APPROACHING MATHEMATICAL DISCOURSE:
TWO ANALYTICAL FRAMEWORKS AND THEIR RELATION TO
PROBLEM SOLVING INTERACTIONS

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Sammanfattning

Det övergripande syftet med de två studier som presenteras i avhandlingen är att undersöka hur begreppsförståelse och problemlösning kan bli en naturlig del av matematikundervisningen, och därigenom också en del av studenternas matematiska kunskapsbildning. Närmare bestämt syftar studierna till: 1) att karaktärisera klassrumssdiskursen i två olika problemlösningskurser inom ramen för en lärarutbildning och 2) att utforska och vidareutveckla två analytiska ramverk - ett kommunikativt (the communicational approach) och ett dialogiskt (the dialogical approach) - som används för att studera matematiska diskurser. Data har samlats in genom ljudupptagningar och fältanteckningar vid undervisningssituationer i ingenjörs- och lärarutbildningen. I relation till det första syftet visar avhandlingen att båda problemlönsningskursernas klassrumssdiskurser kan karaktäriseras i termer av ämnesinriktade, didaktikinriktade och problemlösningsinriktade diskurser. Av analysen framgår dock att fördelningen av dessa typer av diskurser skiljer sig åt mellan de två kurserna. Det föreslås att införandet av explicita begreppsliga ramverk i undervisningen kan få stor betydelse för diskursens innehåll och för lärarstudenternas möjligheter att medverka aktivt i matematiskt produktiva diskurser. I relation till det andra syftet visar avhandlingen att de analytiska ramverken kan utvecklas genom införandet av en kontextualiseringssteori och teorier om matematisk lärande. Avhandlingen mynnar ut i en diskussion av de teoretiska och praktiska implikationerna av resultaten, vilka kan vara av intresse för forskare som studerar matematiska diskurser och för lärare och lärarutbildare som vill utveckla undervisningen om matematisk problemlösning.
List of papers


Chapter 1

Introduction

1.1 Background

Typically, people equate knowledge in mathematics with the ability to calculate mathematical tasks. Skills in calculating are definitely an important aspect of mathematical knowledge but there are also several other competencies that are crucial aspects of mathematical knowledge, such as, logical reasoning, conceptual understanding, and problem solving (cf., e.g., Kilpatrick, Swafford, & Findell, 2001; Niss & Jensen, 2002 ). That is, in knowing mathematics it is not only important to be able to perform mathematical calculations but also to know, for instance, when to use a certain mathematical method, how to create mathematical models of real-world situations, or to be able to reflect upon the plausibility of an answer. However, from my own experiences, both as a student and as a teacher, competencies such as conceptual understanding and mathematical problem solving are emphasized far too little in the mathematical classroom. Therefore, the driving force behind conducting the two studies presented in this thesis is to examine ways that such competencies could be a part of the mathematical teaching, and through that, a part of students’ mathematical knowledge. Expressed differently, by conducting the research presented in this thesis, I intend to contribute to the discussion of how to help students develop mathematical competencies such as conceptual understanding and problem solving.
Therefore, the first study is focused on engineering students’ learning of Linear Algebra and especially their ways of handling the huge amount of concepts introduced in the course, which students typically find difficult (cf. Dorier & Sierpinska, 2001). My practical pedagogical idea in the first study was that the technique of concept mapping could be one way for students to communicate about the many languages, representations, and concepts they meet in Linear-Algebra courses. The results from the study are presented in, first and foremost, Ryve (2004, in press).

The second study constituting the basis for this thesis is concerned with problem solving in teacher education. The practical concern underlying this study is that problem solving is an extremely important aspects of mathematical knowledge, and that it is absolutely crucial that prospective teachers develop ways of teaching mathematical problem solving. The second study is presented in Ryve (2006) but also further discussed below. Therefore, I present here a number of arguments supporting the claim that it is of importance to study problem solving for prospective teachers.

First, problem solving is regarded as an important component of mathematical teaching and learning in many curriculums (e.g., National Council of Teachers of Mathematics, 2000; Swedish National Agency for Education, 2000) as well as an important strand of mathematical proficiency (Kilpatrick et al., 2001). However, if “the term problem solving has become a slogan encompassing different views of what education is, of what schooling is, of what mathematics is, and of why we should teach mathematics in general and problem solving in particular” (Stanic & Kilpatrick, 1989, p. 1), one may argue that there are good reasons to study what problem solving actually is in teacher educations. In relation to this, Wyndhamn and Säljö (1997) argue that “a major point of interest for educational research will be to document what counts as problem-solving activities in the school” (p. 363). Second, even though problem solving, whatever is included in this term, in Swedish teacher education has been the focus of some studies (e.g., Lingefjärd, 2000; Wyndhamn, Riesbeck, & Schoultz, 2000), no studies have focused on the classroom
discourses. Therefore, one purpose of this thesis is to elaborate on the characteristics of the classroom discourse of two problem solving courses for prospective teachers.

The elaboration of the problem solving discourses is conducted by means of analytical approaches. In this thesis these analytical approaches become an explicit object of study. Below I briefly introduce some arguments why it is of interest to explicitly examine analytical frameworks used for studying mathematical discourses. For several decades there has been an interest in studying individual language use in mathematics education but recently there has been a shift of focus from language towards mathematical discourse (Sierpinska, 1998). This shift is partly dependent on the social turn in mathematics education (Lerman, 2000b). That is, the sociocultural perspective has influenced researchers within the field of mathematics education to study mathematical discourses as such, not just as a way of capturing individual cognition (e.g., Kieran, Forman, & Sfard, 2001). However, taking the mathematical discourse as the unit of analysis has turned out to be a complex enterprise. For instance, the term discourse signals an interrelationship between the individual and the social as well as between cognition and communication. A number of analytical approaches have been introduced to account for these interrelationships but “we must realize that when it comes to tools and techniques that would match this endeavor, we have yet a long way to go” (Kieran et al., 2001, p. 8). Subsets of productive approaches may be found in other scientific disciplines but there is a need to develop analytical approaches crafted to fit the special demands of mathematics education (Bartolini Bussi, 1998; Even & Schwarz, 2003; Kieran et al., 2001). Further, for instance Ryve (2004) shows that approaching the same datum with different set ups of analytical tools generates contradictory results, that is, whether the discourses are to be seen as mathematically productive or not. To sum up, there are good reasons to examining analytical approaches used for studying mathematical discourse. Therefore, I will focus on two promising analytical approaches for studying mathematical discourses: the communicational approach to cognition (Kieran 2001; Sfard, 2001; Sfard & Kieran, 2001a)
and Linell’s (1998, 2005) *dialogical approach to discourse, cognition, and communication*.\(^1\)

Both Sfard and Kieran as well as Linell emphasize *empirical* motives for developing these analytical approaches. For instance, Linell (1998) states “the detailed *empirical* studies ... provide the most important and convincing evidence in support of dialogism” (p. xii, italic added). Moreover, Sfard’s (2001) reasons for elaborating the communicational approach regard the need for “more penetrating theories of mathematical thinking and learning” (p. 18). That is, the cognitively orientated research of thinking and learning needs to be complemented and related to theories of interaction, communication, and discourse. Linell, coming from a linguistic tradition, notices that “the description and explanation of language and language use must be based on a theory of human actions and activities in cognitive and interactional contexts” (p. 35). So while Sfard searches for ways to complement the cognitive approach with communicational features, Linell aims at connecting language use to cognitive theories. Hence, both Linell and Sfard, as well as many other scholars (e.g., Cobb, Stephan, McClain, & Gravemeijer, 2001; Lerman, 2001b), recognize a need for elaborated theories connecting communication and cognition.

