on the ray between tangent point and interception with expected return coordinate represents combination of different amounts possible to lend or borrow to combine with our optimal risky portfolio on intersection of tangent line and efficient set.

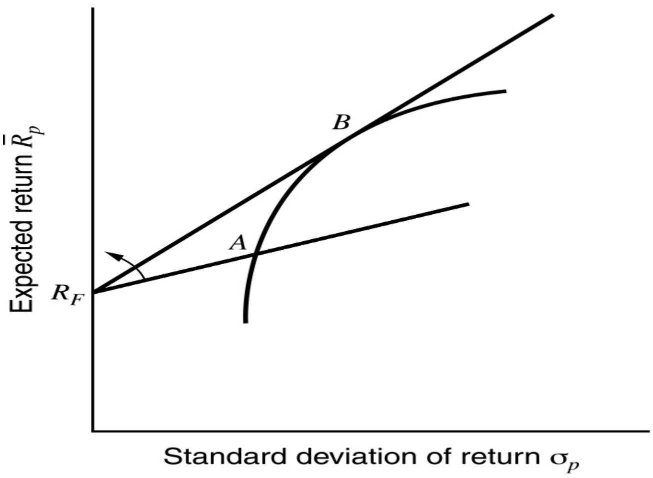


Figure 3 - Combinations of the risk less asset in a risky portfolio (Gruber et al. [9])

As we mentioned above, the ray discussed has the greatest slope. It can help us to determine the ray. The slope is simply the return on the portfolio, R_P minus risk-free rate divided by standard deviation of the portfolio, σ_P. Our task is to determine the portfolio with greatest ratio of excess return to standard deviation θ. In mathematical terms we should maximize θ (Later so called Sharpe ratio).

$$\theta = \frac{\bar{R}_P - R_F}{\sigma_P}$$

This function is subject to the constraint,

$$\sum_{i=1}^{N} X_i = 1$$

Where X_is are the samples members, also can be random variables. The constraint can be expressed in another way, Lintnerian, which considers an alternative definition for short sales. It assumes that when a stock sold short, cash did not received but held as collateral. The constraint with Lintner definition of short sales is[6],

$$\sum_{i=1}^{N} |X_i| = 1$$

The above constrained problem can be solved by Lagrangian multipliers. We consider an alternative solution, by substituting the constraint in the objective function, where it will become maximized as in unconstrained problem. By writing R_F as R_F times one,

$$R_F = 1R_F = \left(\sum_{i=1}^{N} X_i \right) R_F = \sum_{i=1}^{N} (X_i R_F)$$

By stating the expected return and standard deviation of the expected return in the general form we get,

$$\theta = \frac{\sum_{i=1}^{N} X_i (\bar{R}_i - R_F)}{\left[\sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} X_i X_j \sigma_{ij} \right]^{1/2}}$$

Now we have the problem constructed and ready to solve. It is a maximization problem and solved by getting the derivatives of the function with respect to different variables and equating them to zero. It gives us a system of simultaneous equations,

[6] Gruber et al. [9].

$$1.\frac{d\theta}{dX_1}=0$$

$$2.\frac{d\theta}{dX_2}=0$$

$$\vdots$$

$$N.\frac{d\theta}{dX_N}=0$$

Let's consider here the Lagrange theorem[7],

> Let X be open in R^n and $f,g:X\to R$ be functions of class C. Let $S=\{x\in X\,|\,g(x)=c\}$ denote the level set of g at highest c. Then if $f|s$ (the restriction of f to S) has an extremum at a point $x_0\in S$ such that $\nabla g(x_0)\neq 0$. There must be some scalar λ such that $\nabla f(x_0)=\lambda\nabla g(x_0)$.

Where λ is called a Lagrange multiplier. Now we show how it proceeds and then its application on our case;

1. Form the vector equation, $\nabla f(x)=\lambda\nabla g(x)$.
2. Solve the system,

$$\begin{cases}\nabla f(x)=\lambda\nabla g(x)\\ g(x)=c\end{cases}$$

For x and λ. By extension of this problem we have *n+1 equation* in *n+1* unknown $x_1,x_2,x_3,\ldots,x_n,\lambda$,

$$\begin{cases}f_{x_1}(x_1,x_2,\ldots,x_n)=\lambda g_{x_1}(x_1,x_2,\ldots,x_n)\\ f_{x_2}(x_1,x_2,\ldots,x_n)=\lambda g_{x_2}(x_1,x_2,\ldots,x_n)\\ \vdots\\ f_{x_n}(x_1,x_2,\ldots,x_n)=\lambda g_{x_n}(x_1,x_2,\ldots,x_n)\\ g(x_1,x_2,\ldots,x_n)=c\end{cases}$$

Where the solution for $x=(x_1,x_2,\ldots,x_n)$, along with any other point satisfying $g(x)=c$ and $\nabla g(x)=0$, are candidates for extrema for the problem.

[7] James Stewart [10].

3. Finally we determine nature of *f* (as maximum, minimum or neither) at the critical points found in step 2.

As you see this method reduces a problem in *n* variables with *k* constraints to a solvable problem in *n+k* variables with no constraint. This method introduces a new scalar variable, the Lagrange multiplier, for each constraint and forms linear combination involving the multipliers as coefficients.

Before we start to mention Lagrange theorem we got to the point that in order to solve the maximization problem we need to take derivatives of the ratio θ. We rewrite θ in the following form;

$$\theta = \left[\sum_{i=1}^{N} X_i (\bar{R}_i - R_F) \right] \left[\sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} X_i X_j \sigma_{ij} \right]^{-1/2}$$

As it is written above, the ratio consists of multiplication of two functions. To derivate this ratio we need to use Product Rule and as the second term suggests where it has power $-1/2$ another rule of calculus, the Chain Rule must be applied. After applying the chain rule, we use product rule and we get,

$$\frac{d\theta}{dX_k} = \left[\sum_{i=1}^{N} X_i (\bar{R}_i - R_F) \right] \left[\left(-\frac{1}{2} \right) \left(\sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} X_i X_j \sigma_{ij} \right)^{-3/2} \times \left(2 X_k \sigma_k^2 + 2 \sum_{\substack{i=1 \\ j \neq k}}^{N} X_j \sigma_{kj} \right) \right]$$
$$+ \left[\sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} X_i X_j \sigma_{ij} \right]^{-1/2} \times \left[\left(\bar{R}_k - R_F \right) \right] = 0$$

If we multiply the derivative by

$$\left(\sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} X_i X_j \sigma_{ij} \right)^{1/2}$$

And rearrange, then;

$$-\left[\frac{\sum_{i=1}^{N} X_i (\bar{R}_i - R_F)}{\sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} X_i X_j \sigma_{ij}} \right] \left(X_k \sigma_k^2 + \sum_{\substack{i=1 \\ j \neq k}}^{N} X_j \sigma_{kj} \right) + \left[\left(\bar{R}_k - R_F \right) \right] = 0$$

Where we define λ as Lagrange multiplier,

$$\frac{\sum_{i=1}^{N} X_i(\bar{R}_i - R_F)}{\sum_{i=1}^{N} X_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} X_i X_j \sigma_{ij}}$$

Yields,

$$-\lambda\left(X_k\sigma_k^2 + \sum_{\substack{i=1 \\ j\neq k}}^{N} X_j\sigma_{kj} \right) + \left[\left(\bar{R}_k - R_F\right)\right] = 0$$

By multiplication,

$$-\left(\lambda X_k\sigma_k^2 + \sum_{\substack{i=1 \\ j\neq k}}^{N} \lambda X_j\sigma_{kj} \right) + \left[\left(\bar{R}_k - R_F\right)\right] = 0$$

Now by extension

$$\frac{d\theta}{dX_i} = -(\lambda X_1\sigma_{1i} + \lambda X_2\sigma_{2i} + \cdots + \lambda X_i\sigma_i^2 + \cdots + \lambda X_{N-1}\sigma_{N-1i} + \lambda X_N\sigma_{Ni}) + \bar{R}_i - R_F = 0$$

We use a mathematical trick, where we define a new variable $Z_k = \lambda X_k$. The X_k are fraction to invest in each security, and Z_k are proportional to this fraction. In order to simplify we substitute Z_k for λX_k and move variance covariance terms to the left,

$$\bar{R}_i - R_F = Z_1\sigma_{1i} + Z_2\sigma_{2i} + \cdots + Z_i\sigma_i^2 + \cdots + Z_{N-1}\sigma_{N-1i} + Z_N\sigma_{Ni}$$

The solution of the above statement involves solving the following system of simultaneous equations,

$$\begin{aligned}
\bar{R}_1 - R_F &= Z_1\sigma_1^2 + Z_2\sigma_{12} + Z_3\sigma_{13} + \cdots + Z_N\sigma_{1N} \\
\bar{R}_2 - R_F &= Z_1\sigma_{12} + Z_2\sigma_2^2 + Z_3\sigma_{23} + \cdots + Z_N\sigma_{2N} \\
\bar{R}_3 - R_F &= Z_1\sigma_{13} + Z_2\sigma_{23} + Z_3\sigma_3^2 + \cdots + Z_N\sigma_{3N} \\
&\vdots \\
\bar{R}_N - R_F &= Z_1\sigma_{1N} + Z_2\sigma_{2N} + Z_3\sigma_{3N} + \cdots + Z_N\sigma_N^2
\end{aligned}$$

Now we have N equations with N unknowns. By solving for Zs we can get X_k, which are the optimum proportions to invest in stock *k*,

$$X_k = Z_k \Big/ \sum_{i=1}^{N} Z_i$$

Up to here we calculate the weights for the general form, where short sales are allowed and lending and borrowing is possible. For other form of portfolio constructions, we follow the same pattern but there might be other kinds of constraints defined.

Diversification

Despite what kind of role we have in finance world all of us might have heard this old English proverb "Do not put all your eggs in one basket" by the character Sancho Panza in Miguel de Cervantes Don Quixote[8]. It is simply what we call it here diversification. More specifically, diversification is a risk management technique that mixes a wide variety of investments within a portfolio. It is done to minimize the impact of any security on the overall portfolio performance. A great reason for anybody to choose mutual funds is because they are said to be well diversified. In order to have a diversified portfolio it is important that the assets chosen to be included in a portfolio do not have a perfect correlation, or a correlation coefficient of one.

Diversification reduces the risk on a portfolio, but not necessarily the return, and though it is referred as the only free lunch in finance. Diversification can be loosely measured by some statistical measurement, intra-portfolio correlation. It has a range from negative one to one and measures the degree to which the various asset in a portfolio can be expected to perform in a similar fashion or not.

Portfolio balance can be measured by some of these intra-portfolio correlations. As the sum approaches negative one the percentage of diversifiable risk eliminated reaches 100%. It is why it is called weighted average intra portfolio correlation. It is computed as[9]

$$Q = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j \rho_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j}$$

Where,

Q is the intra-portfolio correaltion

X_i is the fraction invested in asset i.

X_j is the fraction invested in asset j.

ρ_{ij} is the crrealtion between assets i and j.

n number of different assets.

Table one shows how diversifiable risk eliminated in relation with intra-portfolio correlation.

[8] http://www.riskythinking.com/articles/article13.php#herbison

[9] Internet Reference: http://www.freepatentsonline.com/20030088492.html

Intra-portfolio correlation	Percentage of diversifiable risk eliminated
1	0,0%
0,75	12,5%
0,5	25,0%
0,25	37,5%
0	50,0%
-0,25	62,5%
-0,5	75,0%
-0,75	87,5%
-1	100,0%

Table 1 - Percentage of the diversifiable risk eliminated[10]

Now let's come back again to diversification. In order to understand how to diversify a portfolio we should understand the risk. According to Ibbotson et al. [11] risk has two components, systematic and unsystematic. Where market forces affect all assets simultaneously in some systematic manner it generates Systematic risk or what so called, undiversifiable risk. Examples are Bull markets, Bear markets, wars, changes in the level of inflation. The other component of risk is unsystematic one, or so called diversifiable risk. These are idiosyncratic events that are statistically independent from the more widespread forces that generate undiversifiable risk. The examples of a diversifiable risk are Acts of God (Hurricane or flood), inventions, management errors, lawsuits and good or bad news affecting one firm.

As defined above, Total risk of a portfolio is the result of summation of systematic and unsystematic risks. On average, the total risk of a diversified portfolio tends to diminish as more randomly selected common stocks are added to the portfolio. But, when more than about three dozen random stocks are combined, it is impossible to reduce a randomly selected portfolio's risk below the level of undiversifiable risk that exists in the market. Figure 4 shows the graphical interpretation of this. The straight line separates the systematic risk from unsystematic one, the systematic or undiversifiable risk lies under the straight line.

[10] These figures from Table 1 is taken from; M. Statman, "How Many Stocks Make a Diversified Portfolio?" *Journal of Financial and Quantitative Analysis 22* (September 1987), pp. 353-64. They were derived from E. J. Elton and M. J. Gruber, "Risk Reduction and Portfolio Size: An Analytic Solution," *Journal of Business* 50 (October 1977), pp. 415-37. Taken from Ross, Westerfield, and Jordan, "Fundamentals of Corporate Finance" 7th Edition (2006-11-14), pp. 406.

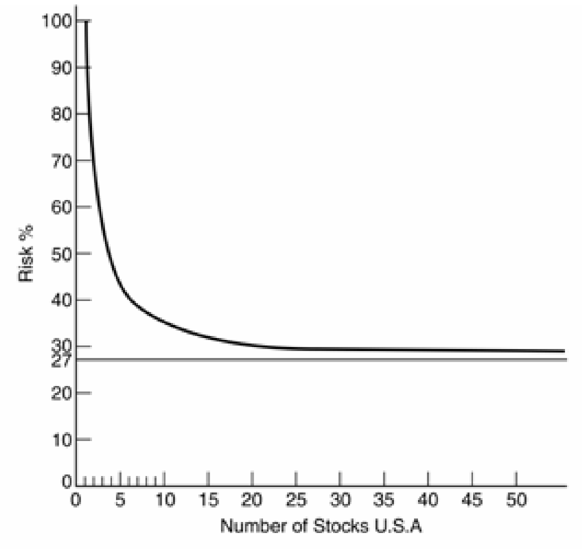


Figure 4 - The effect of number of securities on risk of the portfolio in the United States (Gruber et al [9])

Diversification in Markowitz model

Markowitz model suggests that it is possible to reduce the level of risk below the undiversifiable risk. Ibbotson [11] categorized Markowitz diversification on five basic interrelated concepts,

1. The Weights Sum to One: The first concept requires that the weights of the assets in the portfolio sum to 100%. Simply the investment weights are a decision variable, which is the main task for portfolio manager to determine them.

$$\sum_{i=1}^{N} x_i = 1$$

Where x represents weights or participation level of asset i in a portfolio that contains N assets. When the portfolio involves short sales, weights can be negative, but remember that they should not violate this concept. A portfolio which has negative weights for some assets is called leveraged portfolio or borrowing portfolio.

2. A Portfolio's Expected Return: It is the weighted average of the expected returns of the assets that make up the portfolio, the portfolio's expected rate of return for N-assets portfolio is,

$$E(R_p) = \sum_{i=1}^{N} x_i E(R_i) .$$

Where $E(R_i)$ is the security analysts forecast for expected rate of return from the *i*th asset.

3. The Objective: Investment weights chosen by portfolio managers should add up to an efficient portfolio which is:

- The maximum expected return in its risk-class, or, conversely.
- The minimum risk at its level of expected return.

The set of all efficient portfolios is called efficient frontier. This is the maximum return at each level of risk. The efficient frontier dominates all other investment opportunities.

4. Portfolio Risk: In contrast with expected return of a portfolio which is based on forecast, the risk of a portfolio is calculated from historical data available to the asset manager. The risk of the portfolio, or its variance should be broken into two parts, the variance which represents the individual risks and interaction between *N* candidate assets. This equation (double summation) represents the variance-covariance matrix and can be expanded and written in matrix form.

$$VAR(R_p) = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}$$

Where $\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$ and ρ_{ij} is correlation coefficient between assets *i* and *j*. In order to have a portfolio well-diversified according to Markowitz, the assets included in the portfolio should have low enough correlations between their rates of returns. As shown in the figure 5, a portfolio with correlation coefficient equal to zero gives the same level of return, but with a lower risk level, than a portfolio which the assets including it have a correlation coefficient of one. If an investment or portfolio manager achieves to include securities whose rates of return have low enough correlation, according to Markowitz, he or she can reduce a portfolio's risk below the undiversifiable level.

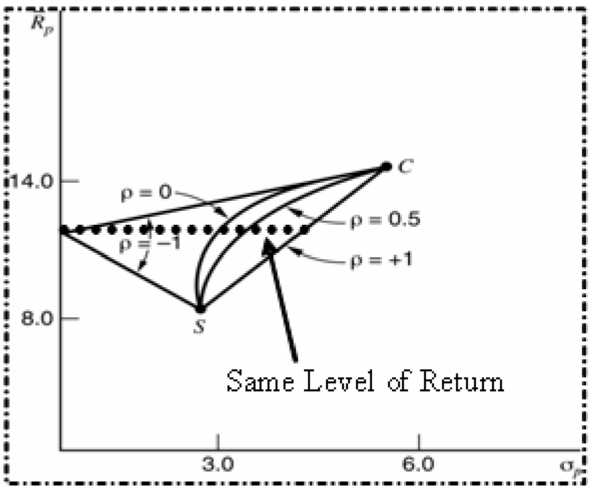


Figure 5 - Relationship between expected return and standard deviation of return for various correlation coefficients (Gruber et al. [9].)

5. The Capital Allocation Line: The last concept to consider on diversification by Markowitz is The Capital Allocation Line. This concept discusses the possibility of lending and borrowing at a risk free rate of interest provided by Markowitz model. An example can be a government treasury bill, where as the phrase risk free interest rate suggests the variance is zero.

Markowitz model gives the opportunity to the asset manager to combine a risky asset or a set of risky assets (a portfolio of risky assets) with a riskless asset. In next parts this concept will be more clarified when we explain all concepts in MPT one by one.

The Risk Free Asset

This asset is said to be a hypothetical asset which pays a risk-free return to the investor, with a variance and standard deviation equal to zero. Usually this type of assets issued by the government and can be referred to as government bond or Treasury bill (T-bill). But then it is also assumed that government dose not go bankrupt. In reality we can also conclude that there is no such thing as a risk-free asset, all financial instruments carry some degree of risk. But also that these risk free-rates are subject to inflation risk. The common notation of the risk-free asset is R_F.

The Security Market Line – (SML)

The Security Market Line is based on the CAPM model, where one believes that the correct measure of risk (systematic risk) is based on the market and called Beta. This means that the SML line is graphed by the CAPM equation.

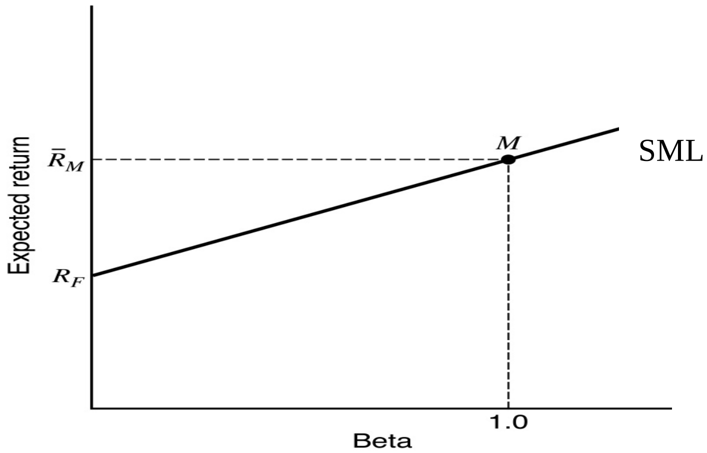


Figure 6 - The Security Market Line (Gruber et al. [9])

Here in the graph we can see that as the expected return increases so dose the risk (Beta). The SML line is based on the risk free rate R_F. We can then also see that since R_F is risk free it has a zero beta. When you go to the right of the graph, you will come to the market portfolio (M). The market portfolio is a hypothetical portfolio, consisting of all the securities that are available for an investor. That is why we have a beta of 1. The markets risk premium is determined by the slope of the SML line.

The Capital Market Line – (CML)

The difference in the Capital Market Line compared to the SML Line is that all investors on the market are taking some position on the CML line, by lending, borrowing or holding the market portfolio (A). The market value for the equity an investor holds is the same as for any other investor. Both of them own the same portfolio namely the market portfolio. The CML line considers the equity risk, standard deviation σ , while the SML line considers the market risk beta β . The CML line is derived by the following expression:

$$E(R_P) = R_F + \sigma_P \frac{E(R_M) - R_F}{\sigma_M}$$

The CML line also represents the highest possible Sharpe ratio possible. The CML line is derived by drawing a tangent line from the intercept point of the efficient frontier (or the optimal portfolio) to the point where the expected return equals the risk-free rate R_F .