As described above, Linell and Sfard have arrived at this conclusion from two different positions. This is both interesting and strengthens the arguments for the importance of developing fruitful theories and methodologies, connecting cognition and communication. As shown in this chapter, the work of developing such theories has been initiated by several scholars but both Linell and Sfard stress that much work has to be done before fully comprehensive analytical approaches could be presented. One important purpose of this thesis is to contribute to such a development.

1.2 **Aims of the thesis**

As indicated above, the driving force of conducting the research presented in this thesis is to contribute to the knowledge about the question: What are important features of mathematical teaching if we are about to help students develop conceptual understanding and

\(^1\)Often I use the abbreviated expressions: the communicational approach and the dialogical approach, respectively.
problem solving competencies? This overall purpose is operationalized into two research aims introduced below.

The first aim of the thesis is to analyze and discuss the characteristics, and plausible reasons for those characteristics, of the discourses of two problem solving courses for prospective teachers. Three fairly broad research questions are connected with this aim: What is the the topical focus of those discourses? How are the discourses co-constructed? What plausible reasons are there for the characteristics found?

The process of examining the analytical approaches focuses on two approaches; the communicational approach to cognition (Kieran 2001; Sfard, 2001; Sfard & Kieran, 2001a) and the dialogical approach to discourse, cognition, and communication (Linell, 1998, 2005). More precisely, the second aim of this thesis is to examine possibilities to develop the communicational approach and the dialogical approach, as they are used for studying mathematical discourses.

In relation to the first aim, the analysis shows that the discourses of both courses could be characterized in terms of three broad categories; subject oriented, didactically oriented, and problem solving oriented discourses. However, the analysis also shows that the distribution of the discourses in those categories differs substantially between the two courses which, in turn, could be related to the teacher educators’ contextualization of arranging such a problem solving course. This finding, together with the characteristics found, then serve as a basis for two discussions. First, the importance of not only talking about mathematics in teacher programs in quantitative terms (how much mathematics should prospective teachers study) but also in qualitative terms (what to include in those

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2 The reason for analyzing these approaches is simple, I have myself used them for studying mathematical discourses. There are other researchers that have made important contributions to the study of mathematical classrooms discourses such as Alrø and Skovsmose (2001), Bartolini Bussi (1998), Cestari (1997), Cobb, Stephan, McClain, and Gravemeijer (2001), Forman and Ansell (2001), Lampert (2001), Lerman (2001b), Moschkovich (2004), O’Conner (2001), and van Oers (2000, 2001), to mention but a few.

3 One may argue that it is complicated to elaborate on two separate aims in one thesis. However, the two aims of this thesis should not be seen as separate but rather as reflexively related since when elaborating on one of them the background is constituted for the discussion of the other aim, and vice versa.
mathematics courses). Second, I discuss a number of practical aspects one should consider when designing or teaching a problem solving course for prospective teachers. The analysis of the second aim shows that there are considerable possibilities to develop both analytical approaches for improving their usefulness in approaching mathematical discourse.

To be able to introduce, specify and develop the general conclusions just presented, I need to introduce and discuss several lines of research. In the next section an overview of that research is presented.

1.3 How to read this thesis

As indicated above, this piece of work is centered around three papers. These papers are attached at the end of this volume. This volume also consists of what we in Sweden denote a *coat*, which I here prefer to call the thesis\(^4\). The purpose of writing the thesis is not only to introduce and relate the papers to each others, but also to deepen and extend the conclusions of the papers. In conducting this enterprise, I choose to stress and develop *certain* features of the papers in this thesis. There now follows a disposition of the thesis.

An extended summary of the three papers (Ryve, 2004, in press, 2006) will be presented in chapter 2, at which point I also discuss the relation between, and especially the progression of, the papers.

The theoretical background of the thesis is presented in chapter 3. First, a framework divided into two dimensions is presented in order to describe key components of mathematical problem solving. The problem solving framework will serve as a background for the discussions of the characteristics found in the problem solving discourses in teacher education. In 3.2, I discuss research on mathematics teacher education and mathematics teacher knowledge. These discussions should be seen as a background to, rather than a theoretical framework for, the analysis of the second aim of this thesis.

\(^4\)Other suggestions may be; introduction, summary, or synopsis. However, none of them captures what I want to accomplish by writing these pages.
Chapter 4 serves the purpose of setting the thesis into an ontological and epistemological discussion. First, I discuss the relation between the concepts of method and methodology. Second, I introduce the philosophical grounds for the thesis by means of Moschkovich and Brenner (2000). I conclude the chapter by discussing the rationale for the research design of the two aims of this thesis.

In chapter 5 the analytical approaches under scrutiny are introduced. It is here important to stress that the communicational approach and the dialogical approach have got two functions in this thesis. On the one hand, both analytical approaches should be seen as units of analysis in relation to the second aim, and in relation to the first aim the dialogical approach should be seen as a theoretical framework used for analyzing discourses. Chapter 5 starts with a clarification of how certain technical concepts are related to each other in this thesis. I then introduce the communicational approach (Kieran 2001; Sfard, 2001; Sfard & Kieran, 2001a) followed by an introduction of the dialogical approach (Linell, 1998, 2005). After these two presentations I introduce the frameworks of contextualization and intentional analysis. These frameworks have been used in Paper II and Paper III to complement the two above mentioned approaches.

In chapter 6 I analyze the aims of the thesis on the basis of what has been presented in chapters 1-5. Expressed in more detail, the first aim of examining the characteristics of the discourse of mathematical problem solving courses for prospective teachers is related to plausible reasons for the characteristics found. The second aim of examining the two approaches is conducted in terms of possibilities to develop them for studying mathematical discourses.

In chapter 7 the results of the thesis are discussed. This discussion includes three themes. First, to highlight the theoretical and methodological relevance of this thesis by discussing its contribution to important topics in mathematics education. Second, to derive practical pedagogical implications from the results. Third, to introduce suggestions of further research.
Chapter 2

Summary of papers

In this chapter I summarize the papers (Ryve, 2004, in press, 2006). Before going into each paper I first want to indicate in what ways the papers are related to each other. Paper I and Paper II are closely related to each other in that they are written in relation to the same empirical data. More precisely, in Paper I the communicational approach was used for studying the discourses of engineering students constructing concept maps. When using this approach I realized that there were possibilities to develop it, which was the aim of Paper II. Paper III, on the other hand, is based of the empirical data of the discourse of two problem solving courses for prospective teachers. However, my explicit interest in analytical approaches for studying mathematical discourses is reflected in Paper III and the analytical approach taken in this study is under scrutiny in this thesis.

2.1 Paper I

Title: Can collaborative concept mapping create mathematically productive discourses?


As will be argued in the thesis, the study of mathematical discourses is of particular interest in the research field of mathematics education. In this paper, I focus on the
discourse of engineering students working in groups. More precisely, four groups (three engineering students in each group) were videotaped while constructing concept maps in Linear Algebra. My practical pedagogical idea is that concept maps could be a device for helping the students to reflect upon which concepts they use for solving mathematical assignments and how these concepts are related to each other. Concept maps are written hierarchical structures consisting of concepts and labeled links forming statements (Novak & Gowin, 1984).

Figure 2.1: A sample of the students’ concept map taken from Paper I.