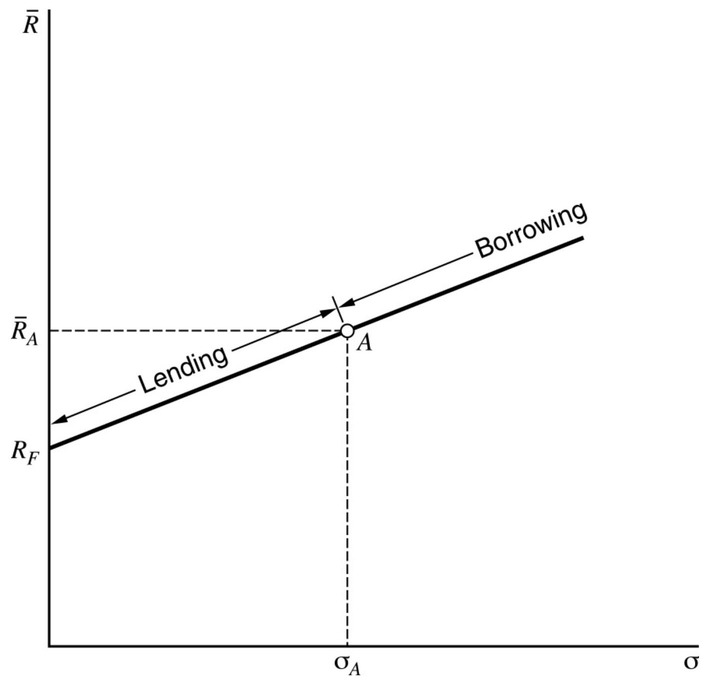


Figure 7 - The Capital Market Line (Gruber et al. [9])

The Security Characteristic Line – (SCL)

The SML line is a line of best fit through some data points. But statisticians call it a time-series regression line. The model uses a one period rate of return from some market index in time period t , it's denoted as $R_{M,t}$. Then to explain some rate of return from some asset, we de denote it by the index i for the ith asset. The characteristics line is used by many security analysts, in the form of estimating the undiversifiable and diversifiable risk of an investment. The Security Characteristic line is denoted by the following regression equation.

$$r_{i,t} = \alpha_i + \beta_i R_{M,t} + e_{i,t}$$

where,

$\alpha_i =$ The lines intercept/the alpha coefficient.

$\beta_i =$ The slope of the line/the beta coefficient.

$e_{i,t} =$ The unexplained residual return from asset *i* that occurs in time period *t*.

$R_{M,t} =$ Rate of return from some market index in the time period *t*.

From this expression we can rearrange and have the following explanation that $\alpha_i + e_{i,t}$ is the diversifiable return in time period t, and that $\beta_i R_{M,t}$ is the undiversifiable return in time period t.

The Capital Asset Pricing Model – (CAPM)

The CAPM model, evaluates the return on the asset in relation to the market return and the sensitivity of the security to the market. The CAPM model is also based on a set of axioms and concepts that are based on the theory of finance. Also the price of the risk in the CAPM model is defined by the difference between the expected rate of return for the market portfolio, and the return on the risk-free rate. This risk measure is called beta, which is defined as follows:

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

As we can see the beta is equal to the covariance between the return on asset *i* and the return on the market portfolio σ_{iM}, which is divided by the variance of the market portfolio σ_M^2. This also means that the risk-free rate has a beta of zero, the market portfolio a beta of one. We can then define the CAPM model as follows:

$$E(R_i) = R_F + \beta_i \underbrace{\left(E(R_M) - R_F\right)}_{\textit{Market}\ \text{Risk Premium}}$$

where,

$E(R_i)$ = Expected return on asset i

R_F = Risk-Free rate

β_i = Risk of asset i

$E(R_M)$ = Expected return on the market

From the CAPM model, we can also establish that at equilibrium the return on asset, less the risk-free rate; have a link to the return and the market portfolio which is linear. Also to note is that the market portfolio is built according to the Markowitz principles. The graphical representation of the CAPM is the security market line.

The Efficient Frontier and Market Portfolio

The efficient frontier emphasizes a geometric interpretation of asset combinations. In this frontier a market portfolio will be allocated which should be preferred by all investors, under the assumptions that all investor exhibit risk avoidance and prefer more return to less. This is also the fundamental of the Markowitz theory. The efficient frontier is also referred to as the "Markowitz frontier".

The efficient frontier is also subject to different assumptions about lending and borrowing, with the constraint combination of the risk-free asset. It also contains alternative assumptions about short sales. When considering these different assumptions to apply, there are also several different constraints that need to be handled. These assumptions are important concepts in portfolio theory if you want to find the optimal market portfolio.

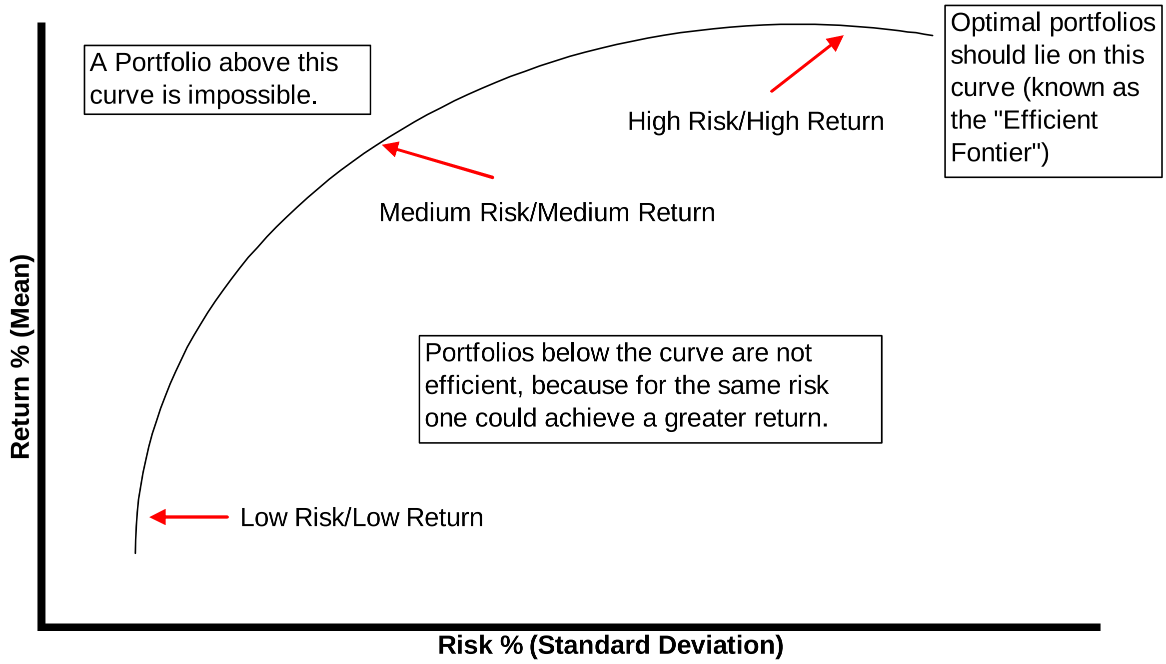


Figure 8 - The Efficient Frontier (Gruber et al. [9])

In the figure we can se that the efficient frontier will be convex. The explanation is that there is a risk and return characteristics of the portfolio that will change in a non-linear fashion as its component weighting are changed. The case is also that the portfolio risk is a function of correlation of the components assets, which also changes in a non-linear fashion as the weighting of the component assets change.

The next step is finding the optimal market portfolio by connect some chosen risk-free asset to the frontier, and then applying the Sharpe ratio which should be maximized. These two properties will give you two points on the graph, which you then make a straight line from. This line represents the lending part of a possible investment on the left side of the market portfolio. If you draw the line straight to the right also, you will be able to borrow and invest more in the market portfolio. This line that is connected to the efficient frontier is called the capital allocation line (CAL).

The following figure shows a graphical view of what I just described. Here the $E(R)$ stands for return, σ is the standard deviation, R_F is the risk-free asset and M stands for the market portfolio.

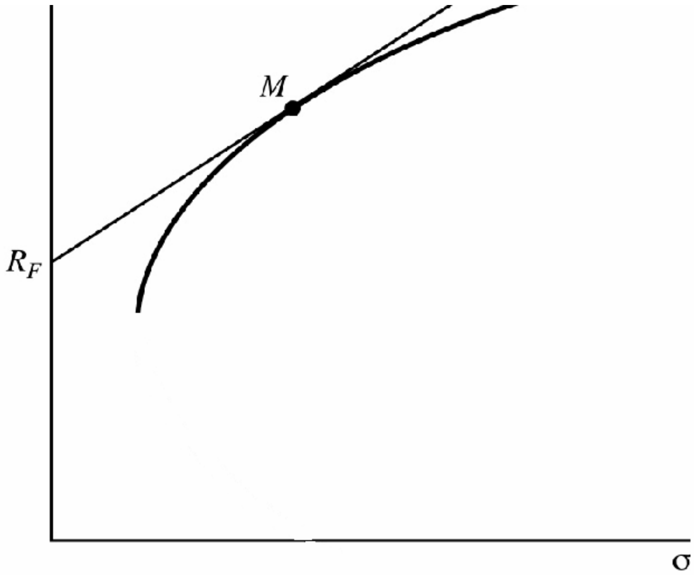


Figure 9 - Efficient Frontier with R_F and M (Gruber et al. [9])

The Sharpe Ratio

This ratio is a measurement for risk-adjusted returns and was developed by William F. Sharpe[11]. This is where the name the Sharpe ratio comes from. The Sharpe ratio is defined by

$$S_P = \frac{E(R_P) - R_F}{\sigma(R_P)}$$

where,

$\mathrm{E}(\mathrm{R_P}) =$ denotes, the expected return of the portfolio;

$\mathrm{R_F}$ = denotes, the return on the risk-free asset; and

$\sigma(\mathrm{R_P}) =$ denotes, the standard deviation of the portfolio returns.

This ratio measures the excess return, or the risk premium of a portfolio compared with the risk-free rate, and with the total risk of the portfolio, measured by the standard deviation. It is drawn from the capital market line, and it can be represented as follows:

$$\frac{E(R_P) - R_F}{\sigma(R_P)} = \frac{E(R_M) - R_F}{\sigma(R_M)}$$

This relation indicates that at equilibrium, this means that the Sharpe ratio of the portfolio to be evaluated and the Sharpe ratio of the market portfolio are equal. The Sharpe ratio

[11] William F. Sharpe won the Nobel Prize in economics for his development of the CAPM in 1990.

corresponds to the slope of the market line. If the portfolio is well diversified, then its Sharpe ratio is close to that of the market.

The Sharpe ratio in Portfolio theory

The Sharpe ratio provides a good basis for comparing portfolios, and is widely used by investment firms for measuring portfolio performance. In isolation, it dose not mean much. This even when managers speak of “good” and “bad” Sharpe ratios, they are speaking only in relative terms. E.g. if portfolio manager A has the highest Sharpe ratio of several managers, then he or she has the highest risk-adjusted return of the managers for that period.

Skewness

Skewness is a parameter that describes asymmetry in a random variable’s probability distribution. In other words a distribution is skewed if one of its tails is longer than the other. Skewness can be positive; this means that it has a long tail in the positive direction. It also can have a negative value, where it is called a negative Skewness. Consider the figure below where two distributions are plotted by the same mean μ, and standard deviation σ, but the one to the left is positively skewed (skewed to the right) and the one on the right is negatively skewed (skewed to the left).

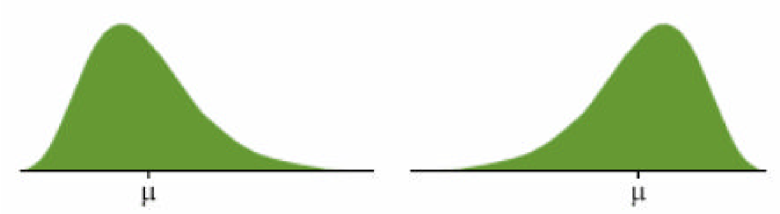


Figure 10 - PDFs with the same expectation and variance. [12]

Skewness is equal to zero where we have a perfect asymmetry. Mathematically the *k*th standardized moment, ψ_k, is defined by Finch [12] as,

$$\psi_k = \mu_k / \mu_2^{k/2}$$

Consequently the third moment becomes,

$$\psi_3 = \mu_3 / \mu_2^{3/2} = \mu_3 / \sigma^{3/2}.$$

[12] Internet Reference: www.riskglossary.com

Kurtosis

The fourth standardized moment according to the general form presented in the last section is the Kurtosis defined by Finch [12] mathematically as,

$$\psi_4 = \mu_4 / \mu_2^2$$

Or equivalently it becomes,

$$\psi_4 = \mu_4 / \mu_2^2 - 3 = \mu_4 / \sigma^4 - 3.$$

In probability theory Kurtosis is the measure of peakedness of the probability distribution of a real valued random variable. The "minus 3" at the end of this formula is often explained as a correction to make the kurtosis of the normal distribution equal to zero. A high kurtosis distribution has a sharper peak and fatter tails, while a low kurtosis distribution has a more rounded peak with wider shoulders. Figure 11 shows different sorts of Kurtosis, Mesokurtic curves take place when kurtosis is zero which means we have a normal distribution. Leptokurtic case happens when data are fat-tailed, we say so that we have a positive kurtosis. The last type is Platykurtic Curve, which the kurtosis is less than zero. Last two cases are not normal distributions.

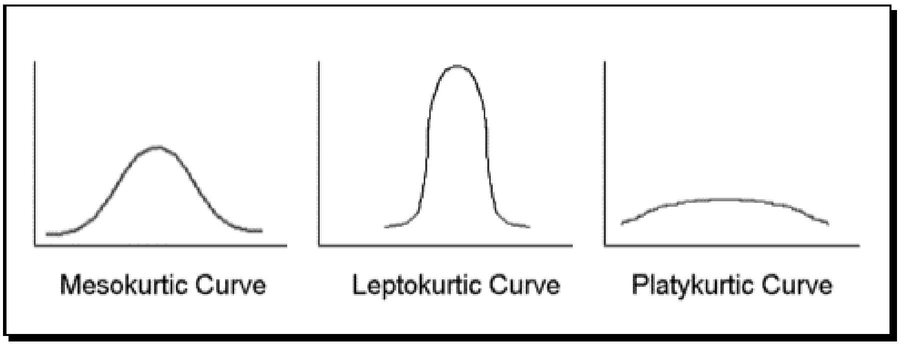


Figure 11 - Different form of Kurtosis[13].

[13] Internet Reference: http://allpsych.com/researchmethods/distributions.html

Construction of the Model on Excel

Excel modules for portfolio modelling

In the following part, we will specify the excel formulas that we have used for the portfolio spreadsheet modeling. We will do this both by the mathematical notation and its equivalent excel notation. When working in a spreadsheet, it is always easer to have expressions for the portfolio risk and return that are easy to enter. Excel is not suited for quadratic computations, but since excel is built on columns and rows we can do the necessary computations by applying linear algebra. For the complex computations our approach will be to observe cell formulas based on excels vector and matrix multiplication, also the defined built in functions that are in excel.

Portfolio Risk and Return

In excel we could express the expected return and the portfolio weights by column vectors (denoted **e** and **w** respectively, with row vector transposes $\mathbf{e}^{\mathbf{T}}$ and $\mathbf{w}^{\mathbf{T}}$), and the variance-covariance matrix is denoted by the matrix notation **V**. From this we can write the expression of the portfolio risk and return in matrix and excel format as follows.

	Matrix notation	*Excel formula*[14]
Portfolio return:	$\mathbf{w}^{\mathbf{T}}\mathbf{e}$	=SUMPRODUCT(**w**, **e)**
Portfolio variance:	$\mathbf{w}^{\mathbf{T}}\mathbf{V}\mathbf{w}$	=MMULT(TRANSPOSE(**w**),MMULT(**V, w**))
Portfolio sigma	$\sqrt{w^T V w}$	$=\sqrt{MMULT(TRANSPOSE(w), MMULT(V, w))}$

NOTE: that when computing the following models, the user needs to press ctrl+shift+Enter for it to be executed.

For calculating the portfolios risk and return, we also need to compute some other parameters that these are based on. These are the arithmetic mean, geometric mean, variance population, standard deviation of population and the variance-covariance matrix. These are implemented mainly by using excel user-defined functions already implemented in excel.

	Mathematical notation	*Excel formula*
Arithmetic mean	$\frac{1}{n}\sum_{i=1}^{n} x_i$	=AVERAGE(arrays)
Geometric mean	$\left(\prod_{i=1}^{n} a_i\right)^{1/n}$	=(GEOMEAN(arrays))-1

[14] The model construction in Microsoft Excel by Jackson [13].

Variance of population	$\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2$	=VARP(numbers)
Sigma of population	$\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2}$	=STDEVP(numbers)
Covariance	$\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right)$	=COVAR(array1;array2)

When it comes to calculating variance-covariance matrix for a large sample of different categories, in our case different equity returns, there exists a fast and simple way in excel to do this. This you do by accessing **"Tools"** in the excel work sheet, and then choose **"Data Analysis"** if it is not enabled, you need to go to the **"Tools"** > **"Add-Ins"** and add it. In the "Data Analysis" screen, you pick **"Covariance"** to generate a variance-covariance matrix.

Using Solver to optimize efficient points

When focusing on the efficient sets of portfolios, we want to find some split across the asset that achieves the target return by minimizing the variance of return. This problem is a standard optimization problem, which Excels Solver can solve. It contains a range of iterative search methods for optimization. Then for this case of the portfolio variance which is a quadratic function of the weights, and for this we will be using Solver for quadratic programming.

The Solver requires *changing cells*, a *target cell* for minimization and the specification of *constraints*, which acts as restrictions on feasible values for the changing cells. The target cell to be minimized is the standard deviation of return, for the portfolio. Also that the *changing cells* should be the cells containing the weights.

The steps in using solver are:

1. Excess solver by choosing **Tools** > **Options** > **Solver**.
2. Specify in the Solver Parameter Dialog Box:
    - The target cell to be optimized
    - Specify *max* or *min*
    - Choose changing cells
3. Choose Add to specify the constraints then OK. This constraint ensures that it must meet the target cell selected.
4. Click on Options and ensure that Assume Linear Model is not checked.
5. Solve and get the results in the spreadsheet.

The following figure shows how the Solver screen looks like.

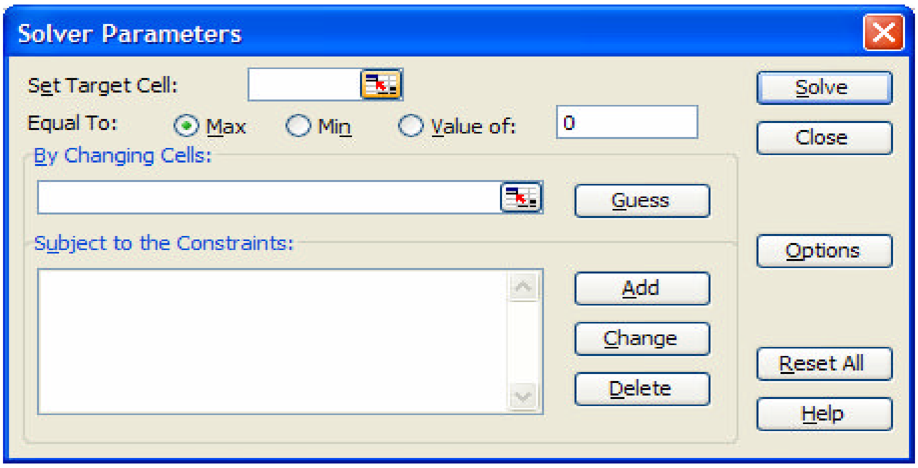


Figure 12 - The Solver Optimizer.

Further excel implementations

For the other parts of the portfolio modeling implementation, we choose to edited formulas directly in to the cells. E.g. when we implemented the CAL we applied the formula in the cell, since it is a linear model, there will be no complications for excel to compute it. This is similar for the computations of the stock returns from the bid prices, calculated by taking today's price minus yesterday's price divided by yesterday's price.

When the risk-free asset is added in to the model we will start working with the CAL. At this point one will start combining the efficient frontier and the risk-free asset. To do so one uses the Sharpe ratio by maximizing it. To maximize the Sharpe ratio we use again the Solver in Excel. The different here then previous, is that now set the target cell to be the Sharpe ration, by changing the cells "*weights*". This will give you the optimal weighted portfolio reachable on the efficient frontier. Since the Sharpe ratio is the slope of the tangent portfolio, we can then draw a line from the risk-free asset and through the tangent portfolio on the efficient frontier. We do this by writing a linear equation for these combinations. Also buy using Solver, which is connected to the risk and return of the portfolio, will give you the best possible return and the lowest risk for the market portfolio.

Next up is to graph the efficient portfolio with the CAL. To do this we mainly need 3-4 portfolios with different risk and returns. The first two portfolios that we need are the maximum return portfolio and the minimum risk portfolio. The minimum risk portfolio is usually denoted as the GMV (Global Minimum Variance) portfolio. Next is the optimal tangent portfolio, which one obtains by maximizing the Sharpe ratio. The optional one is the minimum return portfolio, which will give you a full concave figure of the graph. The final thing is to draw the CAL from the risk-free rate and through the tangent portfolio. The following figure shows how it could be illustrated.

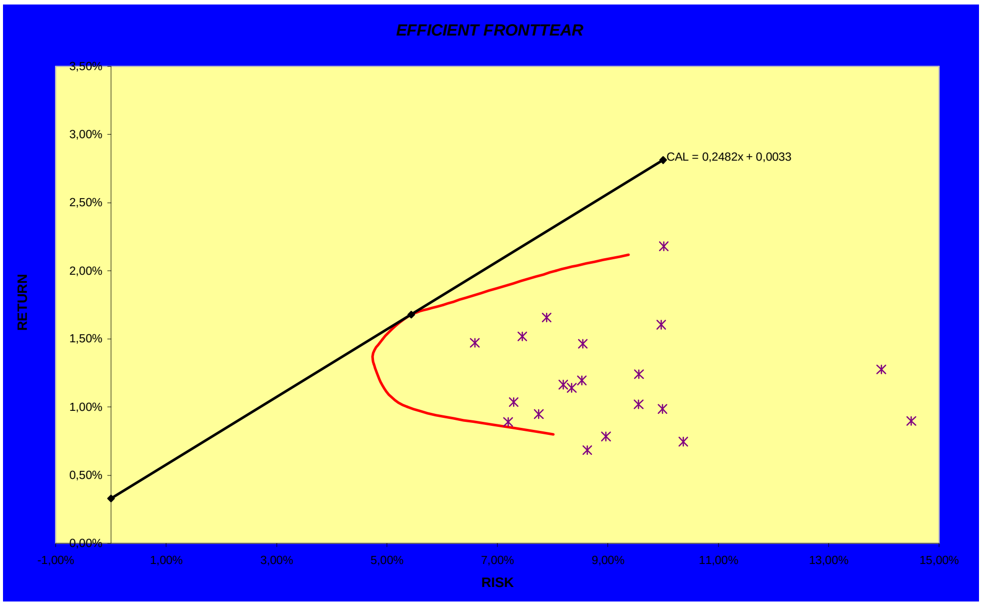


Figure 13 - The Efficient Frontier.

Implementing the Portfolio VaR in Excel

According to Cormac [14], the conventional way to calculate the VaR for a portfolio is that if one wishes to calculate the standard deviation of the portfolio and hence the VaR at the 95% level with a significant of 5%. The details can be outlined as follows:

	A	B	C	D
59			Number of assets	10
60			Port. Return	0,09%
61			Port. Std Dev	1,11%
62			Port. Variance	0,000122351
63			Annual Port. Return	25,806%
64			Annual Port. Std Dev	22,151%
65			Annual Risk-Free	3,83%
66			Annual Risk-Free (Risk)	0,194%
67			Modefied Sharpe	0,6031059
68	Set up for Modefied Sharpe		Value input	
69	Confidence level		95%	
70	No. Of standard deviations		1,644853627	
71	Annual Standars deviation of Portfolio		22,15%	
72	Value at Risk of Portfolio		0,36434	

Figure 14 - Portfolio VaR.

The essential inputs that are needed in this model are under the label "Set up for the Modified Sharpe". Then the first thing needed is the portfolio variance which is 0.000122351, next the annualized portfolio standard deviation (Risk) 22.15%. Then after this we look up how many standard deviations are needed to calculate VaR at the 95% level. This is obtained from the normal distribution tables which will give you 1.64485, or in excel you use the built in function NORMSINV (probability). This returns the inverse of the standard normal cumulative distribution, which has a mean of zero and a standard deviation of one. Thereafter, we multiply this by the portfolios standard deviation of 22.15%. This then means that we are

95% confident that our losses will not exceed 36.43% of our portfolio value. Below is the spread sheet formula behind VaR calculation that was shown in figure 15.

Set up for Modefied Sharpe	Value input
Confidence level	0,95
No, Of standard deviations	=NORMSINV(C69)
Annual Standars deviation of Portfolio	=D64
Value at Risk of Portfolio	=C70*C71

Figure 15 - Spreadsheet fromula behind VaR calculation.

Empirical Investigation

This part under title Empirical Investigation tries to answer to some questions and use some statistical methods to motivate these answers. In the last part of this paper we are going to study some parameters on a group of constructed portfolios with up to forty two assets by Markowitz model. Up to today lots of the financial models including Markowitz model were subject to a series of assumptions. The pioneering work of Harry Markowitz in Modern Portfolio Theory was not an exception; neither the semi-variance introduction could minimize the damage of these assumptions. He defined the reward as expected return and the risk as the standard deviation or variance of the expected returns. Rachev [15] claims since Markowitz assumes the returns are normally distributed, the expectation operator is linear and the portfolio's expected return is simply given by the weighted sum of the individual assets' expected return. The variance operator, however, is not linear. This means that the risk of a portfolio, as measured by the variance, is not equal to the weighted sum of risk of the individual assets.

Before any further steps in analyzing the data we will examine the distributions' normality of our stream of data. There exist different statistical methods to do such a test. Some of them are computational and it is easier to construct a Null Hypothesis Testing with the help of them, and some others can only confirm our claim by visual evidence. We will here examine the stream of data in two ways, Jarque-Bera test and QQ-plot.

Another interesting result of constructing a portfolio with Markowitz model was the amazingly unrealistic results for the Sharpe ratio maximization. The problem with Sharpe ratio is that it is accentuated by investments that do not have a normal distribution of returns. As it is clear here, for a risk manager that tries to guard against large losses, the deviation from the normality can not be neglected.

"In the case of testing the hypothesis that a sample has been drawn from a normally distributed population, it seems likely that for large samples and when only small departures from normality are in question, the most efficient criteria will be based on the moment coefficients of the sample, e. g. on the values of $\sqrt{\beta_1}$ and β_2.[15]"

E. S. Pearson, 1935

The Jarque Bera test of Normality

It is a goodness-of-fit measure of departure from normality, based on the sample kurtosis and skewness. We mentioned earlier that the normal or Gaussian distribution is the most popular distribution family used in modelling finance. When it comes to stock market, it is assumed that a return or change in the stock price is the result of many small influences and shocks and thus the return can be treated as a normal random variable. But is it really true? There are different methods to test for normality such as Jarque-Bera test, Kolmogorov-Smirnov and

[15] $\sqrt{\beta_1}$ and β_2 representing skewness and kurtosis respectively denoted by Finch[12].

Anderson-Darling introduced by Rachev et al [16]. Here we preferred to use Jarque-Bera test of normality. It is simple to calculate and it considers the higher moments which we are concerned about, skewness and kurtosis.

The formula for JB-test presented by Rachev et al [16];

$$JB = \frac{n}{6}\hat{\gamma}^2 + \frac{n}{24}(\hat{\kappa} - 3)^2$$

Where,

$$\hat{\gamma} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{\hat{\sigma}}\right)^3 \text{ The sample skewness}$$

$$\hat{\kappa} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{\hat{\sigma}}\right)^4 \text{ The sample kurtosis}$$

$$\hat{\sigma} = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 \text{ The sample variance}$$

The result shows that under the hypothesis that x_i is independent observations from a normal distribution, for large *n* the distribution of the JB-test statistic is asymptotically Chi-square distributed. This will help us to do a test on normality. If we have a large sample, and we calculate the JB-test statistic on it and compare it with the null hypothesis that the data represents a normal distribution, while we know that in 95% of the cases the value of the JB-test will be smaller than 5,99 for the normally distributed samples. Consequently we reject the hypothesis of normality if the value of JB-test statistic exceeds this amount.

The Result of Jarque-Bera Test on Our Portfolio Assets

In order to see if we can reject the normality of the data set, we performed a JB-test on the data sets. As mentioned before, our study compares 5 different sorts of data on OMX stock exchange, daily, weekly, monthly, quarterly and yearly. The data provides a long term investment of 10 years and what we did was to separate it into two 5-year period data for all categories of data. In order to perform comparisons and analyze the results, we treated the first period as the historical data and the second as the future one. Let's consider these categories closer;

Daily returns are the longest set of the data we analyzed. The size of the data seems to have a big impact on the JB-test. This claim becomes more touchable when we compare it more in depth with other categories of data. As it is shown in the table taken from our empirical investigation shown in Appendix 6, where the marked cells means that the null hypothesis of normality is rejected we see that the statistic values for the JB-test are notably higher than other categories in comparison with the daily returns. But more and less the number of stocks that their normality can be rejected by this test is equal in the first three categories of data sets, daily, weekly and monthly for both periods, historical and future one.

Surprisingly the quarterly data set has a larger number of normally distributed assets, which can be due to the lack of data (the length of the data set is shorter than the latter categories).

But still it does not mean that we can apply models with normality assumptions on these data sets, since almost 50-60 percent of the assets included in this category are not normally distributed. The last category is the yearly data set, most of the assets successfully pass the JB-test, but it can not be a reliable result considering the number of data in each data set. We considered 10 years data, for two periods which will result in an analysis of a data set of five. How convincing can the result of such a study be?! So we exempt this category from our normality test by JB-test method.

Although JB-test rejects the normality for the data categories just mentioned with a good level of significance, but in order to present a more decent result we decided to hire another method for testing the normality. There is a visual method so called Q-plotting for normality, where the data sets' normality will be tested by plotting the data set against a normal distributed one.

Using Plots to Motivate the Non-Normality of Asset's Return Data

There are different types of tests for normality, where we can determine if a data set is normally distributed or not? As we examined our data sets by the JB-test, it became clear to us that the majority of the asset returns under our investigation are non-Gaussian distributed. There are other methods mentioned above which will give us a statistic value and they can determine normality of a data set by doing a hypothesis test. But when we are working with financial data sets, it is interesting to observe the behaviour of the market visually. For this purpose charts, diagrams and graphs are strong tools.

What we did on our data was a large scale empirical investigation where we plotted the histograms for forty four stocks on the Large Cap list of Stockholm Stock Exchange for two 5-year periods where data was sorted in 5 groups, daily, weekly, monthly, quarterly and yearly. It gave us interesting results; the shape of the histograms did not support what we expected; to have a nice bell shaped normal distribution. Instead we got all other possible shapes. It was the reason why we started to calculate the 3^{rd} and 4^{th} statistical moments, the parameters which are essential in forming the shape of the distributions. Here we present some results, but the whole data analysis is available in the excel file provided by this report. These histograms visualize distribution of data for four assets included in our empirical investigation from the historical period (1997-2002).

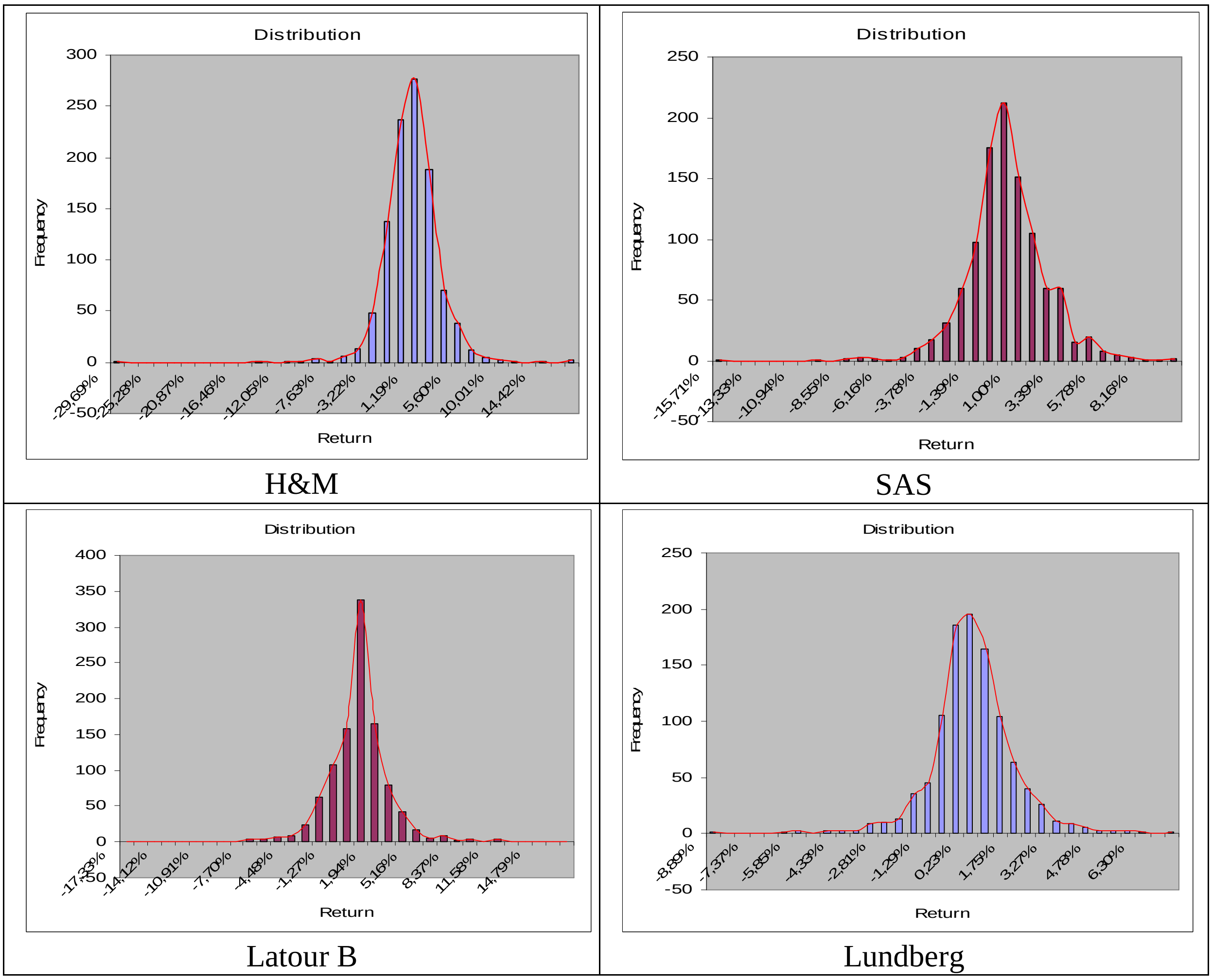


Figure 16 - Asset Returns' Distribution - Daily data (year 1997-2002) [16].

As the figures show clearly, some of the tails are showing fat-tail and no strong visual sign for asymmetric distributions. Some of the stocks like *Latour B* show an excess Kurtosis and heavy tails while others like *SAS* shows an unsymmetrical distribution. But still we need a more convincing method to become sure whether asset returns under our study are normally distributed or not?

Another visual method used widely by statisticians is normal probability plotting. This can be a good method to visualize the data distribution to study. We consider that the reader of this paper is already familiar with normal plotting. We just mention different cases that we might face in our data plot. Some pictures will be provided in order to help reader to understand the concept better.

Normal Probability Plot for Determining Non-Normality

It is a technique to see whether the data is approximately normally distributed or not. The normal probability plot is constructed on a graph with two axes, where vertical axes are our data and the horizontal is the z-values. With the help of normal probability plot we can answer these two questions;

[16] Reference: Taken from the empirical investigations done under this study.

1- Are the data normally distributed?
2- What is the nature of the departure from the normality (Skewness, Kurtosis, tails or peakedness of the data)?

We will consider the possible cases we might face after plotting the normal probability;

1- Data are normally distributed;

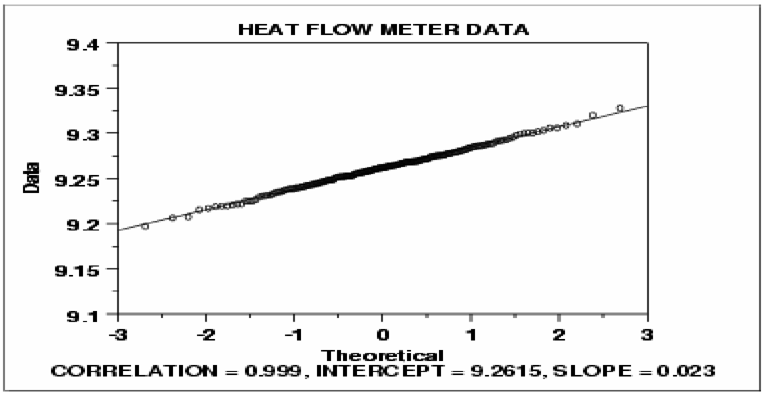


Figure 17 - Data Normally Distributed[17].

We can conclude that the normal probability plot shows a strong linear pattern, where the minor deviations from the line are insignificant. The normal distribution is a good model fitting the subject data.

2- Data have fat-tails;

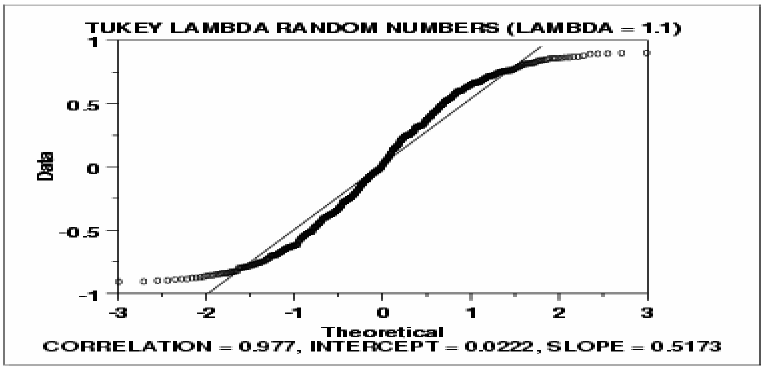


Figure 18 - Data has fat tails[18].

If we face a plot with such a shape we conclude that, the normal probability plot shows a non-linear pattern. Consequently the normal distribution is not a good model for these data. If we take a closer look, both fat and short tail distributed data share some common characteristics. Both showing a S-shaped curve in the middle and both are deviated form the reference line at both ends of the plot.

[17] Internet Reference: http://www.itl.nist.gov/div898/handbook/eda/section3/normprp1.htm
[18] Internet Reference: http://www.itl.nist.gov/div898/handbook/eda/section3/normprp2.htm

3- Data have short-tails;

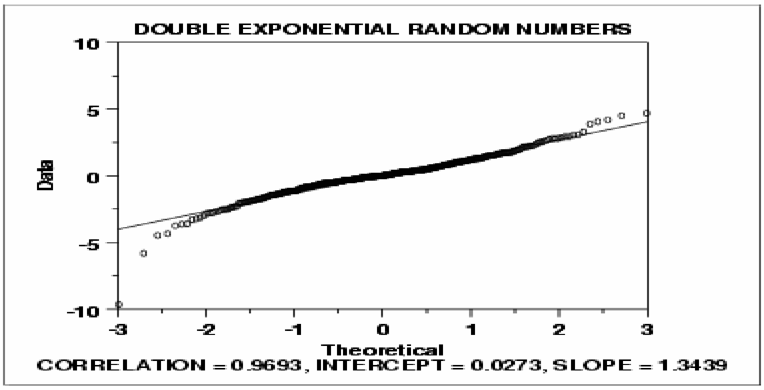


Figure 19 - Data has short tails[19].

In this case the normal probability plot shows a reasonably linear pattern in the middle of the plot but it departures from both ends. As we suggested for long fat-tailed one, another distribution other than normal would be reasonable for this case.

4- Data are skewed right;

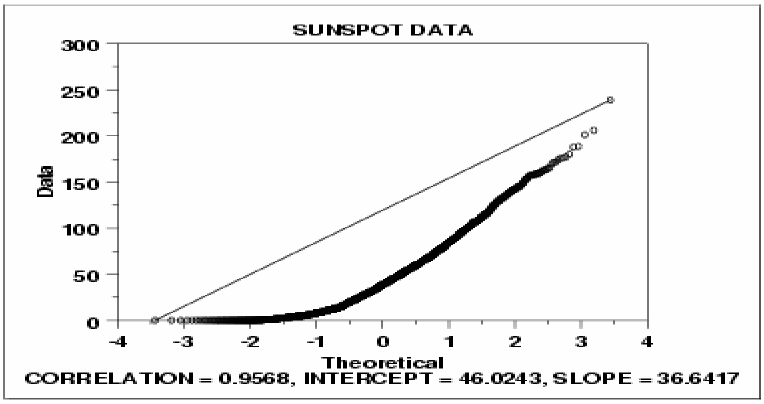


Figure 20 - Data is right skewed[20].

Here we have a strong non-linear pattern, where normal distribution is not a good pattern for this data set.

The Result of Normal Probability Plot on OMX Large Cap List

After introducing the possible cases, we plotted our data on OMX Large Cap list, for 10 years period where data is categorized in 4 different groups, daily, weekly, monthly and quarterly. It was not far from our expectation that almost all the plots confirmed the nonlinearity and the fact that these data are not normally distributed. Here we present the normal probability plots of the stocks we showed histogram plots for earlier. We can take a look at the shapes and analyse them in relation with the histograms just mentioned.