Figure 2.1 shows a sample from a concept map created by three engineering students during a Linear-Algebra course. This concept map is quite limited regarding the numbers of concepts and their connections but hopefully it illustrates the main principles of this technique. The concepts are supposed to be connected to each other by linking words. The concepts and the linking words will together form mathematical statements.

Concept maps have been used for several purposes in mathematics education (see Ryve, 2003) and in this paper this technique is used for stimulating communication about mathematical concepts and their relations. There are two aims of this paper. The first is to characterize the discourse in the groups by addressing the following research questions:
Do the students communicate in an \textit{effective} way? Do the students’ communications contain the elements typical for a \textit{mathematically productive} discourse? When examining these questions I used the communicational approach and the two connected methods of analysis: focal analysis and preoccupational analysis (Kieran, 2001; Sfard, 2001; Sfard & Kieran, 2001a). In short, the focal analysis aims at examining the mathematical foci of the students’ discourses while the preoccupational analysis serves to account for interactional patterns (for further discussions see chapter 5). The second aim of this study is to evaluate this approach and especially the two methods of analysis in terms of their possibilities to answer the research questions of this study.

The analysis shows that the discourses of the four groups are to be seen as mathematically productive. That is, by means of the two types of analyses, focal and preoccupational analysis, and the criteria for mathematically effective and productive discourses derived from the communicational approach, I found that the discourses are mathematically effective and productive. However, the study also shows that several aspects of the methodological framework need to be developed. In Paper II, the possibilities to complement and develop the preoccupational analysis is discussed in more detail.

\section{Paper II}


Sfard and Kieran (Kieran 2001; Sfard, 2001; Sfard & Kieran, 2001a) have developed a methodological framework which aims at characterizing the mathematical discourse of students working in groups. In this paper, I focus on an important aspect of this methodological framework, namely the \textit{interactive flowcharts} which is used as an analytical tool for carrying out the preoccupational analysis. The aim of this paper is to suggest two
complementary analyses for the construction of the interactive flowcharts: an additional analysis by means of the theoretical framework of contextuation (Halldén, 1999) as well as an analysis of types of mathematical discourses by means of an elaborated version of a framework presented in Lithner (2003). The first analysis aims at making explicit the interpretations of students’ immediate intentions while the second analysis aims at differentiating between mathematical discourses as well as making explicit the interpretations of these differentiations. The theoretical framework of contextualization is described in chapter 5 so here I concentrate of the framework presented in Lithner (2003). While Sfard and Kieran (2001a) differentiated between mathematical and non-mathematical utterances, I introduce three different kinds of mathematical discourses as well as a non-mathematical category. The different types of categorizes of mathematical discourses used in the paper are the following:

**Intrinsic Properties (IP)** The discourse is based on intrinsic mathematical properties of the objects. This kind of discourse is typically produced by ‘curious’ students trying to widen their understanding of concepts and the relation between the concepts by asking Why-questions and questions such as ‘What is a set, really?’ Furthermore, students producing such a discourse typically show a will to dig deep into the subject. The depth of the discourse must be related to the students’ abilities as well as to the educational level in which the discourse takes place. Single statements and questions are interpreted as IP utterances if they deal with the intrinsic properties of the objects or have the potential of initiating IP discourse.

**Identification of Similarities (IS)** The discourse/reasoning is based on surface similarities of the elaborated objects, e.g., the only reason for connecting two concepts is that they have similar names.

**Established Experiences (EE)** EE is a category where discourses and utterances are placed if they do not fit into the other mathematical discourse categories. Typically,
the students try to connect the objects to one another by means of discourses based on what directly comes into their minds. For instance, the students could recall that two concept are associated with each other but ignore/fail to consider their intrinsic properties before linking them together. EE discourses in this analysis usually occur when the students seem to remember connections between concepts without further discussing the concepts or the linking words.

**Non-mathematical (NM)** A fourth category will be used to denote non-mathematical utterances/discourses (NM). NM utterances and discourses do not bring the mathematical discourse forward. In this setting, NM utterances are typically about the practical issues of how to extend the concept map and repetitions of utterances.

Based on data from Paper I, I show that the two complementary analyses make the construction of the interactive flowcharts more coherent and transparent, and hence, more reliable. That is, the theory of contextualization helps the researcher to put the agents’ interaction into a wider perspective and by doing so make more elaborate analyses of whether or not they are communicating with each other. Expressed differently, by examining the students’ contextualizations of the task of constructing concept maps, it is easier to understand the students’ rationale for their type of engagement in the discourse. Furthermore, by differentiating between types of mathematical discourses a nuanced picture of the discourse is accomplished. Such a nuanced picture of the discourse was not possible to accomplish within the original methodological framework since just two categories for types of mathematical discourses were available. The differentiation turned out to change the picture of whether the discourses were mathematically productive or not. Before the differentiation was conducted the discourses were looked upon as mathematically productive (cf. Paper I), while after the differentiation I would not say that the discourses were mathematically productive due to the low frequency of sections of discourses interpreted to be based on intrinsic properties (IP).
2.3 Paper III

**Title:** What is actually discussed in problem solving courses for prospective teachers? *Manuscript submitted for publication.*

In this study I take a closer look at two problem solving courses for prospective teachers. More precisely, the aim of the study is to characterize the classroom discourse, especially the prospective teachers’ presentations of tasks at the chalkboard, in terms of what the discourse is about and how it is constructed. The characterization then serves as a basis for discussing more practical aspects of what to consider when developing a problem solving course for prospective teachers. By means of the dialogical approach combined with the theory of contextualization, I show that the characteristics of the two courses substantially differ. That is, from the analysis of the courses three broad categories of discourses emerged; subject oriented discourses, didactically oriented discourses, and problem solving discourses. All three types of discourses could be found in both courses but it is the distribution among them that differs. More precisely, subject oriented discourses dominated in Course 1 while the two others dominated in Course 2. In the paper it is argued that these differences could be given reasonable explanations by referring to the act-activity interdependence of the dialogical approach. This means that the teacher educators’ contextualization of the task of giving a problem solving course strongly determines the characteristics of the discourses through the selection of mathematical tasks, introduction of frameworks for analyzing the task presentations of the prospective teachers, and the specific directions given to the prospective teachers when presenting the tasks (see also 6.1).

The characterization and the elaboration of plausible reasons then function as grounds for discussing implications for mathematical teacher education. In short, the need to discuss mathematics in teacher education in both quantitative and qualitative terms,
ways of choosing mathematical tasks and the consequences of this, the importance of developing, selecting, introducing, and using explicit conceptual frameworks in teaching are all stressed. These issues are discussed in more detail in 7.2.
Chapter 3

Theoretical background

3.1 Problem solving

Schoenfeld (1992) shows that the ways mathematicians and college students approach non-routine mathematical problems substantially differs. These differences can not be explained in terms of the number of method or algorithms that the mathematicians and college students master but rather relate to the capacity of handling mathematical problems. These capacities of approaching mathematical problems will be discussed below. I will distinguish two diverse views of problem solving: the different roles of problem solving in mathematics teaching and the competencies needed for solving problems. But let us first start by defining what a mathematical problem is.

3.1.1 What is a mathematical problem?

I follow Schoenfeld’s (1993) definition of what a mathematical problem is. Schoenfeld states:

For any student, a mathematical problem is a task (a) in which the student is interested and engaged and for which he wishes to obtain the resolution, and (b) for which the student does not have a readily accessible mathematical means by which to achieve that resolution. (p. 71)
Hence, the concept of problem is relative in two ways. First, following Schoenfeld’s definition, a task could be a problem for one person and at the same time not for another since one of them has a ready method for solving it and not the other. Further, following the theory of contextualization presented below, a problem denotes an interpreted task. Hence, different agents may interpret the same task differently.