[19] Internet Reference: http://www.itl.nist.gov/div898/handbook/eda/section3/normprp3.htm
[20] Internet Reference: http://www.itl.nist.gov/div898/handbook/eda/section3/normprp4.htm

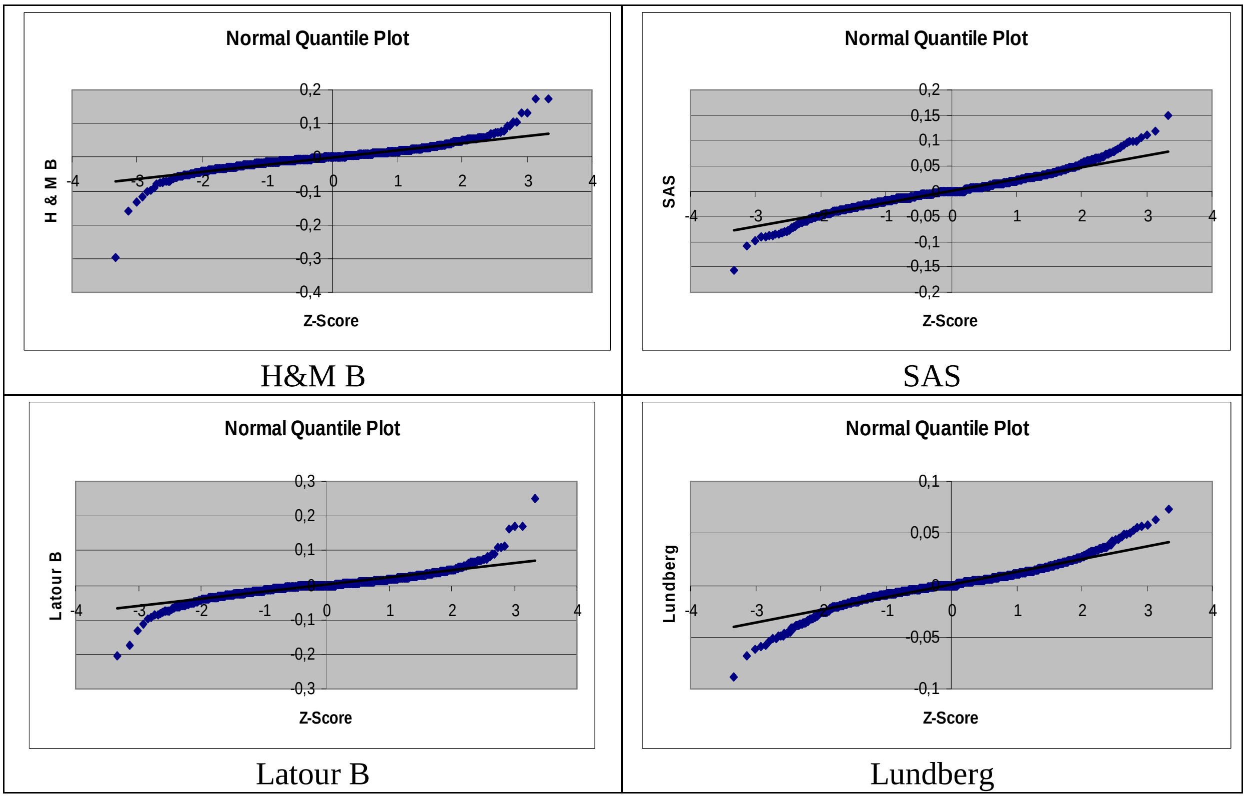


Figure 21 - The normal probability plot for 10 years daily asset returns[21].

The histograms showed a high-peaked data plot and made us suspicious about the asset returns being normally distributed, and lead us to conclude that our data sets are fat-tailed. Also based on the calculations for skewness and kurtosis which are the parameters of statistic value of the JB-test, it is highly convincing to reason and claim the non-normality of the distributions. Now the normal probability plot confirms strongly that the fundamental assumption, normality of the asset returns by Markowitz model is not true. Most of the investment managers are not aware of this fact that each one of this assumptions are made to simplify the calculations, where the normality assumption by Markowitz model makes it easy for almost every investment manager despite the level of knowledge in mathematics or statistics to construct the model. The damage of this assumption often is underestimated by investors or basically can be unknown to them.

The Problem with Sharpe Ratio and the Reason

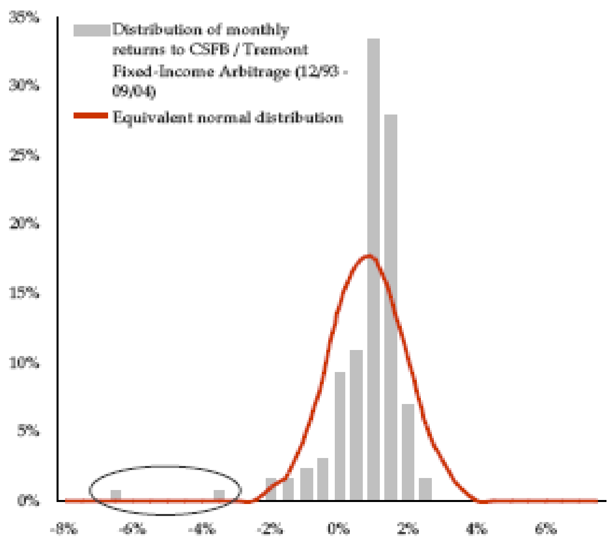
The greater a portfolio's Sharpe ratio, the better is its risk-adjusted performance. Where Lhabitant [17] claims that in contrast, a Sharpe ratio of 1.0 indicates a return on investment that is proportional to the risk taking in achieving return. A Sharpe ratio lower than one indicates a return on the investment that is less than the risk taken.

But what about extreme losses?! This question might seem inappropriate here, but it is not true. An important parameter of the Sharpe ratio is risk, or to be precise the volatility. For a portfolio with normally distributed returns, what the measure of volatility provides can be

[21] Reference: Taken from the empirical investigations done under this study.

used in constructing the portfolio. For a portfolio with assets which are normally distributed the volatility automatically infer the extreme loss risk. This illustrated under a paper work released by *Sinopia Asset Management* member of the HSBC Group by Bertland [18][22]. But, is the asset returns always normally distributed? The answer is No. What we showed so far by this empirical investigation by JB-test and graphs was that the assets' returns are not normally distributed, at least in the case of Large Cap List on OMX.

The following graph compares the distribution of monthly returns of a Fixed-Income Arbitrage instrument and the equivalent normal distribution. Volatility is equal for both cases, while the extreme losses on the Fixed-Income Arbitrage instrument are much higher than the Gaussian one. This shows the danger of relying solely on volatility to measure risk in portfolios.



Source : Sinopia

Figure 22 - Extreme losses in normal VS. Non-normal asset distributions with equal volatility[23]

A feeling which is common to every risk manager is that the perception of investment is naturally related to the potential for extreme losses. Despite the technical analysis and complicated calculations, all investment managers are concerned about the large losses, what we called here extreme losses. By the graph just presented, we are convinced that normal distribution is not a good model to fit the assets returns. Let's consider another graph presented on a study by Rachev et al [19] and see what happened to Dow Jones Industrial Average (DJIA) daily return between 1991 and 2003. In this graph presented in Figure 23, the volatility plotted in the interval just mentioned. As you see this graph indicates that extreme losses happened where it will never occur under normal distributions assumptions.

Now let's see how these facts can affect Sharpe ratio. Because of the way volatility is defined, returns that are 5% above average will inevitably be treated in the same way as those 5% below average. When we consider a normal distribution since its skewness is zero and perfectly symmetrical it can not cause any problem regarding the results. But where the symmetrical distribution is not the case, it can be a source of problem. Consider the two extreme losses occurred at left tail of the distribution shown in Figure 22. It shows the

[22] Internet Reference: Source: http://www.sinopia.fr/docs/NewRiskIndicator.pdf
[23] Internet Reference: Source: http://www.sinopia.fr/docs/NewRiskIndicator.pdf

extreme losses, now considers these extreme occurrences that are on the positive side of the mean, and then the investment has extreme gains. Compared to normally distributed ones presented in the same graph but still the risk indicated by the volatility is equal for all three cases. Our conclusion would be that the volatility is not an adequate measure for risk. In the same way accuracy of Sharpe ratio can be questioned!

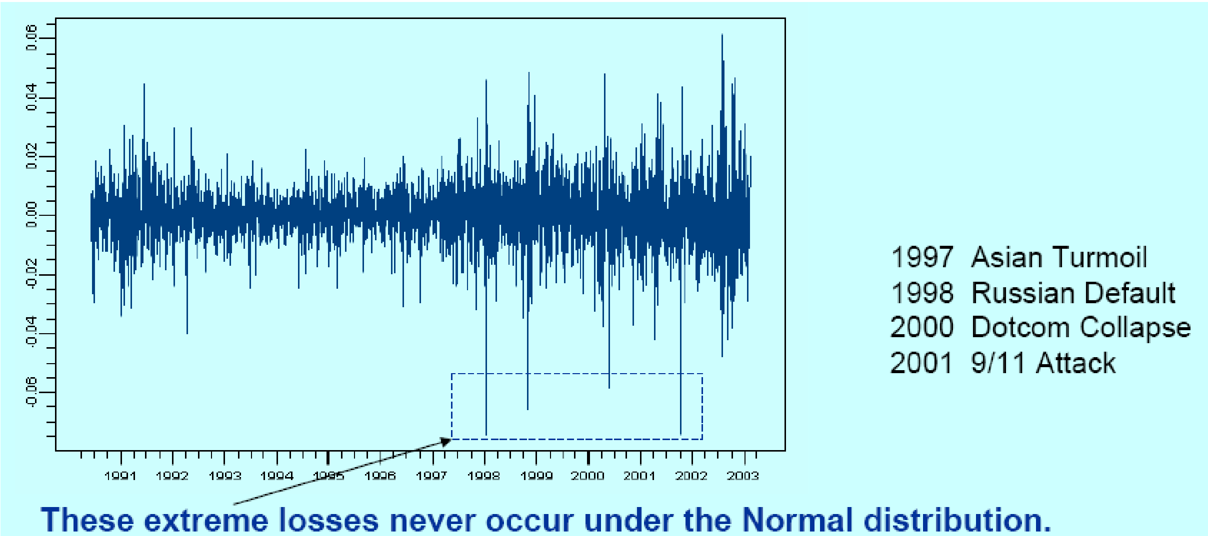


Figure 23 - Volatility and extreme losses of DJIA daily returns[24].

Adjustments to the Risk Regarding Higher Moments' Effects

Since we proved that our asset's returns are not normally distributed, and we explained how it affects the Sharpe ratio measure for performance, and finally the concept of extreme falls, now we can apply some adjustments to the Sharpe ratio. After comparing some methods for measuring performance we choose modified Sharpe ratio as a good measure in order to compare it to our benchmark, conventional Sharpe ratio. The reason to do so was the result of the study by Greg et al. [20]. He claims that due to non-normality of returns, a mean-variance framework suffers from some limitation. Some adjustments should be done on the original Sharpe ratio through the use of a new measure for risk. He introduces the Modified Sharpe Ratio where the standard deviation or risk factor of the model is replaced by Modified Value at Risk (MVaR), a measure which takes in the higher statistical moments discussed earlier.

$$\text{Modified Sharpe ratio} = \frac{(R_p - R_f)}{MVaR}$$

Where,

$$MVaR = W[\mu - \{z_c + \frac{1}{6}(z_c^2 - 1)S + \frac{1}{24}(z_c^3 - 3z_c)K - \frac{1}{36}(2z_c^3 - 5z_c)S^2\}\sigma],$$

And,

[24] Internet Reference: http://www.statistik.uni-karlsruhe.de/download/Sofia_conference.pdf

R_p Portfolio return

R_f Risk-free ratio

MVaR Modified VaR

σ Standard Deviation

S Skewness

K Kurtosis

z_c is the critical value of the probability (1-α)

The benefit of involving the higher moments in our calculation is that we want to avoid underestimating risk. MVaR is introduced here since the conventional value at risk exhibit the same shortcoming as the standard deviation (based on normal distribution assumptions for asset returns). For derivation of the formula you can refer to Favre and Galeano [2002][25].

Construction of the Portfolio with New Adjustments to Sharpe Ratio

The steps to construct a portfolio with the new optimization factor, Modified Sharpe ratio is as the pattern for the tradition one. The only difference between these two models is the measure of the risk, which in the traditional Markowitz model is the volatility or standard deviation of the asset returns and in the new one is the MVaR. After replacing the new risk measure we can use our optimizer tool[26] to maximize the new measure for the Sharpe quote.

Analysis of the Empirical Investigation

In this part of the empirical investigation we will sum up our study by answering some questions. These questions were loosely thought of in our minds from when we started to construct the model by two methods (traditional and modified Sharpe). But here we want to formulate them and answer them with regards to the result of the study done on the available data. The answers of these questions will be given in the following parts.

The Questions to Answer;

1. What will be the difference between two optimized portfolios when;

   - The first one is optimized by traditional mean-variance with the Sharpe model, and then by sorting out the stocks' skewness and kurtosis and study the importance of these parameters.
   - Second case, when we optimize the portfolio considering a new risk measure, MVaR. Then sorting out data by skewness and kurtosis and perform the same study done already on the last group.

[25] Internet Reference: http://www.gloriamundi.org/picsresources/rb-fg.pdf

[26] Excel tools, Add-Ins; Solver

Then compare these results with future data.

2. Compare the portfolios calculated in part one with other portfolios that have different time series (monthly, daily, etc).

Daily Portfolio

In the first place, everybody would like to compare risk and returns between these two types of optimized portfolios and by doing so we will find out that the figures are slightly equal. So what is the point of doing this analysis? We will try to find out the answer of this question by considering the results from traditional and modified Sharpe ratio.

Many investment managers, who intend to prepare statistical reports for their clients and the board of directors, prefer to use the traditional Sharpe quote. It is a ratio which is used vastly, to measure the investments' performance. But why is it so that the traditional Sharpe quote is so popular?

Comparing the results from our study on the daily data showed that we had the same results concerning return and risk with optimization for two different Sharpe ratios, Traditional and modified one. The definition of these was mentioned earlier in this paper. The drawback of the traditional Sharpe ratio is that it does not distinguish between upside and downside risk, but rather penalized upside risk specifically as downside risk. The other case is the way these treat the extreme losses. The traditional Sharpe quote considers extreme or irregular losses as those which are repeated regularly. By extreme or irregular losses we mean the events such as IT crash or 11^{th} September 2001 incident. This had extreme impact on the world's stock markets. The use of Modified Sharpe ratio proposed to count for cancelling out the impact of such deficiencies by the traditional model.

As the Table 2 shows, our claim holds. Optimization of Sharpe ratio gives results in the same stocks picked by the same proportions with almost equal risk and return. This shows an overestimation of traditional Sharpe, compared to modified Sharpe ratio.

Comparing the portfolios constructed to study extra parameters, skewness and kurtosis, the only portfolio which performed acceptably was the portfolio constructed with positive skewness. If we look at the traditional Sharpe ratio, it is still highest and greater than one, which means we are accepting less risk for getting more return. But the Modified Sharpe ratio is still less than one. Among other portfolios still this category claims for the highest Sharpe ratio calculated by these two methods, and still risk and return are best performed. Comparing the results with future data, it is still this category that gives the best risk, return and Sharpe ratios results in connection with each other.

The other category which performs acceptably is where we have stocks with only Kurtosis greater than 3. The risk and return are relatively high and it performed a good Sharpe quote for both methods, and this is still the second best performance. In our ranking it comes second after the portfolio with only positive skewness. When it comes to future data it gives the highest return, but also second high risk. So the ranking for this case will be a little bit tricky and it depends on the importance of the risk in the second time period, future data.

The other category which ranked by us as the third best performance is the portfolio optimized without considering the higher statistical moments. The return of this category

becomes first while the Sharpe quote stands at second place after the portfolio with stocks sorted out by positive skewness. When considering the future data risk, this category comes as best as a result of diversification (access to more assets to be included in the portfolio).
Based on this study the worst performance is for the category which hires stocks with both Kurtosis grater than 3 and positive Skewness. Comparing Sharpe ratios calculated by both methods for both periods this category stand for the lowest results. This category is ranked last.

Daily Portfolio - Sharpe						
	Historical Data		Future Data		Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	0,99202	0,60311	0,45249	0,27509	**-54,39%**	**-54,39%**
Optimization withPositive Skewness and Kurtosi Greater than 3	0,86085	0,52336	0,33160	0,20185	**-61,48%**	**-61,43%**
Optimization with Positive Skewness	1,04398	0,63470	0,49298	0,29988	**-52,78%**	**-52,75%**
Optimization with Kurtosis Greater than 3	0,88005	0,53503	0,45455	0,27637	**-48,35%**	**-48,34%**

Daily Portfolio - Return						
	Historical Data		Future Data		Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	25,81%	25,81%	10,37%	10,37%	**-59,82%**	**-59,82%**
Optimization withPositive Skewness and Kurtosi Greater than 3	26,83%	26,83%	9,56%	9,57%	**-64,37%**	**-64,34%**
Optimization with Positive Skewness	27,03%	27,03%	11,35%	11,35%	**-58,01%**	**-57,99%**
Optimization with Kurtosis Greater than 3	24,89%	24,89%	11,50%	11,51%	**-53,78%**	**-53,78%**

Daily Portfolio - Risk						
	Historical Data		Future Data		Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	22,15%	22,15%	17,08%	17,08%	**-22,92%**	**-22,92%**
Optimization withPositive Skewness and Kurtosi Greater than 3	26,72%	26,71%	20,85%	20,85%	**-21,95%**	**-21,96%**
Optimization with Positive Skewness	22,22%	22,22%	17,66%	17,66%	**-20,53%**	**-20,53%**
Optimization with Kurtosis Greater than 3	23,93%	23,93%	19,49%	19,49%	**-18,55%**	**-18,55%**

Table 2 - Daily Portfolio

Weekly Portfolio

In the same pattern we analyzed the data for the daily portfolios constructed, we can look at weekly ones. We have the same table for Sharpe ratios, return and risk of portfolios when different factors, skewness and kurtosis considered in our optimization.

Let's consider the first portfolio constructed by the traditional Sharpe ratio where skewness and kurtosis effect were not considered by the original model. Not surprisingly the traditional Sharpe ratio is almost double the Modified one in the first case, but by referring to the excel file we will see that the stocks chosen and their weights are identical for both portfolios. This case introduces the second highest return for the historical portfolio and the lowest risk. But it will be interesting to compare the results with the future portfolios. In this case the Sharpe still has the highest value and risk and return kept their positions.

The next portfolio is the one with both positive skewness and kurtosis greater than 3. In this case we have the lowest Sharpe ratio for both historical and future portfolios. Returns are second best, but considering the high risks they are not worth to consider. But it is interesting to consider the velocity of losing value of the returns from the historical portfolio to the future one, from 23,21% to 9,12%.

The third portfolio is the one with just positive skewness. It has still a traditional Sharpe ratio greater than 1, highest return and simultaneously lowest risk. The most interesting case, but let's see if these characteristics remain the same when we go to the future portfolio. Sharpe is still relatively high. The return is not the highest but the risk managed to be the lowest for the future data.

The fourth portfolio that we analyzed is the portfolio with stocks which have kurtosis greater than 3. As it is predictable by looking back again in the second case it is not a good method to construct a portfolio. Low Sharpe ratio for both periods, and the risk which is high for both periods and the return which is not so high compared with other cases for the first period, but interestingly not diminished as much as other portfolios for the second period.

Again as the daily analysis, the worst case seems to be the second case. But it is difficult to distinguish between the first and third case as a candidate for the best case.

Weekly Portfolio - Sharpe						
	Historical Data		Future Data		Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	1,12922	0,68652	0,706234	0,429416	**-37,46%**	**-37,45%**
Optimization withPositive Skewness and Kurtosi Greater than 3	0,52150	0,31705	0,251157	0,152692	**-51,84%**	**-51,84%**
Optimization with Positive Skewness	1,08602	0,66025	0,611901	0,372110	**-43,66%**	**-43,64%**
Optimization with Kurtosis Greater than 3	0,55674	0,33848	0,58705496	0,357137817	**5,44%**	**5,51%**

Weekly Portfolio - Return						
	Historical Data		Future Data		Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	24,90%	24,91%	13,67%	13,68%	**-45,09%**	**-45,10%**
Optimization withPositive Skewness and Kurtosi Greater than 3	23,21%	23,21%	9,12%	9,12%	**-60,71%**	**-60,71%**
Optimization with Positive Skewness	25,17%	25,17%	12,59%	12,60%	**-49,97%**	**-49,96%**
Optimization with Kurtosis Greater than 3	20,24%	20,24%	15,34%	15,35%	**-24,22%**	**-24,16%**

Weekly Portfolio - Risk						
	Historical Data		Future Data		Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	18,63%	18,64%	15,76%	15,77%	**-15,37%**	**-15,39%**
Optimization withPositive Skewness and Kurtosi Greater than 3	37,10%	37,10%	26,21%	26,21%	**-29,37%**	**-29,37%**
Optimization with Positive Skewness	19,62%	19,62%	16,43%	16,43%	**-16,24%**	**-16,24%**
Optimization with Kurtosis Greater than 3	29,42%	29,42%	21,81%	21,81%	**-25,88%**	**-25,86%**

Table 3 - Weekly Portfolio

Monthly Portfolio

In the following data set, we can se that values for the two portfolios with “Skewness and Kurtosis” and “Kurtosis greater than 3” is not included. The reason for this is that the numbers of stocks after sorting out for these portfolios were not reaching the desired level for an efficient diversification. This is one of the most important concepts of portfolio construction.

Considering the first available category which is the portfolio constructed with the traditional Markowitz model where only mean and variance are considered, the Sharpe ratio is the highest for both methods of calculation, modified and traditional Sharpe. When we move to future portfolios for the same category the Sharpe ratios almost became half. When we are analyzing the Sharpe ratios, it would make more sense to look at risk and return closely. Return is still highest for this category while representing the least risk. But surprisingly while the return became almost half of the historical portfolios the risk is decreased only by 6%. The next category is where we have stocks included in the portfolio with only positive skewness. In this category we have almost the same figures as the last case, but in general 1-2 percent less.

The interesting result is where we compare the modified Sharpe ratio by the traditional one where the portfolio constructed by the traditional Markowitz model. In this category we have a minimization of only 43 percent for modified Sharpe ratio against 52 percent of the traditional case. In order to analyze this result, you can compare these figures with the case of considering stocks with positive skewness. In the case of constructing a portfolio with only positive skewness, the figures for both methods of calculation of the Sharpe ratios are identical. It clarified that the modified Sharpe ratio considers the positive skewness even in the case of traditional Markowitz model.

Monthly Portfolio - Sharpe						
	Historical Data		Future Data		Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	1,41441	0,85990	0,67654	0,48224	**-52,17%**	**-43,92%**
Optimization withPositive Skewness and Kurtosi Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
Optimization with Positive Skewness	1,32204	0,80374	0,62795	0,38177	**-52,50%**	**-52,50%**
Optimization with Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A

Monthly Portfolio - Return						
	Historical Data		Future Data		Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	25,62%	25,62%	12,33%	12,33%	**-51,89%**	**-51,88%**
Optimization withPositive Skewness and Kurtosi Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
Optimization with Positive Skewness	24,67%	24,67%	11,99%	11,99%	**-51,41%**	**-51,42%**
Optimization with Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A

Monthly Portfolio - Risk						
	Historical Data		Future Data		Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	15,38%	15,38%	14,45%	14,45%	**-6,03%**	**-6,02%**
Optimization withPositive Skewness and Kurtosi Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
Optimization with Positive Skewness	15,74%	15,74%	15,03%	15,03%	**-4,50%**	**-4,50%**
Optimization with Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A

Table 4 - Monthly Portfolio

Quarterly Portfolio

Based on the same reason we mentioned on the last type of the portfolio, we have only two categories to analyze for quarterly portfolio.

The first category is where we have the general model applied. Looking at Sharpe ratios and their developments we will see that this category has the highest Sharpe ratios both traditional and modified while the development for the traditional case is worse compared with other categories, a figure equal to almost 80%.

Compared to the case of the portfolio with positive skewness, the return considering the risk for the same category is not at all satisfying 14% of return versus 15% of risk. The development of the risk in this category considering the development of its counterpart is not good at all.

Considering the portfolio with stocks which has only positive skewness, we have a good Sharpe ratio for both methods compared with the first category analyzed where the return is almost 26% and risk is relatively low, only 16%. Development of the figures from the historical portfolios to future is interesting. While return diminished, the risk has risen for both categories.

The other phenomena explained above, about the development of modified Sharpe ratio for the original method of the construction of the portfolio is touchable here too. The development of the Modified Sharpe ratio remained almost constant for the case of the portfolio with positive skewness while it has fallen for the first category.

Quarterly Portfolio - Sharpe						
	Historical Data		Future Data		Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	1,49323	0,90782	0,30437	0,42635	**-79,62%**	**-53,04%**
Optimization withPositive Skewness and Kurtosi Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
Optimization with Positive Skewness	1,39841	0,85017	0,61989	0,37686	**-55,67%**	**-55,67%**
Optimization with Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A

Quarterly Portfolio - Return						
	Historical Data		Future Data		Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	25,78%	25,78%	8,52%	8,53%	**-66,93%**	**-66,90%**
Optimization withPositive Skewness and Kurtosi Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
Optimization with Positive Skewness	26,66%	26,66%	14,82%	14,82%	**-44,41%**	**-44,41%**
Optimization with Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A

Quarterly Portfolio - Risk						
	Historical Data		Future Data		Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	15,38%	15,38%	19,63%	19,65%	**27,64%**	**27,79%**
Optimization withPositive Skewness and Kurtosi Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
Optimization with Positive Skewness	16,30%	16,30%	19,80%	19,80%	**21,44%**	**21,45%**
Optimization with Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A

Table 5 - Quarterly Portfolio

Yearly Portfolio

For this category, since the time series in not long, we can not construct portfolios with reasonable structures. So we won’t consider this category in our investigation. The reason for this is the unrealistic results of Sharpe ratio calculations.

Analysis for the different type of time series for constructing a portfolio

Daily time series:

We have constructed 4 different types of portfolios by combining two additional parameters, skewness and kurtosis. The time series’ length is 10 years that is divided into two periods. These two periods as mentioned before are called historical and future, each of length of 5 years. In this part we will compare the development of Sharpe ratio, risk and return from historical period to future one for these 4 types of portfolios. The development parameterized by ratios presented in Table 3.

The first portfolio is constructed by standard Markowitz model, and the first part of the table shows the traditional Sharpe ratio development against modified Sharpe ratio. For the daily portfolio we have a traditional Sharpe of 0.99 which is relatively high. The development is about 54% decrease comparing with weekly portfolio, which has a traditional Sharpe ratio of 1.12 for the historical period and a negative development of 37%. This can be due to the level of diversification, but surprisingly we see that the number of the stocks included in both portfolios is the same. By looking at risk it can be explained why we have such a development. Since the number of data is less in the case of weekly portfolio, and probably most of the extreme events happened during the week and this data are not available at weekly time series to analyze, we have a lower risk for weekly portfolio versus the daily one and consequently a higher traditional Sharpe ratio for the weekly portfolio. To see if it is really true we can compare the modified Sharpe ratio of the two portfolios. The historical daily portfolio has a Sharpe ratio of 0.6 while the weekly has a modified Sharpe ratio of 0.68. By looking at Value at Risk for both portfolios we can see that the daily portfolio has a higher value compared with the weekly portfolio, 0.36 versus 0.3. This explains the differences in the modified Sharpe ratios obtained by these two data sets and the reason is that the daily portfolio as a matter of fact considers all the events in the time series, including positive extreme events. So as a result we have a higher return for the daily portfolio compared with weekly one, but also higher risk. It will not obviously violate the concept of the utility. This result becomes more touchable if the reader understands fully what we explained under subtitle “The Problem with Sharpe Ratio and the Reason”.

The second row of the table shows the development for Sharpe ratios on a portfolio constructed considering two additional parameters, positive skewness and a kurtosis greater than three. Since the number of stocks is limited under such a constraint the traditional Sharpe ratio has a low value. It is due to the level of diversification for this category. The annual portfolio risk is 26% for the daily portfolio which assigns weight to only 8 stocks to the portfolios out of 19, while in the weekly case it is 37% which is the result of a portfolio of only 2 stocks out of 11. Since the weekly portfolio has such a high risk and low Sharpe consequently and the construction of the portfolio is impossible for other categories, a deeper analysis of this type of portfolio (with positive skewness and kurtosis greater than 3) seems to be useless. But among all other portfolios if we consider the development of Sharpe ratio, risk and return, still this type of portfolio is the worst one, with the highest decrease of Sharpe ratio and return for the two time periods versus the lowest decrease of the risk. So we can conclude that this type of portfolio not only in Daily time series but also in other time series can not be a considerable investment. So we will omit this category from our analysis for further time series.

The third row representing a portfolio constructed with only positive skewness. The development among the other portfolios can be claimed to be moderate. This category enjoys a better change in Share ratio and particularly the return for the two periods compared with the traditional Markowitz model. In the case of a daily time series, the traditional Sharpe ratio is 1,04, and greater than one which is good. And still in a weekly time horizon the Sharpe ratio have a value grater than 1. Considering the values of the return and risk, they show still the best performance compared with other portfolios in this time horizon.

The fourth type of portfolio is the one with only kurtosis greater than three. Before looking into the Sharpe ratio and the return and risk, we will start by considering the number of stocks included in this type of portfolio to have a rough estimation of the level of diversification. Among the stocks on the Large Cap list on OMX, there are 29 stocks that have kurtosis greater than 3. After optimization for this category, the weights are assigned to only 9 of them. This resulted in a high return but also high risk. Explanation for the high return is due to the positive Kurtosis which results from fat-tail distributed asset returns and this can be seen when we apply the new Sharpe ratio by a modified risk measure, resulting in a lower Sharpe ratio, with the same risk, return and weights for the portfolio. But the question is that if this category is a good type of investment. Answering this question is difficult, since we have a relatively low negative development in Sharpe ratio compared with other portfolio types in the same time horizon but also still a high level of risk. This can be considered as the last choice to an investment manager since it has the lowest traditional or modified Sharpe ratio. So in order to compare, we can not only take a look at development ratios, but also the magnitude of these parameters are essential to our analysis.

The last interesting result which is true for all 4 different portfolios is the result of optimization for maximizing both traditional and modified Sharpe ratio. Although we get the same weights, portfolio risk and return for these cases, the modified Sharpe ratio is smaller than the traditional. This confirms our claim about the overestimation of the Sharpe ratio calculated by the traditional method.

Optimization	Daily Portfolio - Sharpe Increase/Decrease %		Weekly Portfolio - Sharpe Increase/Decrease %		Monthly Portfolio - Sharpe Increase/Decrease %		Quarterly Portfolio - Sharpe Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	**-54,39%**	**-54,39%**	**-37,46%**	**-37,45%**	**-52,17%**	**-43,92%**	**-79,62%**	**-53,04%**
Positive Skewness and Kurtosi Greater than :	**-61,48%**	**-61,43%**	**-51,84%**	**-51,84%**	**N/A**	**N/A**	**N/A**	**N/A**
Positive Skewness	**-52,78%**	**-52,75%**	**-43,66%**	**-43,64%**	**-52,50%**	**-52,50%**	**-55,67%**	**-55,67%**
Kurtosis Greater than 3	**-48,35%**	**-48,34%**	**5,44%**	**5,51%**	**N/A**	**N/A**	**N/A**	**N/A**
	Daily Portfolio - Return Increase/Decrease %		**Weekly Portfolio - Return** Increase/Decrease %		**Monthly Portfolio - Return** Increase/Decrease %		**Quarterly Portfolio - Return** Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	**-59,82%**	**-59,82%**	**-45,09%**	**-45,10%**	**-51,89%**	**-51,88%**	**-66,93%**	**-66,90%**
Positive Skewness and Kurtosi Greater than :	**-64,37%**	**-64,34%**	**-60,71%**	**-60,71%**	**N/A**	**N/A**	**N/A**	**N/A**
Positive Skewness	**-58,01%**	**-57,99%**	**-49,97%**	**-49,96%**	**-51,41%**	**-51,42%**	**-44,41%**	**-44,41%**
Kurtosis Greater than 3	**-53,78%**	**-53,78%**	**-24,22%**	**-24,16%**	**N/A**	**N/A**	**N/A**	**N/A**
	Daily Portfolio - Risk Increase/Decrease %		**Weekly Portfolio - Risk** Increase/Decrease %		**Monthly Portfolio - Risk** Increase/Decrease %		**Quarterly Portfolio - Risk** Increase/Decrease %	
	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio	Sharpe Ratio	Modified Sharpe Ratio
Traditional Markowitz Model	**-22,92%**	**-22,92%**	**-15,37%**	**-15,39%**	**-6,03%**	**-6,02%**	**27,64%**	**27,79%**
Positive Skewness and Kurtosi Greater than :	**-21,95%**	**-21,96%**	**-29,37%**	**-29,37%**	**N/A**	**N/A**	**N/A**	**N/A**
Positive Skewness	**-20,53%**	**-20,53%**	**-16,24%**	**-16,24%**	**-4,50%**	**-4,50%**	**21,44%**	**21,45%**
Kurtosis Greater than 3	**-18,55%**	**-18,55%**	**-25,88%**	**-25,86%**	**N/A**	**N/A**	**N/A**	**N/A**

Table 6 - Difference Ratios for Different Time Horizons

Weekly time series:

This time series is divided into two periods as mentioned above for the latter time series, and four types of portfolios constructed considering combinations of 2 additional parameters, skewness and kurtosis. As we mentioned above, we are not going to consider the case of a portfolio with positive skewness and kurtosis greater than 3, since the number of the stocks available is limited and the diversification can not take place.

The traditional Markowitz model shows the lowest decrease in Sharpe ratio which is due to the low decrease in return and the lowest decrease in risk of the portfolio for the two periods. This case was compared with other portfolios has the highest Sharpe ratio in this time horizon, also the least risk and a high return.

The next category is where we have a portfolio of positive skewness. 34 stocks out of our 42 in the sample have this characteristic. This obviously gives a good level of diversification to us. The decrease of the Sharpe ratio both traditional and modified seems to be moderately low compared with other categories. Despite the last case, with only kurtosis greater than 3 we have a positive development! Later we discuss the reason for this. A high traditional Sharpe ratio of 1.08 simultaneously an annual portfolio return of 25% followed by a risk of 19% makes this investment attractive for its time horizon. As we see the difference in this category is not much from the traditional Markowitz model, but still there is slight differences, due to effect of diversification, and extreme events happened during the week.

The last case is interesting, the only case with positive development of the Sharpe ratio. The reason is that the decrease in risk of the portfolio is greater than the decrease for the return of the portfolio. And this is due to a low Sharpe ratio, less than 1. So this case will be not worth to consider as investment opportunity.

Monthly Time series:

The next category to analyze is the monthly time series. Before considering this category we should mention that two types of portfolios were not possible to establish, the portfolios with positive skewness and kurtosis greater than 3, and the one with only kurtosis greater than 3. The reason for this was the lack of data, a limited number of stocks to perform a portfolio and consequently low level of diversification.

Let's consider the first row of the data for the monthly portfolios. It is the portfolio based on traditional Markowitz model and for the first time compared with latter portfolios we see that the level of decrease in traditional Sharpe and the modified one is not equal. This can be another reason to why we use modified Sharpe ratio. Modified Sharpe ratio gave us a more stable result in these two periods which was not observed under the traditional Sharpe ratio. One explanation to this can be the low level of decrease in the risk associated with this type of portfolio. Monthly portfolio in compare with daily and weekly portfolio has a lower risk in association with almost the same level of return with these portfolios. In other two cases we have such a radical decrease in the risk measure in these two periods while the level of decrease for return remained almost constant for these two types of portfolios. One might conclude that it is a good sign. More decrease in the level of risk for two periods, associated with almost the same level of return might be attractive to anybody! But there is a question of stability too. Is this radical decrease really reliable? Our answer is No!

Here we need more inter analysis of the risk for these four types of time series. One of the most obvious characteristic of the behaviour of the risk is as the time series of the events become more frequent, which means that if we are going to have longer time series, obviously we have a higher level of risk, due to discrepancies. It is touchable under daily and weekly time series. On the other hand if the data sets become less frequent but of the same time length the omission of the volatility during the intervals can cause a decrease in the level of risk, which happened in the case of monthly and quarterly time series. For more detailed data you can refer to risk and return tables followed at the end of this analysis.

Now after this analysis on the behaviour of the risk, we can see that we should be more careful to accept and interpret the results of studies.

Sharpe Ratio/Modified Sharpe Ratio		Daily Portfolio	Weekly Portfolio	Monthly Portfolio	Quarterly Portfolio
	Traditional Markowitz Model				
Historical	Sharpe Ratio	0,992020696	1,12922393	1,414414883	1,493233205
	Modified Sharpe Ratio	0,603105761	0,686519981	0,859903226	0,907821329
Future	Sharpe Ratio	0,45248599	0,706233886	0,676540412	0,304372119
	Modified Sharpe Ratio	0,275087437	0,429416123	0,48223651	0,426347861
	Positive Skewness and Kurtosi Greater than 3				
Historical	Sharpe Ratio	0,86085468	0,521497735	N/A	N/A
	Modified Sharpe Ratio	0,523362572	0,317048111	N/A	N/A
Future	Sharpe Ratio	0,331603596	0,2511566	N/A	N/A
	Modified Sharpe Ratio	0,201847344	0,15269236	N/A	N/A
	Positive Skewness				
Historical	Sharpe Ratio	1,043982831	1,08602251	1,322041672	1,39841282
	Modified Sharpe Ratio	0,634696239	0,6602549	0,803744036	0,850174664
Future	Sharpe Ratio	0,49297677	0,611900981	0,627952737	0,619887812
	Modified Sharpe Ratio	0,299879543	0,372109973	0,381771894	0,376864504
	Kurtosis Greater than 3				
Historical	Sharpe Ratio	0,880049544	0,556742372	N/A	N/A
	Modified Sharpe Ratio	0,535032139	0,338475322	N/A	N/A
Future	Sharpe Ratio	0,454549224	0,587054964	N/A	N/A
	Modified Sharpe Ratio	0,276372552	0,357137817	N/A	N/A

Table 7 - Sharpe Ratio

The second portfolio in this time category is constructed with stocks which have only positive skewness. As it was not far from imagination, we have again the same level of decrease in the both portfolios, optimized by traditional Sharpe ratio or by the modified Sharpe ratio. This can be the result of our choice of stocks, the stocks with only positive skewness. It can be seen that the Modified Value at Risk used to measure the risk for constructing the modified Sharpe ratio can consider the right skewed effect, since still giving the same level of the risk and return for both of the portfolios optimized by traditional or modified Sharpe ratio. You can compare the results for Sharpe ratios by table 7.

Quarterly Time Series:

The last time series we are going to analyze more in detail is the quarterly time series. The data to construct two types of portfolios was not available to us as indicated in the table by N/A. This type of time horizon is quiet different from other time horizons. The reason is the release of quarterly reports by companies. Almost all companies try to clean up their financial losses and show a good performance, although it might come quiet late into the analysis of investors, but it has its impact on stock markets, both on liquidity and volatility of the market.

As it is shown in table 6, the first row where we have a portfolio constructed by traditional Markowitz model for quarterly time horizon, you'll see that the decrease in traditional Share ratio is the highest change ever in our research. The reason can be clarified by looking more closely into the risk factor. For this time horizon the difference in return of the portfolios is almost in the same range of the other time horizons, that is why we exempt this parameter and go directly to the risk for finding out the reason for this dramatic decrease in traditional Sharpe ratio.

The pattern of changes in the difference of ratios for risk which started from daily time horizon just turned the sign and became an increase for quarterly time horizon. This increase in the risk can be due to release of the quarterly reports by corporations and of course followed by an increase in trade for stocks. This results in more liquidity in the market. The other reason can be the cumulative return of the stocks during the quarter, while we ignore the volatility of the market in this period. We should also consider positive or mostly overestimated effect of these reports; the annual returns based on figures deviated long from the mean, and the annual risk based on the not so frequent return statistics, but cumulated and long away from the mean.

Now let's take a look at the second portfolio in this category. It takes only stocks with positive skewness. For historical time horizon we have almost the same risk, return and Sharpe ratio for these two portfolios. These figures are available in tables 7, 8 and 9 for comparison. But the interesting results come into eyes when we look at developments of these two portfolio types in future time horizon. The portfolio with positive skewness has a less difference in risk development in compare with the Markowitz model and also a much less difference in return's developments. This consequently follows by a less difference in traditional Sharpe ratio. In contrast with the latter portfolio, the one with skewness shows the same development for traditional vs. modified Sharpe ratio.

To conclude one more time we can claim that the portfolio constructed by stocks with positive skewness can generate better results for future periods.

Yearly Time Series

This data series can not be used to construct a portfolio, since the value obtained by solver for optimized Sharpe ratio is irrelevant. A Sharpe quote of almost 600 obtained for this time horizon.

Risk		Daily Portfolio	Weekly Portfolio	Monthly Portfolio	Quarterly Portfolio
	Traditional Markowitz Model				
Historical	Sharpe Ratio	22,15%	18,63%	15,38%	15,38%
	Modified Sharpe Ratio	22,15%	18,64%	15,38%	15,38%
Future	Sharpe Ratio	17,08%	15,76%	14,45%	19,63%
	Modified Sharpe Ratio	17,08%	15,77%	14,45%	19,65%
	Positive Skewness and Kurtosi Greater than 3				
Historical	Sharpe Ratio	26,72%	37,10%	N/A	N/A
	Modified Sharpe Ratio	26,71%	37,10%	N/A	N/A
Future	Sharpe Ratio	20,85%	26,21%	N/A	N/A
	Modified Sharpe Ratio	20,85%	26,21%	N/A	N/A
	Positive Skewness				
Historical	Sharpe Ratio	22,22%	19,62%	15,74%	16,30%
	Modified Sharpe Ratio	22,22%	19,62%	15,74%	16,30%
Future	Sharpe Ratio	17,66%	16,43%	15,03%	19,80%
	Modified Sharpe Ratio	17,66%	16,43%	15,03%	19,80%
	Kurtosis Greater than 3				
Historical	Sharpe Ratio	23,93%	29,42%	N/A	N/A
	Modified Sharpe Ratio	23,93%	29,42%	N/A	N/A
Future	Sharpe Ratio	19,49%	21,81%	N/A	N/A
	Modified Sharpe Ratio	19,49%	21,81%	N/A	N/A

Table 8 - Portfolio Risk in Different Time Horizons.