3.1.2 The roles of problem solving in mathematics teaching

Wyndhamn et al. (2000) have studied the Swedish mathematical curriculums and their way of using the concepts of problem and problem solving. The curriculum (Lgr 62; Lgr 69; Lgr 80; Lpo 94) differ regarding the relation between problem solving and the learning of mathematics. In the curriculum of 1962 and 1969, an implicit assumption seems to be that the students’ mastery of the mathematical techniques will lead to competence in solving problems. That is, mathematics is learnt for solving problems. In the curriculum of 1980, problem solving becomes an explicit topic that should be taught. Hence, there should be teaching about problem solving. Teaching about problem solving should mainly be focused on the abstract mathematical aspects of problem solving, rather than real world aspects of the tasks. In the curriculum of 1994, rather than learning about problem solving, the students should learn mathematics through problem solving. In general, seeing problem solving as a vehicle for learning mathematics is strongly accentuated in current research literature (see, e.g., Lester & Lambdin, 2004; Stein, Boaler, & Silver, 2003).

3.1.3 Competencies for solving mathematical problems

Schoenfeld (1985) extends Pólya’s (1945) work of problem solving directing us toward five competencies that are needed for becoming a successful problem solver. That is, five interrelated categories including resources, heuristics, control, beliefs, and practices are introduced as cornerstones for teaching and analyzing mathematical problem solving. These categories will be complemented to fit the purposes of this thesis.
The term *resource* is used to characterize the mathematical ‘tools’ the problem solver could use when approaching a task. These resources include facts, procedures, and skills which Schoenfeld (1985) chooses to call mathematical knowledge, which I find somewhat strange. Facts, procedures, and skills are parts of mathematical knowledge but one of the major ideas of Schoenfeld’s book is that there is so much more included in mathematical knowledge. In Schoenfeld (1992), the category is instead called *knowledge base*, which I find more appropriate.

Schoenfeld (1992) denotes several problem-solving strategies (heuristics) including, e.g., *working backwards, searching for analogies, decomposing and recombining, specialization*. Mason and Davis (1991) also recognize specializing as an important heuristic and relates it to the process of *generalizing*. Further, on the same line as Bjuland (2002), I also regard *questioning and visualization* as important heuristics.

In comparing successful problem solvers with less successful, Schoenfeld (1985) finds that the degree of *control* has a major impact of the problem solvers’ success. Control is, in this case, closely related to the terms of *metacognition* and *self-regulation* which have been elaborated extensively in the research literature (e.g., De Corte, Verschaffel, & Op’t Eynde, 2000; Schoenfeld, 1987). Typical control activities “include making plans, selecting goals and subgoals, monitoring and assessing solutions as they evolve, and revising or abandoning plans when the assessments indicate that such actions should be taken” (Schoenfeld, 1985, p. 27). Control decisions have an impact on the solution at a global level where typical considerations are: Why should one plan be implemented and not the other? Should I stop and try another plan or not? How much time should I spend on different parts of the problem solving process? Schoenfeld (1985) found that there were major differences between successful problem solvers and less successful when considering control activities. Successful problem solvers put much more effort in assessing their choice of plan and how the solution process proceeded than less successful. Furthermore, successful problem solvers were much more inclined to test different kinds of promising
approaches popping up during the solution attempt, and not necessarily sticking to their initial plan.

One may think that resources, heuristics and control should cover the problem solving activities. Nevertheless, research shows (e.g., Schoenfeld, 1992) that formal and relevant knowledge is simply ignored in ‘real-world’ situations. This is the reason why Schoenfeld (1985, 1992), among others, has introduced a fourth category called beliefs. The beliefs of the students, related to mathematics and mathematical learning, are often divided into three categories: beliefs about the self in connection to the learning of mathematics and problem solving, beliefs about contexts, and beliefs about mathematics as a discipline (De Corte et al., 2000; McLeod, 1992). Schoenfeld (1985) describes three typical beliefs that students express about problem solving which strongly affect their way of approaching the problems. (1) “Formal mathematics has little or nothing to do with real thinking and problem solving” (p. 43). (2) Mathematical problems could be solved in 10 minutes or not at all. (3) Only tremendously smart human beings could discover or create mathematics. In short, one’s beliefs about mathematics strongly affect the way we do mathematics.

The fifth category is called practice and refers to students’ enculturation into the mathematics practice. More specifically, students need to be guided into productive dispositions, habits, and approaches to think about mathematics. These processes could be seen as a kind of socialization into doing mathematics.

### 3.2 Teacher knowledge and Mathematics teacher education

If learning mathematics is a complex enterprise (e.g., Niss, 1999), learning to teach mathematics may be even more complicated (cf. Kilpatrick et al., 2001; Sullivan & Mousley, 2001). Therefore it seems natural that there has been a distinct increased interest in the practice of mathematics teachers and mathematics teacher education over the last ten to twenty years (Adler, Ball, Krainer, Lin, & Novotna, 2005; Lerman, 2001a; Sfard, 2005b;
Wood, 2005). The presentation below of mathematics teacher knowledge should, first and foremost, be seen as a background for the discussions of the practical pedagogical implications of this thesis (see 7.2), while the survey of research in mathematics teacher education functions as an introduction to the first aim of the thesis and Paper III.

3.2.1 The knowledge base for teaching mathematics

Before going into details about what mathematics teachers need to know we must first, of course, say a few words about what we want students to learn. Kilpatrick et al. (2001) present five interrelated components of mathematical proficiency, central for successful mathematics learning. Conceptual understanding refers to the students’ ability to connect concepts and methods in a coherent whole as well as their ability to represent mathematical ideas in multiple ways. Procedural fluency is characterized by efficiency and accuracy in knowing how to perform procedures. Abilities included in strategic competence are; formulating, representing, and solving mathematical problems. Adaptive reasoning is strongly interrelated with the three former categories and refers to reasoning based on logic, patterns, analogy as well as capacities to justify and explain statements in mathematically legitimate ways. And finally, productive disposition is characterized by students’ beliefs of mathematics as something important, both for the society and for themselves.

Adler et al. (2005) state that “more teachers and better mathematics teaching are needed if mathematical proficiency is indeed to become a widely held competence” (p. 360). Here I use a framework involving three categories for discussing the knowledge base for teaching mathematics: mathematical knowledge, knowledge of students, and knowledge of instructional practice (Kilpatrick et al., 2001).
Mathematical knowledge for teachers does not only include mathematical proficiency as presented above but also knowledge about mathematics as a discipline with its norms and ways of producing truths. In addition, teachers should be able to explicitly reflect on the goals of mathematics, including what knowledge is in mathematics (Davis, 1999). Let me here note in passing that there is a strong correlation between teacher’s beliefs about mathematics and how they teach mathematics (Thompson, 1992; Wilson & Cooney, 2002). Further, knowing mathematics for oneself is one thing, to teach that mathematics also requires deep conceptual understanding (Ma, 1999), including a vision of how mathematical ideas are connected to other areas of mathematics and, for instance, an ability to tease out embedded mathematical properties in tasks (cf. Lester & Lambdin, 2004). The concept of mathematical knowledge shortly presented above must be seen in relation to numerous studies showing that the number of university courses of mathematics is not connected to successful teaching in terms of students’ performance (e.g., Monk, 1994; Begle, 1979). This suggests that we should not only discuss teachers’ mathematical knowledge in quantitative terms, thus in number of university courses, but also in qualitative terms, thus in terms of what to include in those courses (see also 7.2).

The category of knowledge of students refers to knowledge of individual students as well as students’ learning in general. So, besides knowing typical possibilities and difficulties of students’ mathematical learning, the teacher should be able to place individual students in relation to this in order to support the students in appropriate ways (Kilpatrick et al., 2001). For instance, Sullivan and Mousley (2001) notice that teaching based on ideas from the ‘zone of proximal development’ (Vygosky, 1978) or ‘conjectures of learning trajectories’ (Cobb & McClain, 2001) must be based on “potential spaces and pathways between children’s current understandings and higher levels of knowledge” (Sullivan & Mousley, 2001, p. 149). Further, the Standards (National Council of Teachers of Mathematics, 1989, 1991, 2000) include new ways of conceptualizing learning and teaching.

\footnote{However, criticism against cognitive-belief research has been presented by, e.g., Lerman (2001a) and Potter (1996).}
of mathematics which have not fully been implemented into the classrooms due to the insufficient understanding of the dynamics of learning in the reform classroom (Sfard & Kieran, 2001b). To engage prospective teachers in reflections of students’ knowledge and possible ways of knowing is therefore an important task for teacher educator programs (Shulman, 1986; Tirosh, Stavy, & Tsamir, 2001) which demands, and hopefully produces, content pedagogical knowledge (Ball & Bass, 2000). That is, Ball and Bass (2000) show how analyses of simple tasks, and possible answers to those, have the potential to engage prospective and in-service teachers in deep discussions of students’ mathematical knowledge.

Knowledge of instructional practice includes knowledge of, and competencies in interpreting, curriculum goals and how to make them teachable through planning, implementation, and assessment. Further, teachers need to develop skills in orchestrating the mathematical classroom practice by, for instance, devoting a suitable amount of time for discussing a particular task, engaging students in classroom communications, asking mathematically challenging questions (Lester & Lambdin, 2004). Without going into details, the knowledge of instructional practice could also be understood along the dimensions of ‘action-reflection’ and ‘autonomy-networking’ (Krainer, 2001). In short, traditionally there seems to be a lot of autonomy and action in the mathematical classroom but teachers also need time to reflect on their practice together with others. Therefore Krainer suggests that “promoting reflection and networking seems to be a powerful intervention strategy in the professional development of teachers” (p. 288).

To sum up, becoming a competent mathematics teacher, in relation to what has been discussed above, requires knowledge of many areas. One way to conceptualize this complex enterprise is to view ‘teaching as problem solving’ (Carpenter, 1989). In parallel to mathematical problem solving, it is absolutely essential to stress that factual knowledge about those areas is not enough, teachers should also know how, when, and why to implement that knowledge (cf. Lampert & Ball, 1999; Mason & Spencer, 1999). Therefore,
one can conclude that mathematics teacher programs are faced with a challenging task of helping prospective teachers develop those competencies.

3.2.2 Mathematics teacher education

Adler et al. (2005) analyzed 282 papers focused on mathematics teacher education published between 1999 and 2003\(^5\). Their study shows that small-scale studies conducted by English speaking researchers examining their own practice dominate the field of research. Moreover, many articles are focused on developments initiated by teacher educators (who are also the researchers) aiming at educating pre-service and in-service teachers in relation to the reform movement (e.g., National Council of Teachers of Mathematics, 2000). At the other end of the spectrum, Adler et al. observe that there are few studies dealing with teaching outside reform classrooms, teachers’ learning from teaching experiences, and teacher learning related to handling inequality and diversity in the mathematics classroom. In addition, large-scale studies, cross-case studies, and longitudinal studies are lacking.

In the case of Sweden, Bergsten et al. (2004) find that “the lack of research on mathematics teacher education is astonishing” (p. 22). I here present some of the few studies that have focused on Swedish mathematics teacher education and that are of particular relevance for this thesis: Lingefjärd (2000) analyzes prospective teachers’ modeling of mathematical tasks using technology; in a longitudinal study Grevholm (e.g., 1999, 2000) as well as Hansson and Grevholm (2003) examine the conceptual development of prospective teachers by means of concept maps; Wyndhamn et al. (2000) study prospective teachers’ views of problem solving; Bergsten and Grevholm (2004) focus on the tension between mathematical and pedagogical knowledge within the teacher education.

To conclude, in relation to the overview of research in mathematics education, especially in Sweden, one may argue that there is a distinct need for further studies of

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\(^5\)To be specific, the papers in *Journal of Mathematics Teacher Education* were published between 1998-2003 while the rest of the papers were published between 1999-2003.
mathematics teacher education. The elaboration of the second aim of this thesis as well as Paper III may be seen as such contributions.

3.3 Concluding remarks

In this chapter the frameworks of mathematical problem solving, mathematics teacher knowledge, and mathematics teacher education have been introduced serving as background for discussing the first aim as well as the practical pedagogical implications of this thesis.
Chapter 4

Methodological considerations

In writing the methodological section of the thesis, I have been influenced by several scholars, in particular Leone Burton and Paul Ernest. Burton (2002) observes that “in many of the Ph.D. theses that I have read, the chapter headed Methodology has dealt, in fact, with the methods used by the researcher to undertake their research” (p. 1). Hence, there seems to be a confusion about the relation between the concepts of method and methodology and I want to accentuate that there are "important differences between method and methodology" (Ernest, 1998, p. 34). In this thesis, I follow Ernest’s (1998) definition of methodology which he describes as “a theory of methods - the underlying theoretical framework and the set of epistemological (and ontological) assumptions that determine a way of viewing the world and, hence, that underpin the choice of research methods” (p. 35).

Therefore, I will not only describe the methods used in this thesis but also scrutinize my rationale for using them. Expressed in another way, I will describe how, and argue why (cf. Burton, 2002), specific methods were used for elaborating the aims of the thesis.

Below I will introduce the philosophical grounds for the thesis and discuss how those have influenced the research performed in connection to this thesis. At the end of the chapter I outline an overview of the research design in relation to the two aims of this thesis.
4.1 Philosophical considerations

There are two kinds of results in this thesis; results that indicate quantitative relations between different types of discourses and results in the form of conceptual frameworks that help both researchers and teachers to structure their thinking about the mathematical classroom practice. The latter kind of result is dominant and one may therefore argue that this thesis belongs to the *interpretative paradigm* (Carr & Kemmis, 1986). The interpretative paradigm is also referred to as the *qualitative research paradigm* (Ernest, 1998) and the *naturalistic research paradigm* (Lincoln & Guba, 1985). The concept of paradigm should, in this case, be understood as a specific way of viewing the world, both ontologically and epistemologically (Kuhn, 1970). Several different paradigms exist within the field of education. However, the qualitative research paradigm has established itself as the dominant paradigm within the field of mathematics education (Ernest, 1998).

Below, I will argue for the general theoretical perspective on knowledge and research of this thesis, which is in tune with the interpretative research paradigm. However, within the interpretative paradigm there are a number of possible methodological approaches, each with its own views of the world and arguments about how the unit of analysis is related to these views (Teppo, 1998), which makes it necessary to go beyond the paradigm to specify the methodological approach taken in each study of this thesis. These specifications will partly be conducted in this chapter but will be discussed further in chapter 5 and in each paper.