Return		Daily Portfolio	Weekly Portfolio	Monthly Portfolio	Quarterly Portfolio
	Traditional Markowitz Model				
Historical	Sharpe Ratio	25,81%	24,90%	25,62%	25,78%
	Modified Sharpe Ratio	25,81%	24,91%	25,62%	25,78%
Future	Sharpe Ratio	10,37%	13,67%	12,33%	8,52%
	Modified Sharpe Ratio	10,37%	13,68%	12,33%	8,53%
	Positive Skewness and Kurtosi Greater than 3				
Historical	Sharpe Ratio	26,83%	23,21%	N/A	N/A
	Modified Sharpe Ratio	26,83%	23,21%	N/A	N/A
Future	Sharpe Ratio	9,56%	9,12%	N/A	N/A
	Modified Sharpe Ratio	9,57%	9,12%	N/A	N/A
	Positive Skewness				
Historical	Sharpe Ratio	27,03%	25,17%	24,67%	26,66%
	Modified Sharpe Ratio	27,03%	25,17%	24,67%	26,66%
Future	Sharpe Ratio	11,35%	12,59%	11,99%	14,82%
	Modified Sharpe Ratio	11,35%	12,60%	11,99%	14,82%
	Kurtosis Greater than 3				
Historical	Sharpe Ratio	24,89%	20,24%	N/A	N/A
	Modified Sharpe Ratio	24,89%	20,24%	N/A	N/A
Future	Sharpe Ratio	11,50%	15,34%	N/A	N/A
	Modified Sharpe Ratio	11,51%	15,35%	N/A	N/A

Table 9 - Portfolio Return in Different Time Horizon.

Summary of the Results Touched by Empirical Investigation

To sum up shortly the answers to the questions that we have investigated, we have the following result.

> *1. What will be the difference between two optimized portfolios when;*
>
> *-The first one is optimized by traditional mean-variance with the Sharpe model, and then by sorting out the stocks' skewness and kurtosis and study the importance of these parameters.*
>
> *-Second case, when we optimize the portfolio considering a new risk measure, MVaR. Then sorting out data by skewness and kurtosis and perform the same study done already on the last group.*
>
> *Then compare these results with future data.*

Daily Portfolio

For the comparison of the two different Sharpe ratios for optimising the portfolios, we found no differences when comparing return and risk or even the weights of these two Portfolios. But also that the modified Sharpe quotes is approximately half of the traditional one. The reason for this as we mentioned was that the traditional one dose not distinguish between upside and downside risk, while the use of modified Sharpe ratio proposed to count for cancelling out the impact of such deficiencies by the traditional model.

When looking also at the different combinations of skewness and kurtosis for portfolios. We could reasonably easy see that the portfolio of choose would be the on optimized with only by positively skewed stocks. This portfolio had the highest Sharpe in both cases and also preformed best when looking at the risk and return outcome in the future. So this portfolio had the best combination of these three parameters.

It was also quit easy to distinguish that the combination of optimizing portfolios with both positive skewness and kurtosis greater then 3, was the worst performer of these four portfolios.

Weekly Portfolio

Then again we see the same case of equal weights when using different Sharps, also the same risk and return. But also that the traditional Sharpe is almost doubled compared to the modified one.

In this case when comparing the different combinations of portfolios, it is not that easy to rally distinguish which one is the best. The ones that we could be indifference with is the portfolio optimised by traditional Markowitz model or positively skewed. But from the result we have obtained we can say that the best portfolio leans more to the traditional Markowitz model based on our result.

But to distinguish which portfolio(s) are not clear investment objectives was not as difficult. The least attractive investment would be the on optimised by assets which are both positively skewed and have a kurtosis grater then 3.

Monthly Portfolio

For this data set, we did not include portfolios with “Skewness and Kurtosis” and “Kurtosis grater then 3”. This is due strongly to the inefficiency of diversification possibility.

Then when we only head two portfolios to look at, traditional Markowitz and positively skewed. There is not really a clear distinction between optimized Markowitz model or the positively skewed one. But still again it leans a little more against the Markowitz model.

But also to notice an interesting finding in this part of the investigation is that the traditional Sharpe ratio and the modified one differs, but only for the traditional Markowitz model and not for the portfolio with positive skewness.

Quarterly Portfolio

In this part we omitted the two portfolios mentioned also in the monthly data. Since the diversifiable amount of assets is not acceptable.

Then for the traditional Markowitz portfolio we see that it has the highest Sharpe ratio compared to the positively skewed one. We also saw in this case that the traditional Sharpe had an 80% in difference compared to the modified one of 53%. While for the skewed portfolio was the same in both cases.

One could also notice from our investigation that the risk for both portfolios has risen when we look in to the future. While at the same time the return has decreased considerably more for the Markowitz model then the skewed portfolio. But looking at the historical figures only, would make it difficult to determine which portfolio performed the best. But looking at the whole picture of the future and history, we can say that the positively skewed one is the best choose.

Yearly Portfolio

For this type of series we have conclude from the attempt to implement the different models, that we are not able to construct reasonable portfolio structures with this data. The reason is the unrealistic result of the Sharpe ratio calculations.

2. Compare the portfolios calculated in part one with other portfolios that have different time series (monthly, daily, etc).

Daily time series

We can see in this data set that the standard Markowitz model has a Sharpe of 0.99 while weekly has a Sharpe of 1.12, this is higher then the daily. But the number of stocks in the portfolio is the same. We found that the explanation for this lies under the risk comparison. Since the data set is larger in the daily series, more extreme events can be registered, then during the weekly data. This is showed by the daily data having a higher risk and weekly a lower one. We could also find the effect on the modified Sharpe, which had daily Sharpe of 0.60 and weekly 0.68, and the VaR was 36% and 30% respectively. This shows that the difference is smaller since VaR accounts for extreme events.

In our time series investigation we have noticed that by building portfolios during a then year period divided by two five years period, and adding on the different combinations of skewness and kurtosis. That the smaller amount of time series you have to work with the less diversification possibility you will have when adding on extra constraints to the Markowitz model. This is also because it limits the amount of stocks to be included in the optimisation. This is the reason why we omitted some portfolios, because they did not follow the diversification principle.

The different portfolios have different levels of impact depending on the time series. Mostly because the methods we used had effects on the assets at different time series events.

We were also able due to the time series to conclude that, for our 4 types of portfolios that although the weight, return and risk, modified Sharpe ratio is smaller than the traditional one. This shows that our claim about the overestimation of the traditional Sharpe ratio is true.

Weekly time series

In this time series we found that the Sharpe ratio for the Markowitz model had the smallest change, as we can see in table 6 comparing all other portfolios and time series. The weekly time series shows an overall smallest change in Sharpe compared to the other time series.

We were able to find one portfolio compared to other time series and portfolios, which had an increase in the future, Sharpe ratios. The reason for this was that the decrease in risk in the portfolio was greater then the decrease in the return. The portfolio in this case was the one with a kurtosis greater then 3.

The positively skewed and the Markowitz portfolio is the two clear chooses in this data set. But when comparing to the different time series, we could se that the Markowitz portfolio had the lowest return compared to the other time series for this portfolio, but had the second highest risk. The skewed portfolios had the second highest risk but the third highest return. In the future data for these portfolios the Markowitz model had the third highest risk and the highest return, compared to its other time series. For the skewed one we had the third highest risk and the second highest return. So in this case it would lean a little more in favour for the Markowitz model.

Monthly time series

In this data series we omitted the portfolios with positive skewness and kurtosis greater than 3, and the one with only kurtosis greater than 3. The reason for this was that it did not fill the concept of diversification.

What we found here was that, for the traditional Markowitz model when looking ahead was that the level of decrease in the traditional and modified Sharpe is not equal. But also the modified Sharpe showed a more stable result. A reason for this could be the low level of risk associated with that type of portfolio. The monthly portfolio has a lower level of risk compared to the other time series daily and weekly, whereas the return was approximately the same.

There was also an obvious pattern realised on the characteristics of the behaviour of risk. This was that the bigger frequency of time series we had, the higher the level of risk, due to discrepancies. In the other way around it could decrease the level of risk, which it did for the monthly and quarterly data.

For the portfolio with positive skewness, we obtained again the same level of decrese in both portfolios, optimizing using traditional and modified Sharpe ratio. It was shown that the Modified Value at Risk used to measure the risk for constructing the modified Sharpe ratio can consider the right skewed effect, since it still gives the same level of the risk and return for both of the portfolios optimized by traditional or modified Sharpe ratio.

Quarterly time series

We realized that one of the underlying reasons for the large shifts in the stock returns is, from how the stock markets are affected by the speculators and analysts that play the market during quarterly reports.

We can find that in this time series the decrease in the traditional Sharpe is the highest. And that it had the highest Sharpe ration in the historical time series than the other once. It is also interesting to see in the investigation that the risk when looking into the future has a positive change (increase), then all the other time sets. We believed that the markets speculations on the quarterly reports had an impact of large effects on the stock prices. But that the increase in the future risk, is affected of other things mentioned.

Conclusion

After introducing the model and constructing it on excel we started our empirical investigation on the model. The results were amazingly satisfying in this section. The tests on the normality of the assets' return showed a strong support for our claim about the abnormality of the assets' return with a high level of statistical significance. The next part as a result of the abnormality of data sets leads us into finding a new measure for the performance in order to consider these inefficiencies. The new measure introduced and compared with the original model also in combination with some new parameters, like skewness and kurtosis. Hiring these parameters changed the results of the future portfolios and we presented above which factor had the greatest influence or how they could make an investment manager better in his investment.

Form our empirical research we are able to draw the following conclusion of the study we made. The description will be brief since a deeper explanation of what we conclude in our investigation could be found in the empirical and analysis part of the paper.

- The concept of diversification on portfolio selection showed its importance in the mean-variance optimisation approach, due to the balancing of risk and reward.

- Incorporating higher statistical moments in decision-making has shown both weaknesses and strengths. The incorporation of Skewness has shown slightly better effect on the mean-variance optimisation compared to future portfolios.

- The data set which replicated best for the future portfolios was the monthly time series. It showed a moderate accurate estimate of the future, when risk and return was taken into account.

- In general the traditional Sharpe model showed an inconsistent estimation compared with modified version when two time periods collated. This was mainly due to extreme events.

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Appendices

Appendix 1 – Proof of Expected Value (Mean)[30]

Assume some population X and some random sample of that population $x_1, x_2, ..., x_n$. Then we can define another random variable Y, which is he mean of the sample:

$$Y = \frac{1}{n}\sum_{i=1}^{n} x_i$$

For population in statistics one defines the mean of X as μ. We can then prove that the mean of Y is:

$$\begin{aligned} E[Y] &= E\left[\frac{1}{n}\sum_{i=1}^{n} x_i\right] \\ &= \frac{1}{n}E\left[\sum_{i=1}^{n} x_i\right] \\ &= \frac{1}{n}\sum_{i=1}^{n} E[x_i] \\ &= \frac{1}{n}\sum_{i=1}^{n} \mu \\ &= \frac{1}{n}n\mu = \mu \end{aligned}$$

This means then that since the expected value of Y is μ, this then is referred to as an unbiased estimator of μ. One can also refer to it as, if we don't know μ, but we know the sample mean, $\bar{x}$, then we should use $\bar{x}$ to estimate μ.

[30] Wackerly et al. [5]

Appendix 2 – Proof of Variance[31]

This is a proof of why s^2 is an unbiased estimator of the population variance. To show that s^2 is unbiased, we will show that $E\left(s^2\right)=\sigma^2$. We need to assume that the population which the $x_1, x_2, \ldots\ldots, x_i$ are taken from has mean μ and variance σ^2. Then we can prove that:

$$
\begin{aligned}
E\left(s^2\right) &= E\left(\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2\right) \\
&= \frac{1}{n-1}\sum_{i=1}^{n}E\left(\left(x_i-\bar{x}\right)^2\right) \\
&= \frac{1}{n-1}\sum_{i=1}^{n}\left(\left(\left(x_i-\mu\right)-\left(\bar{x}-\mu\right)\right)^2\right) \\
&= \frac{1}{n-1}\sum_{i=1}^{n}\left\{E\left(\left(x_i-\mu\right)^2\right)-2E\left(\left(x_i-\mu\right)\left(\bar{x}-\mu\right)\right)+E\left(\left(\bar{x}-\mu\right)^2\right)\right\} \\
&= \frac{1}{n-1}\sum_{i=1}^{n}\left[\sigma^2-2\left(\frac{1}{n}\sum_{j=1}^{n}E\left(\left(x_i-\mu\right)\left(x_i-\mu\right)\right)\right)+\frac{1}{n^2}\sum_{j=1}^{n}\sum_{k=1}^{n}E\left(\left(x_j-\mu\right)\left(x_k-\mu\right)\right)\right] \\
&= \frac{1}{n-1}\sum_{i=1}^{n}\left(\sigma^2-\frac{2\sigma^2}{n}+\frac{\sigma^2}{n}\right) \\
&= \frac{1}{n-1}\sum_{i=1}^{n}\frac{\left(n-1\right)\sigma^2}{n} \\
&= \frac{\left(n-1\right)\sigma^2}{n-1}=\sigma^2
\end{aligned}
$$

This proves that s^2 is an unbiased estimator of the population variance. The proof for the standard deviation is the same but instead the whole expression is squared $\sqrt{E\left(s^2\right)}=\sqrt{\sigma^2}$.

[31] Wackerly et al. [5]

Appendix 3 – Table of Skewness[32]

Skewness for Individual Stocks			Historical					Future				
			Daily	Weekly	Monthly	Quarterly	Yearly	Daily	Weekly	Monthly	Quarterly	Yearly
SX-SSVX30.SE	Benchmark	SSVX 30	0,781	0,113	-0,447	-0,741	-1,369	1,957	0,922	0,848	0,904	0,966
RT-SIXRX.SE	Benchmark	SIXRX (TR)	0,053	0,411	0,086	0,313	1,292	0,102	-0,664	-0,523	-1,926	-2,018
JM	JM	Finance	0,071	0,601	0,409	0,483	-0,580	-0,330	-0,011	-0,524	-0,779	-0,605
CAST	Castellum	Finance	0,465	0,504	-0,140	-0,603	0,281	-0,185	-0,345	-0,401	0,169	-0,340
KINV-B	Kinnevik B	Finance	0,187	0,178	0,615	0,749	0,577	0,107	0,225	1,726	0,364	1,156
SWED-A	Swedbank A	Finance	0,150	0,308	0,632	1,116	0,294	-0,021	-0,459	-0,152	-0,737	-1,320
FABG	Fabege	Finance	1,084	0,992	0,313	0,113	-1,050	-1,433	-0,990	-1,123	-0,767	1,466
LATO-B	Latour B	Finance	0,063	0,794	0,888	1,025	0,511	1,085	-0,289	0,346	-1,344	-0,654
LUND-B	Lundberg	Finance	-0,097	-0,451	-1,272	-1,010	-1,893	-0,487	-0,539	0,475	0,108	1,860
OMX	OMX	Finance	3,722	0,733	1,195	1,051	0,156	1,741	1,783	0,683	-0,384	0,385
RATO-B	Ratos B	Finance	-0,008	0,875	-0,086	0,091	-0,889	0,133	-0,443	-0,332	-0,175	1,060
ORES	Öresund	Finance	-1,495	-1,423	-0,755	0,072	0,359	-0,318	0,622	-0,173	-0,256	0,351
SEB-A	SEB A	Finance	0,148	0,749	-1,075	-1,233	0,434	0,527	0,035	-0,534	-0,342	-1,545
SHB-A	SHB A	Finance	0,463	0,721	-0,087	0,313	0,788	0,008	-0,335	-0,084	-0,944	-1,405
INVE-B	Investor B	Finance	0,244	0,073	-0,620	-0,963	-0,530	0,028	-0,391	-0,829	-2,601	-1,267
HUFV-A	Hufvudstaden A	Finance	0,701	1,124	0,237	-0,597	-0,059	-0,181	-0,196	-0,264	-0,397	-0,028
INDU-A	Industryvärden A	Finance	0,040	0,068	-0,108	0,145	1,120	-0,076	-0,539	-0,297	-1,487	-2,011
SKF-B	SKF B	Industry	0,423	0,296	0,214	-0,067	0,912	0,474	0,014	-0,147	0,216	1,619
VOLV-B	Volvo B	Industry	0,218	0,708	0,212	0,282	0,314	0,007	-0,135	0,122	-1,186	-0,952
SKA-B	Skanska B	Industry	-1,962	-1,171	0,117	-0,533	0,767	0,112	0,098	0,360	-1,150	-0,753
TREL-B	Trelleborg B	Industry	0,137	0,914	0,273	-0,420	-0,188	0,144	-0,314	0,035	-0,859	0,811
NCC-B	NCC B	Industry	-0,348	0,099	-0,068	-0,229	1,714	-0,375	-0,289	-0,031	0,236	-0,531
PEAB-B	Peab B	Industry	0,973	0,366	0,415	-0,218	1,438	-0,057	-0,044	-0,105	-0,339	-1,662
HEXA-B	Hexagon B	Industry	-0,318	0,300	-0,092	0,088	0,252	0,331	0,273	0,032	-0,548	1,320
SECO-B	Seco Tools B	Industry	-0,082	0,050	-0,485	-0,167	-0,184	-0,598	-0,471	0,007	0,737	-0,234
ASSA-B	Assa Abloy B	Industry	0,554	0,712	0,663	-0,054	-0,659	0,064	0,296	0,671	-0,733	-0,545
SAS	SAS	Industry	-0,331	-0,950	-0,616	-1,021	0,682	0,450	0,393	0,276	-0,161	0,528
SECU-B	Securitas B	Industry	0,965	0,464	0,553	0,633	0,829	-1,079	-0,393	-0,639	-0,873	-0,715
ATCO-A	Atlas Copco A	Industry	0,415	0,306	0,407	0,242	1,062	0,506	-0,268	-0,049	-1,355	-0,460
SAND	Sandvik	Industry	0,438	0,093	0,412	-0,233	1,827	0,106	-0,357	-0,020	0,925	-0,982
HOLM-B	Holmen B	Material	-1,264	-0,679	-0,900	-0,751	2,211	-0,823	-0,869	-0,922	0,090	-0,446
SCA-B	SCA B	Material	0,320	0,203	0,390	-0,729	0,252	0,381	-0,084	-0,060	-0,078	1,599
SSAB-A	SSAB A	Material	0,494	0,162	-0,134	0,109	0,900	0,209	0,166	0,005	-0,836	0,307
ELUX-B	Electrolux B	Commodities	0,023	0,030	0,211	0,168	-0,589	0,389	-0,205	0,087	-0,743	-0,042
HM-B	H & M B	Commodities	-0,860	-0,007	0,500	0,088	-0,470	0,617	0,055	-0,383	-0,014	-0,832
TEL2-B	Tele2 B	Telecommunication	0,443	0,071	0,520	1,163	0,437	0,267	0,264	1,792	1,514	0,870
ERIC-B	Ericsson B	IT	-0,042	-0,086	-0,078	0,784	1,422	0,356	0,715	2,412	0,115	-0,620
NOKI-SDB	Nokia	IT	0,335	-0,260	-0,216	0,749	0,549	-0,342	0,023	-0,251	-0,405	0,434
GETI-B	Getinge B	Health Care	0,085	0,642	0,286	0,140	1,266	0,407	0,275	0,306	0,308	-0,189
EKTA-B	Elekta B	Health Care	1,171	0,799	1,059	0,566	1,799	-0,051	0,016	-0,503	-0,853	0,434
MEDA-A	Meda A	Health Care	2,330	3,342	3,424	1,485	-0,868	1,676	3,641	2,519	0,298	0,747
AXFO	Axfood	Commodities	1,491	0,804	1,105	0,297	1,457	-0,345	0,485	0,123	-0,224	-0,479
SWMA	Swedish Match	Commodities	0,212	0,635	0,653	0,411	0,807	0,146	-0,172	0,098	1,163	0,334
VOST-SDB	Vostok Nafta	Energy	0,858	1,088	-0,095	1,189	0,487	0,213	0,443	0,541	1,767	1,249

Table 10 - Skewness for stocks and different type of data series.

[32] These tables presented on the appendices are the results of our study and they are available on the excel file provided by this report.