The philosophical grounds of this thesis will be presented by means of a three-principle framework developed in relation to the naturalistic research paradigm (Lincoln & Guba, 1985) and presented in Moschkovich and Brenner (2000). The three-principle framework will be complemented by ideas from other scholars.

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1The interpretative, the qualitative, and the naturalistic research paradigm will be used interchangeable in the rest of the thesis.

2However, more recently, Silver (2004) discusses these issues and notice that there is a need for more research studies conducted within the scientific research paradigm.
The first principle regards the possibility of considering multiple realities when approaching the world (cf. James, 1909/1996). This ontological statement implies that even though there may be an objective world there is no way for us to research it directly. This does not, however, imply relativism but rather that the world existing independently of us partly constrains, but does not determine, our way of understanding it. I just want to stress that there are multiple possibilities of viewing the world. The view of the world as something not directly obtainable has several implications for the research presented in this thesis and I would like to highlight one here, namely the importance of language use. That is, if there are multiple ways of constituting the ‘same event’, our knowledge of the world is strongly dependent on the language we use to describe and constitute it (e.g., Phillips & Jørgensen, 2002; Säljö, 2000). The accentuation on language use for constituting different aspects of the world has affected my research on different levels. First, viewing language in this way supports the claim that the study of mathematical discourses is important within the field of mathematics education. Second, in the analyses, I put great emphasis on the verbal constructs the students are using for constituting their (mathematical) world since to know includes being able to make linguistic distinctions (Säljö, 1999). Third, the emphasis on language use has implications for the possibility of reducing the influences of my own subjective values and beliefs in approaching the data. That is, methodological and theoretical constructs not only guide my analyses toward certain aspects of the data but also direct the way I ascribe meaning to the data. One can therefore say that there is an interplay between my own beliefs and values and the methodological constructs and theoretical constructs in approaching the data.

The second principle relates to theory verification and theory generation. Moschkovich and Brenner (2000) state “combining the verification and generation of theories is most likely to move the field forward” (p. 462). Throughout the work with this thesis there has been a constant process of theory verification and theory generation. In the first study, I approach the data by an already developed theoretical and methodological framework
but during the process of approaching the data I realized that the existing methodological framework had to be developed in order to create better opportunities to answer the research questions (see also Paper I and II). In Paper III, a constant cyclic process between the theories and data was undertaken in order to create a thick description (cf. Geertz, 1973) of the phenomena. More precisely, different theoretical and methodological approaches were applied on the data and finally I decided to use the dialogical approach (Linell, 1998) complemented by the theoretical construct of contextualization (Halldén, 1999). Further, several theoretical frameworks were used to verify the problem solving aspects of the discourses while other theories were generated from the data (see Paper III).

The third principle considers the contextual influence on cognition. In line with many contemporary philosophers (e.g., Wittgenstein, 1953; Lyotard 1984) I view knowledge as dependent on historical and cultural aspects. So, to understand individuals, it is necessary to ascribe meaning to their behaviors by viewing them in a wider situational and cultural setting or as Moschkovich and Brenner (2000) express it “studying cognitive activity in context means not only considering the place where the activity occurs, but also considering how context, the meaning that the place and the practices have for the participants, is socially constructed” (p. 463). In this thesis, this is accomplished by, for example, the introduction of the theoretical construct of contextualization. It is not only important to set the unit of analysis in relation to different contexts but also the researcher must also be seen in relation to contexts, which implies that “in a naturalistic paradigm, the point of view of the researcher becomes explicit” (Moschkovich & Brenner, 2000, p. 461).
4.2 The relation between the researcher and theory

In order to reduce the influences of my own subjective values and beliefs I explicited the use of several theoretical frameworks and analytical constructs when approaching the data (see chapter 5 and each separate paper). These theoretical frameworks and analytical constructs did not only serve as lenses (cf. Lerman, 2000b) when studying the data but they also facilitated the process of making explicit the analysis which is stressed as important by, for instance, Voigt (1995) who states “in order to justify the researcher’s interpretations of an episode, the researcher should explicate ... theoretical concepts” (pp. 166-167). Notice also that the formulation of research questions connected to the aims of the study were conducted within the theoretical and methodological approach.

In line with Furlong and Edwards (1977), I view both the collection and analysis of data as dependent on theoretical underpinnings. When putting such great stress on analytical constructs and theoretical frameworks in approaching the data, one may ask if the conducted analyses just serve to confirm existing theoretical constructs rather than giving opportunities to get fresh insights into the material and to develop theories? As described above, I do think that theoretical constructs function as tools for thinking and hence as tools for approaching the data. However, the relationship between the data and the theoretical constructs should be seen as dynamic and reflexively related. That is, during the process of approaching the data I used certain theoretical constructs, such as communicative project (Linell, 1998) and contextualization (Halldén, 1999), that did not in themselves change during the analysis. Other theoretical frameworks, such as the one used to operationalize the analysis of problem solving discourses (e.g., Pólya, 1945; Schoenfeld, 1992; Wyndhamn et al., 2000), were complemented throughout the analysis of the data\(^3\). At least two different types of elaboration of theoretical frameworks and methodological approaches in relation to the data were conducted. First, one type of elab-

\(^3\)One may argue that the frameworks of, e.g., Pólya (1945) and Schoenfeld (1992) are not theoretical frameworks appropriate for approaching data but rather prescriptive frameworks describing how problem solving should be implemented.
oration aimed at developing already existing theoretical and analytical constructs within the used frameworks, such as the operationalization of the interpretations of students’ immediate intentions (see Paper II for an extensive analysis). Second, other elaborations were conducted in order to complement the existing frameworks, such as the framework introduced to differentiate between four types of mathematical discourses (see Paper II for an extensive elaboration). In fact, the shortcomings of the analytical approaches in relation to the data in each study served to trigger explicit discussions and amendments of these frameworks. More precisely, to discuss and suggest complementary theoretical and methodological constructs became an explicit aim of this thesis. Therefore, I will not further elaborate on the possibilities and limitations of the specific analytical approach here.

4.3 Research design

In mathematics education the object of study are the phenomena of teaching and learning of mathematics (Niss, 1999) and the choice of unit of analysis is dependent upon how one conceptualizes teaching and learning in mathematics. For instance, if one conceptualizes learning by using the metaphor of seeing learning as becoming a participant in a mathematical discourse, the natural unit of analysis is the discourse itself (Kieran et al., 2001). So broadly speaking, the object of study is conceptualized into a metaphor (e.g., learning as acquisition, learning as becoming a participant in a discourse, or teaching as a transfer of knowledge). The metaphor, in turn, guides the researcher in the choice of the unit of analysis. However, the concept of unit of analysis is not trivial and at this stage I would like to say a few words about it before I sketch the research design of the two aims.

For Vygotsky (see Zinchenko, 1985), the unit of analysis was the minimal unit that preserves the properties of the whole phenomenon, here referred to as the object of study.

\footnote{Different ways of viewing and handling the concept of unit of analysis are discussed in, for instance: Cobb (1994, 2002), Davydov and Radzikhovskii, (1985), Lerman (2000a), Roth (2001), and von Glasersfeld (1995).}
Others, such as Patton (2002), refer to the unit of analysis in much broader terms and states “individual people, clients or students are units of analysis” (Patton, 2002, p. 228). My point here is that there seems to be different ideas about what the concept of unit of analysis refers to. Some researchers, such as Vygotsky, use the concept of unit of analysis to refer to the minimal unit, while others use it more loosely to refer to the focus of the study. Below I specify the unit of analysis of this study.