Appendix 4 – Table of Kurtosis

Kurtosis for Individual Stocks			Historical Data					Future Data				
			Daily	Weekly	Monthly	Quarterly	Yearly	Daily	Weekly	Monthly	Quarterly	Yearly
SX-SSVX30.SE	Benchmark	SSVX 30	4,787	4,128	-0,351	-0,048	1,831	4,195	0,104	-0,179	-0,048	-0,285
RT-SIXRX.SE	Benchmark	SIXRX (TR)	2,382	2,683	-0,201	4,861	1,813	3,949	2,668	1,657	4,861	4,180
JM	JM	Finance	5,543	1,234	0,321	0,244	-0,124	4,621	2,833	0,385	0,244	-3,259
CAST	Castellum	Finance	5,958	2,268	-0,232	-0,237	0,859	3,147	1,529	0,421	-0,237	-1,497
KINV-B	Kinnevik B	Finance	5,566	1,976	0,969	0,457	-1,997	7,701	2,721	8,145	0,457	1,656
SWED-A	Swedbank A	Finance	1,251	3,027	3,053	0,894	-2,695	2,790	0,738	0,424	0,894	2,009
FABG	Fabege	Finance	9,722	4,675	-0,028	-0,467	2,506	17,046	3,537	1,644	-0,467	2,326
LATO-B	Latour B	Finance	7,540	3,353	1,389	2,349	1,277	31,342	1,366	0,682	2,349	-2,213
LUND-B	Lundberg	Finance	3,994	2,888	4,572	-0,621	3,529	4,376	2,508	0,596	-0,621	3,440
OMX	OMX	Finance	50,656	1,292	2,165	1,255	-0,875	19,433	12,302	3,090	1,255	1,474
RATO-B	Ratos B	Finance	6,124	8,412	0,651	-0,241	0,464	3,830	1,995	-0,249	-0,241	1,819
ORES	Öresund	Finance	19,747	9,527	1,390	0,124	0,311	6,289	6,457	1,177	0,124	-1,844
SEB-A	SEB A	Finance	1,913	6,637	2,427	0,053	-1,768	4,470	1,577	0,363	0,053	2,962
SHB-A	SHB A	Finance	1,810	3,766	0,042	0,512	-0,348	4,033	2,025	-0,266	0,512	2,311
INVE-B	Investor B	Finance	2,640	0,968	0,843	8,319	-0,244	3,782	2,602	1,292	8,319	2,653
HUFV-A	Hufvudstaden A	Finance	5,552	4,530	0,150	-0,781	-2,134	4,749	3,544	0,917	-0,781	-2,160
INDU-A	Industryvärden A	Finance	1,829	0,992	-0,279	3,175	1,748	3,627	0,682	1,355	3,175	4,314
SKF-B	SKF B	Industry	2,064	0,905	-0,510	-0,329	0,218	5,219	0,398	-0,604	-0,329	2,578
VOLV-B	Volvo B	Industry	3,814	3,588	-0,795	3,465	-2,588	2,828	0,511	1,211	3,465	1,168
SKA-B	Skanska B	Industry	20,582	6,332	0,650	1,358	-2,293	3,689	1,398	1,048	1,358	1,637
TREL-B	Trelleborg B	Industry	4,804	3,736	1,339	0,084	-1,506	3,097	1,054	-0,674	0,084	-1,373
NCC-B	NCC B	Industry	3,631	1,363	-0,460	-0,472	2,698	4,557	1,045	-0,333	-0,472	-1,338
PEAB-B	Peab B	Industry	6,206	0,321	-0,430	0,559	1,802	3,850	1,648	-0,299	0,559	2,652
HEXA-B	Hexagon B	Industry	4,554	2,402	0,990	0,258	-2,883	2,339	0,121	0,457	0,258	2,115
SECO-B	Seco Tools B	Industry	11,574	0,911	-0,249	0,391	1,544	14,096	2,000	1,750	0,391	-1,457
ASSA-B	Assa Abloy B	Industry	3,394	2,565	0,837	-0,186	-2,835	7,577	3,076	2,142	-0,186	-1,138
SAS	SAS	Industry	4,307	7,368	1,632	-0,716	-1,880	2,931	1,099	2,518	-0,716	-0,870
SECU-B	Securitas B	Industry	5,767	2,427	1,592	-0,058	2,300	20,055	6,058	0,779	-0,058	-0,419
ATCO-A	Atlas Copco A	Industry	4,293	0,408	1,447	2,678	-0,571	4,057	0,075	0,193	2,678	0,364
SAND	Sandvik	Industry	2,471	0,414	1,097	0,257	3,749	3,510	0,423	0,043	0,257	-0,369
HOLM-B	Holmen B	Material	17,475	5,718	2,147	0,154	4,911	12,904	3,017	0,901	0,154	-3,022
SCA-B	SCA B	Material	1,471	-0,234	0,091	-0,729	-2,346	4,141	1,192	0,462	-0,729	3,003
SSAB-A	SSAB A	Material	2,593	1,165	-0,004	1,707	0,436	5,471	3,130	-0,699	1,707	-2,063
ELUX-B	Electrolux B	Commodities	1,697	1,309	-0,256	0,689	-0,829	5,648	1,668	-0,554	0,689	-2,629
HM-B	H & M B	Commodities	15,768	3,648	1,641	-0,113	-2,439	7,233	1,568	-0,244	-0,113	-0,100
TEL2-B	Tele2 B	Telecommunication	5,110	1,993	0,639	3,415	-3,034	5,837	2,342	8,576	3,415	0,148
ERIC-B	Ericsson B	IT	2,921	1,929	-0,098	1,298	2,132	7,212	5,377	13,606	1,298	0,637
NOKI-SDB	Nokia	IT	4,618	0,752	0,024	0,421	-3,041	5,751	2,157	1,400	0,421	1,706
GETI-B	Getinge B	Health Care	2,713	1,440	0,621	0,097	1,984	2,133	1,009	0,309	0,097	-0,291
EKTA-B	Elekta B	Health Care	6,510	2,228	2,727	2,443	3,777	5,043	2,633	1,548	2,443	0,168
MEDA-A	Meda A	Health Care	28,517	23,600	17,790	0,245	0,414	14,137	29,876	12,279	0,245	-1,251
AXFO	Axfood	Commodities	10,598	1,871	2,203	-0,680	1,736	12,387	2,835	0,629	-0,680	-3,017
SWMA	Swedish Match	Commodities	1,920	0,445	0,372	2,072	1,223	3,121	0,877	-0,635	2,072	-0,922
VOST-SDB	Vostok Nafta	Energy	8,037	5,954	3,215	5,266	-1,565	4,986	3,212	0,231	5,266	0,636

Table 11 - Kurtosis for stocks and different type of data series.

Appendix 5 – Table of Beta for Individual Stocks

Beta Ratio for Individual Stocks			Historical					Future				
			Daily	Weekly	Monthly	Quarterly	Yearly	Daily	Weekly	Monthly	Quarterly	Yearly
SX-SSVX30.SE	Benchmark	SSVX 30										
RT-SIXRX.SE	Benchmark	SIXRX (TR)										
JM	JM	Finance	0,261	0,313	0,109	0,211	0,432	0,552	0,668	0,797	1,201	1,048
CAST	Castellum	Finance	0,166	0,130	0,129	0,285	-0,193	0,267	0,344	0,656	0,952	0,256
KINV-B	Kinnevik B	Finance	0,957	1,482	1,618	1,788	1,111	1,169	1,211	1,869	2,116	1,841
SWED-A	Swedbank A	Finance	0,604	0,516	0,472	0,426	0,040	0,815	0,815	0,755	0,864	0,718
FABG	Fabege	Finance	0,326	0,370	0,217	0,307	-0,082	0,416	0,517	0,411	0,640	0,155
LATO-B	Latour B	Finance	0,333	0,632	0,708	0,702	0,196	0,523	0,855	0,902	0,947	0,596
LUND-B	Lundberg	Finance	0,294	0,314	0,310	0,225	-0,024	0,378	0,422	0,419	0,523	-0,010
OMX	OMX	Finance	1,019	1,217	1,877	2,044	1,258	1,387	1,782	2,037	1,971	1,771
RATO-B	Ratos B	Finance	0,420	0,686	0,517	0,420	0,144	0,548	0,702	0,710	0,894	0,558
ORES	Öresund	Finance	0,292	0,284	0,473	0,674	0,189	0,323	0,499	0,493	0,703	0,406
SEB-A	SEB A	Finance	0,789	0,966	0,943	0,851	0,212	1,181	1,068	0,755	0,751	0,846
SHB-A	SHB A	Finance	0,497	0,470	0,280	0,289	0,011	0,864	0,796	0,641	0,627	0,594
INVE-B	Investor B	Finance	0,828	0,792	0,704	0,842	0,659	1,196	1,378	1,327	1,281	1,400
HUFV-A	Hufvudstaden A	Finance	0,222	0,271	0,295	0,396	0,208	0,368	0,477	0,673	0,792	0,311
INDU-A	Industryvärden A	Finance	0,960	1,038	1,072	0,995	0,972	1,000	1,075	1,126	1,076	0,983
SKF-B	SKF B	Industry	0,640	0,794	0,674	0,601	1,011	1,028	1,036	0,808	0,639	0,291
VOLV-B	Volvo B	Industry	0,619	0,775	0,521	0,405	0,301	0,989	1,037	0,996	0,880	0,880
SKA-B	Skanska B	Industry	0,522	0,640	0,515	0,380	0,571	0,934	1,076	1,184	1,186	0,836
TREL-B	Trelleborg B	Industry	0,458	0,564	0,416	0,329	0,176	0,828	0,935	1,056	1,149	0,721
NCC-B	NCC B	Industry	0,352	0,570	0,505	0,414	0,772	0,623	0,753	0,671	0,877	0,902
PEAB-B	Peab B	Industry	0,305	0,219	0,122	0,154	-0,408	0,467	0,573	0,687	0,797	0,005
HEXA-B	Hexagon B	Industry	0,439	0,478	0,388	0,280	-0,325	0,617	0,827	1,070	1,159	0,884
SECO-B	Seco Tools B	Industry	0,316	0,397	0,265	0,230	0,190	0,484	0,491	0,575	0,619	0,421
ASSA-B	Assa Abloy B	Industry	0,721	0,872	0,924	0,853	0,770	1,152	1,329	1,225	1,292	0,587
SAS	SAS	Industry	0,312	0,483	0,534	0,456	0,168	0,845	1,100	1,190	1,322	1,008
SECU-B	Securitas B	Industry	0,591	0,669	0,714	0,630	0,598	1,147	1,247	1,231	1,034	0,716
ATCO-A	Atlas Copco A	Industry	0,689	0,797	0,645	0,455	0,601	1,243	1,322	1,383	1,144	1,184
SAND	Sandvik	Industry	0,596	0,620	0,624	0,435	0,973	0,971	1,050	0,581	0,517	0,660
HOLM-B	Holmen B	Material	0,553	0,553	0,618	0,679	0,849	0,667	0,688	0,770	0,672	0,365
SCA-B	SCA B	Material	0,475	0,477	0,382	0,383	0,412	0,657	0,666	0,536	0,564	0,049
SSAB-A	SSAB A	Material	0,597	0,596	0,566	0,532	0,672	0,786	0,930	0,710	0,912	0,790
ELUX-B	Electrolux B	Commodities	0,675	0,731	0,701	0,870	0,656	1,026	1,012	0,924	0,773	0,568
HM-B	H & M B	Commodities	0,901	0,907	0,808	0,647	0,962	0,812	0,736	0,529	0,376	0,552
TEL2-B	Tele2 B	Telecommunication	1,240	1,626	1,658	1,795	1,268	1,156	0,958	1,038	1,031	0,899
ERIC-B	Ericsson B	IT	2,029	2,013	2,246	2,407	2,574	2,165	2,210	2,971	2,516	2,023
NOKI-SDB	Nokia	IT	1,629	1,379	1,367	1,990	2,869	1,299	1,169	1,070	0,611	0,837
GETI-B	Getinge B	Health Care	0,370	0,263	0,220	0,218	-0,435	0,564	0,670	0,802	0,968	0,549
EKTA-B	Elekta B	Health Care	0,552	0,556	0,438	0,227	-0,578	0,615	0,651	0,873	0,973	0,773
MEDA-A	Meda A	Health Care	0,274	0,207	0,182	0,183	0,092	0,515	0,630	0,683	1,220	1,220
AXFO	Axfood	Commodities	0,297	0,322	0,045	0,148	-0,595	0,400	0,554	0,419	0,396	-0,398
SWMA	Swedish Match	Commodities	0,179	0,099	-0,185	-0,193	-0,327	0,272	0,205	-0,097	0,307	-0,067
VOST-SDB	Vostok Nafta	Energy	0,677	0,779	0,789	0,832	0,579	0,940	0,743	1,022	1,375	1,770

Table 12 - Beta for individual stocks.

Appendix 6 – Table of Jarque Bera Test

JB-Test			Historical Data					Future Data				
			Daily	Weekly	Monthly	Quarterly	Yearly	Daily	Weekly	Monthly	Quarterly	Yearly
SX-SSVX30.SE	Benchmark	SSVX 30	245,26	12,95	27,06	8,62	1,85	830,74	115,46	29,21	9,42	3,03
RT-SIXRX.SE	Benchmark	SIXRX (TR)	17,12	7,60	23,12	2,89	1,68	46,75	18,34	6,52	13,72	3,68
JM	JM	Finance	282,49	44,71	17,65	6,40	2,31	151,99	0,28	17,86	7,52	8,47
CAST	Castellum	Finance	418,62	15,19	23,68	8,95	1,02	7,86	25,84	16,41	7,94	4,31
KINV-B	Kinnevik B	Finance	292,84	11,51	12,69	6,53	5,48	1098,89	2,74	86,38	5,25	1,49
SWED-A	Swedbank A	Finance	137,14	3,71	3,60	7,07	6,83	2,27	58,35	15,14	4,95	1,66
FABG	Fabege	Finance	2172,07	65,99	21,51	9,05	0,97	10198,25	41,23	15,48	10,78	1,89
LATO-B	Latour B	Finance	898,32	25,89	12,93	3,47	0,84	40096,46	29,41	13,17	5,73	6,02
LUND-B	Lundberg	Finance	44,65	8,09	20,13	12,89	3,05	141,04	13,73	15,03	9,87	2,92
OMX	OMX	Finance	101300,83	49,59	14,41	5,60	3,15	14002,58	971,71	4,22	2,73	0,61
RATO-B	Ratos B	Finance	424,87	316,78	12,48	7,90	2,00	37,73	17,59	24,74	7,97	1,23
ORES	Öresund	Finance	12600,81	496,50	10,96	6,22	1,61	556,87	132,16	7,75	6,40	4,99
SEB-A	SEB A	Finance	55,27	151,50	11,13	11,08	4,89	162,33	19,86	18,21	6,87	1,99
SHB-A	SHB A	Finance	98,99	26,13	19,76	4,94	2,85	52,97	13,69	24,06	7,32	1,74
INVE-B	Investor B	Finance	16,05	40,65	13,93	24,00	2,43	30,47	7,54	12,74	41,51	1,36
HUFV-A	Hufvudstaden A	Finance	369,19	72,43	18,79	11,79	5,49	158,36	4,39	10,39	11,20	5,55
INDU-A	Industryvärden A	Finance	59,99	39,65	24,29	0,09	1,37	20,67	64,01	6,89	6,66	3,73
SKF-B	SKF B	Industry	69,26	46,42	28,13	8,33	2,30	288,95	66,32	29,42	8,45	2,22
VOLV-B	Volvo B	Industry	37,11	23,01	32,80	0,40	6,59	1,48	61,39	7,33	4,38	1,45
SKA-B	Skanska B	Industry	14129,62	162,39	12,55	2,88	6,33	26,05	25,50	9,74	5,99	0,86
TREL-B	Trelleborg B	Industry	145,06	38,07	6,88	6,91	4,26	4,58	40,92	30,37	8,59	4,53
NCC-B	NCC B	Industry	38,43	26,62	26,98	9,20	2,47	148,18	40,69	25,01	9,21	4,15
PEAB-B	Peab B	Industry	612,13	75,53	28,02	4,61	2,02	36,53	17,98	24,58	4,81	2,33
HEXA-B	Hexagon B	Industry	122,81	7,04	9,17	5,66	7,26	43,42	84,06	14,56	6,54	1,61
SECO-B	Seco Tools B	Industry	3201,82	42,85	25,87	5,19	0,47	6180,56	18,47	3,51	6,73	4,18
ASSA-B	Assa Abloy B	Industry	60,26	21,72	14,48	7,62	7,46	1040,48	3,49	5,71	9,22	3,81
SAS	SAS	Industry	93,42	222,11	7,62	13,48	5,35	40,44	41,42	1,21	10,44	3,35
SECU-B	Securitas B	Industry	495,54	11,66	7,21	8,22	0,67	14665,29	97,62	14,78	9,30	2,86
ATCO-A	Atlas Copco A	Industry	102,81	69,46	6,92	0,25	3,60	106,17	86,61	17,76	5,58	1,62
SAND	Sandvik	Industry	45,65	65,85	9,68	5,81	2,90	15,15	70,02	19,68	8,21	3,17
HOLM-B	Holmen B	Material	9401,69	90,36	8,92	7,77	4,84	5002,06	29,58	17,57	6,10	7,72
SCA-B	SCA B	Material	119,72	104,01	20,40	12,02	6,01	93,46	32,27	14,53	10,45	2,13
SSAB-A	SSAB A	Material	49,73	34,00	20,47	1,29	2,04	311,58	1,24	30,79	3,35	5,42
ELUX-B	Electrolux B	Commodities	73,98	28,03	24,25	4,09	3,34	378,04	19,01	28,48	5,66	6,60
HM-B	H & M B	Commodities	7226,67	4,11	6,41	7,29	6,35	964,70	20,20	24,99	7,27	2,58
TEL2-B	Tele2 B	Telecommunication	228,03	10,13	14,98	4,19	7,75	413,63	6,98	98,87	7,00	2,33
ERIC-B	Ericsson B	IT	0,58	11,53	21,65	4,02	1,84	905,44	75,35	305,45	2,21	1,48
NOKI-SDB	Nokia	IT	133,61	52,11	20,35	6,67	7,85	398,97	6,98	6,33	5,48	0,51
GETI-B	Getinge B	Health Care	4,84	39,95	13,47	6,38	1,55	70,14	41,78	17,14	6,60	2,29
EKTA-B	Elekta B	Health Care	775,34	30,84	10,27	1,19	2,82	207,62	1,33	7,02	2,42	1,83
MEDA-A	Meda A	Health Care	29296,64	4592,58	597,73	12,31	2,02	6712,65	7591,98	250,82	5,96	4,23
AXFO	Axfood	Commodities	2900,46	37,81	12,42	10,42	2,10	4396,55	9,46	12,79	10,31	7,73
SWMA	Swedish Match	Commodities	58,56	79,68	19,38	1,15	1,20	4,97	45,29	29,82	4,70	3,30
VOST-SDB	Vostok Nafta	Energy	1233,23	131,78	0,19	8,09	4,54	204,67	8,12	19,89	13,22	2,46

Table 13 - Jarque Bera Test Results.

Appendix 7 – Nordic Large Cap List[33]

The Nordic list

June 30, 2006.
Explanation: LP = Liquidity Provider, PLUS = Copenhagen PLUS companies, * = Officially listed companies, OBS = Observation segment

NORDIC LARGE CAP		Currency	Last Paid	Change	Best Bid	Best Ask	High 12 Months	Low 12 Months	End Price 2005	Turnover Thousands	Market Cap Millions	Traded Index	Exchange	Notes
Energy														
D/S Torm		DKK	278,00	15,00	278,00	278,50	364,50	228,00	305,00	120 507	10 119		CSE	*
Lundin Petroleum AB		SEK	87,25	-0,50	87,00	87,25	114,00	65,25	84,00	222 676	22 435		STO	
Neste Oil Oyj		EUR	27,54	0,69	27,54	27,62	32,19	20,82	23,88	37 911	7 061	OMXH25	HEL	*
Vostok Nafta, Inv Ltd SDB	LP	SEK	479,00	18,50	478,50	479,00	639,00	149,00	371,00	415 362	22 987		STO	
Materials														
Ahlstrom Corporation Oyj		EUR	21,48	0,20	21,38	21,50	25,45	20,45		307	979		HEL	*
Boliden AB		SEK	132,50	5,00	132,00	132,50	177,50	30,40	65,00	1 322 957	38 353		STO	
Holmen AB ser. A		SEK			300,00	310,00	352,00	220,00	277,00		6 787		STO	*
Holmen AB ser. B		SEK	291,00	4,00	290,50	291,00	339,00	210,00	262,50	72 170	18 081	OMXS30	STO	*
Huhtamäki Oyj		EUR	13,89	-0,06	13,89	13,92	16,73	12,21	13,91	3 896	1 447	OMXH25	HEL	*
Höganäs AB ser. B		SEK	179,00	-1,50	179,00	180,50	220,00	158,00	172,00	10 996	6 107		STO	*
Kemira Oyj		EUR	12,85	0,00	12,85	12,92	14,98	10,18	13,48	3 410	1 605		HEL	*
M-real Oyj A		EUR	4,15	0,11	4,04	4,36	5,67	3,94	4,24	3	151		HEL	*
M-real Oyj B		EUR	3,87	0,14	3,87	3,89	5,62	3,61	4,22	20 455	1 129	OMXH25	HEL	*
Novozymes B		DKK	394,00	-1,00	394,00	395,50	474,00	297,00	345,00	122 875	21 375	OMXC20	CSE	*
Outokumpu Oyj		EUR	18,30	0,70	18,20	18,30	21,30	9,63	12,55	15 714	3 317	OMXH25	HEL	*
Rautaruukki Oyj K		EUR	23,62	1,05	23,61	23,62	33,31	12,16	20,55	18 519	3 280	OMXH25	HEL	*
SSAB Svenskt Stål AB ser. A		SEK	143,50	7,00	142,00	143,50	500,00	129,00	289,00	255 629	28 943		STO	*
SSAB Svenskt Stål AB ser. B		SEK	136,00	8,00	135,50	136,00	466,00	120,50	269,00	120 823	9 669		STO	*
Stora Enso Oyj A		EUR	10,68	0,08	10,71	11,19	13,80	10,16	11,46	6	1 908		HEL	*
Stora Enso Oyj R		EUR	10,92	0,39	10,88	10,92	13,58	10,01	11,44	59 162	6 677	OMXH25	HEL	*
Stora Enso Oyj ser. A		SEK	101,50	2,75	100,00	101,50	128,00	93,50	108,00	1 440	80 138		STO	*
Stora Enso Oyj ser. R		SEK	99,50	1,75	99,50	100,00	127,50	92,50	107,50	69 591	78 559	OMXS30	STO	*
Svenska Cellulosa AB SCA ser. A		SEK	296,50		295,50	297,00	355,50	245,00	297,00	4 917	11 331		STO	*
Svenska Cellulosa AB SCA ser. B		SEK	297,50	1,50	297,50	298,00	351,00	243,00	297,00	315 107	58 554	OMXS30	STO	*
UPM-Kymmene Oyj		EUR	16,85	0,38	16,85	16,90	20,91	15,25	16,56	83 661	8 817	OMXH25	HEL	*
Industrials														
ABB Ltd		SEK	93,25	3,00	93,25	93,50	109,00	48,60	77,00	234 768	189 204	OMXS30	STO	*
Alfa Laval AB		SEK	215,50	6,50	214,50	215,50	275,00	111,00	172,00	138 895	24 065	OMXS30	STO	
A.P. Møller - Mærsk A		DKK	44 800,00	900,00	44 600,00	44 800,00	68 300,00	40 100,00	63 200,00	51 033	98 461	OMXC20	CSE	*
A.P. Møller - Mærsk B		DKK	45 400,00	900,00	45 300,00	45 400,00	69 500,00	41 000,00	65 200,00	568 673	99 780	OMXC20	CSE	*
ASSA ABLOY AB ser. B		SEK	121,00	1,00	120,50	121,00	160,50	97,25	125,00	313 592	41 956	OMXS30	STO	*
Atlas Copco AB ser. A		SEK	200,00	5,00	200,00	201,00	238,00	118,50	177,00	613 816	83 939	OMXS30	STO	*
Atlas Copco AB ser. B		SEK	187,00	4,50	187,00	188,00	220,00	106,50	158,50	185 233	39 103	OMXS30	STO	*
Cargotec Oyj		EUR	34,25	0,18	34,25	34,27	43,50	21,84	29,29	8 824	1 864	OMXH25	HEL	*
DSV		DKK	975,00	40,00	974,00	975,00	1 117,00	518,00	778,00	191 547	20 382	OMXC20	CSE	*
FLSmidth & Co.		DKK	220,00	5,00	219,00	220,00	276,00	117,00	186,00	48 486	16 874		CSE	*
Group 4 Securicor plc		DKK	18,40	0,50	18,40	18,50	21,50	15,70	17,80	57 928	23 393	OMXC20	CSE	*
Hexagon AB ser. B		SEK	262,00	12,50	262,00	263,00	304,00	137,00	237,00	54 187	21 994		STO	*
KONE Oyj B		EUR	32,50	0,31	32,50	32,55	58,80	27,80	33,53	9 232	3 539	OMXH25	HEL	*
Københavns Lufthavne		DKK	1 785,00	-5,00	1 785,00	1 800,00	2 176,00	1 340,00	1 875,00	407	14 009		CSE	*
Metso Oyj		EUR	28,37	0,76	28,37	28,43	34,95	17,67	23,12	22 555	4 019	OMXH25	HEL	*
NCC AB ser. A		SEK	173,00	6,00		175,00	217,00	117,00	142,50	188	8 258		STO	
NCC AB ser. B		SEK	174,00	5,00	173,50	174,00	217,50	116,50	142,50	43 158	10 562		STO	
NKT Holding		DKK	364,50	-0,50	364,50	367,50	425,50	218,00	289,00	44 523	8 930		CSE	*
Peab AB ser. B		SEK	117,00	2,50	116,00	118,50	134,00	81,00	102,00	3 285	9 055		STO	
Rockwool International A		DKK	740,00	15,00	730,00	745,00	800,00	421,00	630,00	1 510	9 674		CSE	*
Rockwool International B		DKK	743,00	5,00	743,00	745,00	798,00	418,00	622,00	30 669	6 614		CSE	*
Sandvik AB		SEK	83,75	0,25	83,75	84,00	530,00	76,00	370,00	432 208	99 352	OMXS30	STO	*
SAS AB		DKK	61,50	0,00	61,00	61,50	90,00	48,10	83,50	16 157	10 117		CSE	*
SAS AB		SEK	75,75	0,00	75,75	76,25	114,00	60,75	104,50	8 476	12 461		STO	*
SCANIA AB ser. A		SEK	323,50	14,50	319,50	324,00	353,00	255,50	285,50	11 302	40 857		STO	*
SCANIA AB ser. B		SEK	327,00	12,00	326,00	327,00	356,00	256,50	287,50	577 059	32 700		STO	*
Seco Tools AB ser. B		SEK	91,00	-3,00	91,00	94,75	562,00	88,50	400,00	1 443	9 279		STO	*
Securitas AB ser. B		SEK	138,00	2,50	138,00	138,50	161,00	115,00	132,00	314 151	48 012	OMXS30	STO	*
Skanska AB ser. B		SEK	111,00	3,00	111,00	111,50	136,50	91,75	121,00	150 728	43 957	OMXS30	STO	*
SKF, AB ser. A		SEK	114,00	3,50	112,50	114,00	138,50	78,00	111,00	1 001	5 739		STO	*
SKF, AB ser. B		SEK	113,50	2,00	113,00	113,50	140,00	77,50	111,50	506 132	45 968	OMXS30	STO	*
SAAB AB ser. B		SEK	183,00	1,00	182,50	183,00	200,00	117,00	170,00	11 844	19 013		STO	
Trelleborg AB ser. B		SEK	123,00	-2,00	123,00	123,50	193,50	113,00	158,50	252 022	9 945		STO	*

[33] Internet Reference: http://www.omxgroup.com/digitalAssets/5643_nordiclist_june30_06.pdf

NORDIC LARGE CAP	Currency	Last Paid	Change	Best Bid	Best Ask	High 12 Months	Low 12 Months	End Price 2005	Turnover Thousands	Market Cap Millions	Traded Index	Exchange	Notes
Uponor Oyj	EUR	21,15	1,00	21,15	21,16	26,40	15,95	18,00	5 283	1 549		HEL	*
Vestas Wind Systems	DKK	159,50	4,50	159,25	159,50	190,00	90,25	103,50	323 526	29 540	OMXC20	CSE	*
Volvo, AB ser. A	SEK	347,00	7,50	347,00	347,50	391,00	295,00	364,50	55 452	47 026		STO	*
Volvo, AB ser. B	SEK	354,00	7,00	353,00	354,00	397,50	305,00	374,50	1 015 844	102 718	OMXS30	STO	*
Wärtsilä Oyj Abp A	EUR	32,50	1,50	32,30	32,70	36,82	20,60	24,84	119	766		HEL	*
Wärtsilä Oyj Abp B	EUR	33,00	1,69	32,96	33,00	37,57	21,75	25,00	15 379	2 342	OMXH25	HEL	*
YIT Oyj	EUR	19,17	0,52	19,17	19,18	45,48	16,65	36,13	11 236	2 407	OMXH25	HEL	*
Consumer Discretionary													
Amer Sports Corporation	EUR	16,33	0,22	16,30	16,33	18,01	14,00	15,73	1 569	1 167	OMXH25	HEL	*
Autoliv Inc. SDB	SEK	405,00	0,00	404,50	405,00	451,00	311,00	359,00	126 196	33 899	OMXS30	STO	*
Bang & Olufsen B	DKK	645,00	22,00	642,00	645,00	778,00	409,00	648,00	89 550	7 309	OMXC20	CSE	*
Electrolux, AB ser. A	SEK			107,00	114,50	240,00	105,00	214,00		1 079		STO	*
Electrolux, AB ser. B	SEK	104,00	1,00	104,00	104,50	234,00	91,50	206,50	301 197	31 139	OMXS30	STO	*
Eniro AB	SEK	75,75	1,25	75,75	76,00	104,00	72,00	100,00	98 803	13 794	OMXS30	STO	
Hennes & Mauritz AB, H & M ser. B	SEK	279,00	0,50	279,00	279,50	306,00	248,50	270,00	898 188	203 764	OMXS30	STO	
Husqvarna AB ser. A	SEK	99,00	4,00	99,00	100,00	117,00	88,00		70	941		STO	
Husqvarna AB ser. B	SEK	86,75	1,75	86,00	86,75	92,50	75,50		244 825	24 876		STO	
Modern Times Group MTG AB ser. A	SEK	360,00			380,00	422,00	226,00	318,00	36	5 596		STO	
Modern Times Group MTG AB ser. B	SEK	378,50	15,50	378,00	378,50	448,00	226,00	331,50	136 461	19 297		STO	
Nobia AB	SEK	234,00	8,00	233,50	234,00	261,00	107,50	161,00	70 487	13 536		STO	
Nokian Renkaat Oyj	EUR	10,28	0,02	10,28	10,32	20,14	9,70	10,65	16 081	1 253	OMXH25	HEL	*
SanomaWSOY Oyj	EUR	18,82	0,20	18,82	18,90	22,45	17,80	19,67	2 352	2 999	OMXH25	HEL	*
Stockmann Oyj Abp A	EUR	31,50	-0,20	31,15	31,50	37,00	29,02		19	774		HEL	*
Stockmann Oyj Abp B	EUR	31,74	0,18	31,61	31,74	36,86	26,95	32,53	850	959		HEL	*
Consumer Staples													
Axfood AB	SEK	207,00	4,00	206,50	207,00	229,00	181,50	222,00	20 526	11 299		STO	
Carlsberg A/S A	DKK	400,00	8,00	395,00	400,00	415,00	284,00	315,00	2 777	13 480		CSE	*
Carlsberg A/S B	DKK	426,50	8,50	425,50	426,50	444,00	308,00	338,00	90 069	18 160	OMXC20	CSE	*
Danisco	DKK	425,00	7,50	423,00	425,00	529,00	376,00	482,50	154 167	20 794	OMXC20	CSE	*
Kesko Oyj A	EUR	29,00	0,90	28,50	29,00	31,50	20,00	24,19	155	920		HEL	*
Kesko Oyj B	EUR	29,98	0,78	29,98	29,99	31,98	20,36	23,95	4 395	1 966	OMXH25	HEL	*
Oriflame Cosmetics S.A, SDB	SEK	239,50	8,50	239,50	240,00	295,00	168,00	229,00	37 306	12 847		STO	
Swedish Match AB	SEK	116,00	1,50	116,00	116,50	119,50	85,00	93,50	248 146	37 653	OMXS30	STO	*
Health Care													
ALK-Abelló B	DKK	798,00	6,00	795,00	796,00	936,00	665,00	674,00	18 298	7 329		CSE	*
AstraZeneca PLC	SEK	434,50	2,50	434,50	435,00	437,00	312,00	388,50	733 548	686 945	OMXS30	STO	*
Capio AB	SEK	129,00	7,00	129,00	129,50	159,00	109,00	141,50	117 113	10 931		STO	
Coloplast B	DKK	433,00	3,00	432,50	433,00	510,00	312,00	391,00	63 267	19 225	OMXC20	CSE	*
Elekta AB ser. B	SEK	122,00	1,00	121,50	122,00	372,00	103,00	118,00	73 743	11 074		STO	*
Gambro AB ser. A (OBS)	SEK	113,50	-0,50	113,50	115,00	125,00	77,25	86,75	18 709	28 440		STO	*
Gambro AB ser. B (OBS)	SEK	114,00	-0,50	113,50	114,00	120,00	78,00	86,50	12 408	10 725		STO	*
Genmab	DKK	188,00	2,00	187,50	188,00	228,00	96,50	135,00	28 841	7 412		CSE	*
Getinge AB ser. B	SEK	122,50	-2,00	122,50	123,00	135,00	97,00	109,50	63 063	23 076		STO	*
GN Store Nord	DKK	67,00	0,00	66,75	67,00	99,50	59,75	82,50	147 380	14 725	OMXC20	CSE	*
Lundbeck A/S	DKK	133,00	1,00	132,75	133,00	170,25	120,50	130,50	75 793	30 235	OMXC20	CSE	*
Meda AB ser. A LP	SEK	150,00	1,00	149,00	150,00	160,00	74,25	108,00	30 880	15 672		STO	
Nobel Biocare Holding AG	SEK	1 708,00	21,00	1 665,00	1 707,00	1 944,00	1 551,00	1 740,00	1 052	44 460		STO	*
Novo Nordisk B	DKK	371,50	5,00	370,00	371,50	410,50	298,50	354,50	206 976	111 803	OMXC20	CSE	*
Orion Oyj A	EUR	15,56	0,59	15,56	15,69	21,01	14,24	15,60	489	878		HEL	*
Orion Oyj B	EUR	15,53	0,62	15,53	15,57	20,89	14,20	15,64	22 749	1 318	OMXH25	HEL	*
William Demant Holding	DKK	436,00	7,00	435,50	436,00	477,00	275,00	348,50	64 489	28 588	OMXC20	CSE	*
Financials													
Castellum AB	SEK	73,75	2,75	73,50	73,75	354,00	56,50	286,00	28 457	12 685		STO	
Codan A/S	DKK	421,50	6,50	421,00	421,50	488,00	300,50	365,00	62 418	19 057		CSE	*
Danske Bank A/S	DKK	222,00	4,50	221,75	222,00	255,00	180,50	222,00	338 059	141 704	OMXC20	CSE	*
D. Carnegie & Co AB	SEK	132,00	5,50	132,00	132,50	188,00	85,00	117,00	56 367	9 125		STO	
Fabege AB	SEK	134,00	1,00	133,50	134,00	177,00	116,50	151,50	173 135	13 477	OMXS30	STO	
FöreningsSparbanken AB ser. A	SEK	189,00	1,50	189,00	189,50	226,00	167,50	216,50	831 367	100 229	OMXS30	STO	*
Hufvudstaden AB ser. A	SEK	55,25	-0,50	55,25	56,00	65,75	47,50	52,00	4 290	11 216		STO	*
Hufvudstaden AB ser. C	SEK			59,00		72,00	60,00			505		STO	*
Industrivärden, AB ser. A	SEK	202,00	0,00	201,50	202,00	265,50	164,50	217,00	36 294	27 123		STO	*
Industrivärden, AB ser. C	SEK	189,50	-1,50	189,00	189,50	250,00	152,00	200,00	31 670	11 154		STO	*
Investor AB ser. A	SEK	131,50	3,00	131,50	132,50	148,50	102,00	138,50	11 646	40 987		STO	*
Investor AB ser. B	SEK	132,00	3,00	132,00	132,50	149,00	103,00	139,00	362 625	60 124	OMXS30	STO	*
JM AB	SEK	114,50	1,50	114,50	115,00	545,00	111,00	352,00	45 668	11 302		STO	*

NORDIC LARGE CAP		Currency	Last Paid	Change	Best Bid	Best Ask	High 12 Months	Low 12 Months	End Price 2005	Turnover Thousands	Market Cap Millions	Traded Index	Exchange	Notes
Jyske Bank A/S		DKK	338,00	9,50	337,00	338,00	399,00	265,00	309,50	67 820	21 328	OMXC20	CSE	*
Kaupthing Bank	LP	SEK	70,00	0,00	69,75	70,25	125,50	63,00	93,00	6 441	46 519		STO	
Kinnevik, Investment AB ser. A		SEK	87,25		85,25	89,25	111,00	57,50	74,00	28	4 380		STO	
Kinnevik, Investment AB ser. B		SEK	88,50	1,50	88,50	89,00	113,00	55,50	74,25	34 921	18 920		STO	
Kungsleden AB		SEK	84,50	3,50	84,50	84,75	311,50	68,25	230,00	38 350	11 534		STO	
Latour, Investmentab. ser. A	LP	SEK			225,00		283,00	162,50	202,00		2 507		STO	
Latour, Investmentab. ser. B	LP	SEK	258,00	0,00	257,50	260,00	296,50	168,00	204,50	1 071	8 719		STO	
Lundbergföretagen AB, L E ser. B		SEK	373,00	8,00	371,00	373,00	415,50	304,00	335,50	7 699	14 228		STO	
National Bank of Greece		DKK											CSE	*
Nordea Bank AB		DKK	69,50	1,75	69,50	69,75	79,50	54,75	66,25	89 793	180 291	OMXC20	CSE	*
Nordea Bank AB		SEK	86,00	1,75	85,75	86,00	99,25	69,50	82,50	952 021	223 093	OMXS30	STO	*
Nordea Bank AB (publ) FDR		EUR	9,30	0,17	9,30	9,40	10,59	7,37	8,83	12 910	25 169	OMXH25	HEL	*
OKO Pankki Oyj A		EUR	11,50	0,13	11,50	11,54	15,06	9,19	11,86	2 637	1 815	OMXH25	HEL	*
Old Mutual Plc		SEK	21,60	-0,10	21,60	21,70	28,20	20,30		305 910	82 944	OMXS30	STO	*
OMX AB	LP	EUR	14,07	1,05	13,95	14,25	17,85	9,30		44	1 667		HEL	*
OMX AB		DKK	103,85	6,35	104,00	106,50	134,00	70,00	87,50	2 819	12 321		CSE	*
OMX AB		SEK	129,00	6,50	129,00	130,00	167,00	87,75	110,50	98 327	15 283		STO	*
Ratos AB ser. A		SEK			101,00	136,00	280,00	101,00	195,50		4 741		STO	*
Ratos AB ser. B		SEK	105,50	0,50	105,50	107,00	280,50	97,00	196,00	15 336	12 556		STO	*
Sampo Oyj A		EUR	14,92	0,26	14,92	14,98	17,99	12,01	14,72	40 321	8 413	OMXH25	HEL	*
Skandinaviska Enskilda Banken ser. A		SEK	171,50	4,50	171,00	171,50	199,50	126,50	163,50	417 619	113 705	OMXS30	STO	*
Skandinaviska Enskilda Banken ser. C		SEK	163,50	2,50	164,00	169,00	194,50	122,00	158,00	3 357	3 961		STO	*
Spar Nord Bank		DKK	130,00	3,50	129,00	130,00	1 650,00	112,00	929,00	7 080	7 419		CSE	*
Svenska Handelsbanken ser. A		SEK	185,50	0,50	185,50	186,50	240,00	157,00	197,00	452 152	120 566	OMXS30	STO	*
Svenska Handelsbanken ser. B		SEK	187,50	2,00	187,00	188,50	238,00	155,00	196,50	5 350	3 692		STO	*
Sydbank		DKK	193,50	2,50	193,00	193,50	225,00	130,00	151,00	27 065	13 545		CSE	*
Topdanmark		DKK	813,00	29,00	808,00	813,00	839,00	422,00	547,00	67 332	16 877	OMXC20	CSE	*
TrygVesta A/S		DKK	364,00	10,00	363,00	364,00	398,00	242,00	318,50	179 783	24 752	OMXC20	CSE	*
Information Technology														
Ericsson, Telefonab. L M ser. A		SEK	23,80	0,70	23,60	23,80	30,90	20,90	27,50	5 422	31 149		STO	*
Ericsson, Telefonab. L M ser. B		SEK	23,80	0,70	23,70	23,80	31,00	20,90	27,30	2 834 152	352 799	OMXS30	STO	*
Lawson Software, Inc.		SEK	48,50	-1,50	48,00	48,50	59,50	43,00		2 501	9 427		STO	
Nokia Abp, SDB		SEK	146,50	3,50	146,50	147,50	178,00	116,00	144,50	250 343	621 918	OMXS30	STO	*
Nokia Oyj		EUR	15,96	0,51	15,96	15,97	19,09	12,42	15,45	633 852	65 338	OMXH25	HEL	*
Nokia Oyj New Shares		EUR					18,45	14,10			2 475		HEL	*
TietoEnator Oyj		EUR	22,58	0,38	22,53	22,58	32,88	20,33	30,85	14 241	1 713	OMXH25	HEL	*
TietoEnator Oyj		SEK	207,50	2,50	207,00	208,00	307,00	187,00	285,50	13 795	15 737		STO	*
Telecommunication Services														
Elisa Oyj		EUR	14,89	0,15	14,89	14,96	18,15	12,40	15,65	13 479	2 473	OMXH25	HEL	*
Millicom International Cellular S.A. SDB	LP	SEK	330,00	0,00	325,00	330,00	375,50	136,50	214,50	382 498	32 605		STO	
TDC A/S (OBS)		DKK	190,00	2,50	190,00	190,25	402,00	179,00	377,50	1 387	37 691		CSE	*
Tele2 AB ser. A		SEK	73,50	0,00	72,25	74,50	98,75	70,00	86,25	37	3 421		STO	
Tele2 AB ser. B		SEK	72,75	0,75	72,75	73,00	98,00	69,00	85,25	303 128	28 929	OMXS30	STO	
TeliaSonera AB		EUR	4,42	-0,02	4,42	4,45	5,43	3,78	4,55	9 906	20 665	OMXH25	HEL	*
TeliaSonera AB		SEK	40,90	-0,10	40,80	40,90	50,75	35,60	42,70	1 082 720	191 217	OMXS30	STO	*
Utilities														
Fortum Oyj		EUR	20,00	0,30	20,00	20,01	23,48	13,04	15,84	42 692	17 654	OMXH25	HEL	*
Fortum Espoo Oyj		EUR			67,45	69,34	67,01	44,00			1 054		HEL	*

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List of Abbreviations

CAL	Capital Allocation Line
CAPM	Capital Asset Pricing Model
CML	Capital Market Line
DJIA	Dow Jones Industrial Average
GMV	Global Minimum Variance
MPT	Modern Portfolio Theory
MVaR	Modified Value at Risk
PDF	Probability Distribution Function
SML	Security Market Line
VaR	Value at Risk