4.3.1 Rationale for research design of the first aim

As discussed above, the objects of study in mathematics education are “phenomena and processes actually or potentially involved in the teaching and learning of mathematics” (Niss, 1999, p. 5). The metaphor of learning which I relate to in this thesis is ‘learning as becoming a participant in a specific discourse’, making it particularly interesting to study mathematical discourses. The first aim of this study is focused on the mathematical discourses of prospective teachers taking a course directed toward mathematical problem solving. So the unit of analysis is the classroom discourse, where discourse is defined in accordance with Linell (1998) who defines discourse as actions (see chapter 5). The classroom discourse of two problem solving courses were recorded on audiotape, approximately ten hours from each course, and I also took field notes. After testing several frameworks, I decided to use the dialogical approach and theoretical construct of contextualization to characterize the topical focus and the co-construction of the discourses, as well as to argue for plausible reasons for the characteristics found. The actual process of approaching the data could best be described as a cyclic process between the audio recordings, the transcripts, the field notes, and the theoretical constructs.

4.3.2 Rationale for research design of the second aim

In the second aim of this thesis I am not interested in mathematical discourses as such, but rather analytical approaches used for studying mathematical discourses. So the units of
analysis in the second aim are the communicational approach and the dialogical approach (see 1.1 and 1.2 for the rationale for this aim and the specific research questions). In elaborating upon these approaches I use results from my three papers. Expressed in more detail, there are two kinds of results used from the papers. First, in Paper II an explicit examination of the possibilities of strengthening the communicational approach are conducted. Therefore, the results from Paper II could be used directly in elaborating upon the first aim. Second, in Paper III the dialogical approach is used for studying mathematical discourses and I use the experiences to propose ways of developing the dialogical approach for studying mathematical discourses.

4.4 Concluding remarks

In this chapter I have described the methodological grounds for the thesis. Further, I have presented an overview of the research design connected to the two aims of this thesis. My hope is that this account will function as one of the contexts which will help the reader to better understand the results and discussions of the thesis.

In the following chapter I present the two analytical approaches and some other theoretical frameworks that have been used for approaching the mathematical discourses of this thesis.
Chapter 5

Analytical approaches

In this chapter I present the analytical approaches used for examining and characterizing the mathematical discourses of this thesis. In Paper I the communicational approach (Kieran, 2001; Sfard, 2001; Sfard & Kieran, 2001a) is used and in Paper II, which is a theoretical paper, this approach is under scrutiny. The dialogical approach (Linell, 1998, 2005), together with the theoretical construct of contextualization, serve as the analytical approaches for analyzing the data in Paper III. It is here essential to stress that the dialogical approach should be seen as theoretical framework in relation to the first aim and that the two approaches should be seen as the unit of analysis in relation to the second aim.

I will start this chapter by clarifying the relation between a number of technical constructs/concepts/terms, such as *analytical approach, perspective, methodology, analytical construct*, which I hope will help the reader to further structure their interaction with the thesis. I then present the analytical frameworks of the communicational approach and the dialogical approach. This presentation is followed by an introduction of the theory of contextualization and intentional analysis.
Figure 5.1: The relation between technical terms

5.1 Terminology

Figure 5.1 portrays a structure of the terms used for denoting certain aspects of the theoretical arsenal used for approaching the data, here called analytical approach or analytical framework. The communicational approach and the dialogical approach are such analytical approaches. The concept of analytical approach has two main strands, perspective and methodology. The term perspective, also called epistemological framework (Linell, 2005), theoretical framework, or conceptual framework (Sfard, 2001), functions to capture underlying epistemological considerations as well as principles and definition. For instance, in this thesis dialogism should be seen as a perspective\(^1\). Further, a principle within dialogism, and hence within the dialogical approach, is, for example, that discourses are always co-constructed, in one way or another. Discussions of the relation between communication and cognition or how to define individuality also falls under the

\(^1\)Bjuland (2002) denotes dialogism as an epistemological approach (cf. perspective) while the connected methodology is called the dialogical approach.
heading of perspective.

The strand of methodology, which is connected to the perspective, is more focused on the process of collecting and analyzing the data. Here, and in the thesis in general, I specifically focus on the processes of analyzing the data. Under the heading of method of analysis fall two concepts; analytical tool and analytical constructs. Interactive flowchart, which is presented below, may serve as an example of an analytical tool. The term of analytical construct denotes the concepts used for structuring (methodological constructs) and interpreting (theoretical constructs) the data.

5.2 The communicational approach to cognition

5.2.1 Principles and definitions

The analytical approach of the communicational approach to cognition (Sfard, 2001), which could be seen as placed both in the cognitive and sociocultural tradition, is built around the metaphor: learning as becoming a participant in a certain distinct discourse\(^2\) (Sfard, 1998). From this metaphor, and by means of the work of, e.g., Vygotsky (1987) and Harré and Gillett (1994), a number of principles are derived. First, learning, cognition, and knowledge are seen as situated. Second, language is not seen as a medium for expressing ones thoughts but “thinking may be conceptualized as a case of communication” (Sfard, 2001, p. 26). Thinking, then, is seen as both dependent on, and informed by, the process of making communication effective, with others or with oneself. This does not mean that thinking is equated with inner speech since the concept of communication is not used just to denote language use but also other semiotic means. The close relationship between communication and cognition leads Sfard (2001) to see individuals as interrelated to the social. In other words, it is not productive, in educational settings, to view the individual

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\(^2\)Sfard (2001) stresses that a whole perspective could not be built around a single metaphor and she states “however, of those metaphors that can be identified, one is usually the most prominent and influential” (p. 23).
5.2 The communicational approach to cognition

and the social as two separated entities3.

In this nexus of individual and social relations the natural unit of analysis becomes the activity itself. More precisely, the unit of analysis is the discourse. The concept of discourse refers to any specific act of communication, “whether diachronic or synchronic, whether with others or with oneself, whether predominantly verbal or with the help of other symbolic systems” (Sfard, 2001, p. 28)4. Since discourses are analyzed as acts of communication, it is of interest to define the concept of communication. Within the communicational approach to cognition, communication is defined as an individual’s efforts to make “interlocutor act, think or feel according to her intentions” (Sfard, 2001, p. 27).

One should however be careful with the concept of intention for two reasons. First, “intention is not meant to be in any way prior to the utterance ... it comes into being in this act” (Sfard & Kieran, 2001a, p. 48). Hence, the concept of intention is not used for denoting something located behind an act but instead within the (speech) act5. Second, the concept of intention is something that the researcher ascribes to the participants (second order construct) in order to produce rational interpretations of the discourses and, hence, nothing that could be directly related to the agents’ intentions (cf. 5.4). However, within the communicational approach “intentions are central to all our decisions, and thus cannot be omitted in any serious attempt at understanding human actions” (Sfard, 2001, p. 32).

In harmony with the principles and the definitions above, learning mathematics is seen as the initiation into mathematical discourse. Sfard (2001) punctuates two factors that are of particular importance in the mathematical discourse: mediating tools, such as mathematical symbols, and meta-discursive rules, such as implicit norms and genre


4 For elaborated discussions of the concept of discourse see, e.g., Fairclough (1992), Gee (1999), Phillips and Jørgensen (2002), and van Dijk (1997c, see also 1997b).

5 The concept of speech act was introduced by Searle (1969). Later on I will account for how Linell (1998) criticizes the idea that one intention is ascribed to an utterance.
specific ways of interacting. Due to the importance given to these two concepts, two types of analyses are developed to capture how they come into play in the mathematical discourse. Below, I go into details about these analyses.

5.2.2 Methods of analysis

Considering the close relation between communication and cognition it seems natural to put emphasis on the specific mathematical sign system. That is, the mediating tools are seen as “part and parcel of the act of communication and thus cognition” (Sfard, 2001, p. 29). *Focal analysis* is used to account for how individuals use those mediating tools in order to examine the effectiveness of the communication. The analysis is performed by means of three theoretical constructs; *pronounced focus*, *attended focus*, and *intended focus*. The pronounced focus is used for taking account of what the interlocutors verbalize while the attended focus serves to capture *what and how* the agent is attending to when communicating and thinking. While the pronounced and attended foci are located in the discourse, the intended focus is mainly private and denotes the “cluster of experiences evoked by the other focal components plus all the statements a person would be able make on the entity in question” (Sfard, 2001, p. 53). As indicated above, these three constructs are used for scrutinizing the effectiveness of the communication which is characterized by the fact that “all the participants seem to know what they are talking about and feel confident that all the parties involved refer to the same things when using the same words” (Sfard, 2001, p. 34).

While the focal analysis is conducted for examining the object-level of the communication, *preoccupational analysis* is performed to account for the meta-level aspects of the communication, hence in relation to the meta-discursive rules. Before going into detail about the preoccupational analysis let me first discuss the theoretical construct of meta-discursive rules. The interest in implicit rules that, to a large extent, guide us in what to do and how to do it, have until recently not been the focus in mathematics education
5.2 The communicational approach to cognition

(Sfard, 2000). While the object-level rules regulate the content of the mathematical discourse the meta-rules dictate what counts as mathematics and how to communicate about it. Hence the meta-rules function as a frame for interpreting the object-level aspects of the discourse. Furthermore, meta-level rules are typically not explicitly discussed, and perhaps never reflected on by the participants of the discourse. For instance, Säljö and Wyndhamn (1993) show that the same task renders totally different ways of solving the task depending on whether it is given in a mathematical classroom or in a social studies classroom. One can therefore assume that there are unwritten rules of what to do in certain discursive practices. Further, Sfard (2000) states that meta-rules are often learned in an implicit way. Put differently, we seldom, for example, explicitly discuss that ‘examining the $f(x) = x^2 + 2$’ does not mean studying the color of the letters (which may be the case in a visual arts class).

The preoccupational analysis is performed to capture aspects of the meta-discursive rules. More precisely, the analytical tool of interactive flowchart is used for studying the engagement of the interlocutors in the communication, and by using this tool “we are able to evaluate the interlocutors’ interest in activating different channels and in creating a real dialogue with their partners” (Sfard & Kieran, 2001a, p. 58). Put differently, are the interlocutors mainly talking to themselves or to each other? In addition, the interactive flowchart creates opportunities to examine whether the utterances are of a mathematical character or not.

Let us now take a closer look at Figure 5.2 to outline the basic features of the interactive flowcharts. A, B and C are three persons interacting with each other. The numbers (1-5) to the left stand for the order of the utterances. A begins in 1 by addressing a proactive utterance to B and C. The term proactive denotes that an interlocutors’ statement or question invites a response. Since A is addressing the two other interlocutors

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6The analytical tool of interactive flowchart could be discussed in terms of a coding scheme derived from the theoretical construct of meta-discursive rules where the researcher has the possibility to categorize utterances along three dimensions: proactive/reactive, interpersonal/intrapersonal. mathematical/non-mathematical. See further 7.3.
the utterance is made in the *interpersonal channel* (thus between interlocutors). The arrows are *solid* and therefore represent *object-level utterances*, hence $A$ is communicating about mathematics. In 2, $C$ makes a *reactive utterance* commenting on $A$’s utterance in 1. Reactive here indicates that the interlocutor is commenting, in one way or another, on a prior utterance. The dotted arrow shows that the utterance is of a non-mathematical character, hence a *non-object-level utterance* (or meta-level utterance). Utterance 2 also belongs to the interpersonal channel since $C$ is addressing a fellow participant. In 3 $A$ comments on $C$’s prior utterance. Since $B$ addresses himself in 4, the arrow is vertical. This symbolizes that the utterance belongs to the *personal channel*.

### 5.3 The dialogical approach to discourse, cognition, and communication

The term dialogical approach\(^7\), or dialogism, is used by many scholars in partly different ways and should therefore not in general be seen as one well defined and coherent approach to the study of communication and cognition (Linell, 2005). This implies that it is essential

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\(^7\)The dialogical approach as presented in Linell (1998, 2005) has been developed by the Bad Homburg Study Group on ‘The Dynamics of Dialogue’ in which Jörg Bergmann, Rob Farr, Klaus Foppa, Carl Graumann, Per Linell, Thomas Luckmann, Ivana Marková, and Ragnar Rommetveit are members.
5.3 The dialogical approach to discourse, cognition, and communication

...to specify which strand of the dialogical approach and dialogism one refers to. In this thesis I use dialogism and the dialogical approach as they are used by, first and foremost, Linell (1998, 2005).

Linell (2005) states that “dialogism must be seen as a ‘counter-theory’ to monologism” (p. 49). Therefore, I first start by outlining some basic assumptions of monologism as presented in Linell (1998) before discussing the dialogical approach. Notice in this case I regard dialogism as a subset of the dialogical approach, namely the specific set of the dialogical approach that forms the epistemological basis for the analytical approach.

5.3.1 Monologism

The monological epistemology of language is predominant in fields such as linguistics, cognitive psychology, computer science, and communicative studies (Linell, 1998). One of its more fundamental assumptions regards the information processes, which, according to the monological approach, take place within the individual’s mind. Furthermore, cognition is interpreted as symbol-based processes in the mind of the individual.

From a monological approach, communication and cognition are seen as two distinct features rather than as two aspects of the same phenomenon. In harmony with this view, the monological approach considers language as a tool used by individuals for expressing their thoughts. These thoughts constitute the communicative intentions which are the basis for the conversation. Normally, there is little interest in examining how these communicative intentions came there in the first place (Linell, 1998).

The monological model of communication typically consists of a sender and a receiver. The sender tries to express his/her communicative intention by sending a message while the receiver interprets the message in order to conceptualize the sender’s intention. A successful communication according to this approach is fulfilled when the sender’s intended messages are in tune with the receiver’s interpretations.

Linell (1998) claims that the monological way of examining language is strongly biased
by how written language has been studied. Some consequences of this bias are: The
overemphasis on sentences in language in relation to other structures in language, the
focus on fixed lexical meanings instead of meaning potentials (cf. meaning/sense), the
overemphasis on individuals in interactions at the expense of social interaction, and the
viewing of communication as something planned, instead of something growing within
the interaction (cf. Linell, 1998, pp. 28-32). The point here is not that monologism is
unproductive for approaching written language but instead that the same theory is less
suitable for analyzing verbal discourses.

One of the grounds for the domination of the monological approach concerns the prior
lack of artifacts facilitating the analysis of face-to-face interaction. Artifacts such as
audiotape recorders and video cameras have created better opportunities for analyzing
interactions. New theories for analyzing discourses have then been elaborated by means
of these artifacts. One of these new theories, the dialogical approach, is developed for
analyzing language use rather than the structure of language.

5.3.2 Principles and definitions

Dialogism is the epistemological basis for the dialogical approach and stresses the human
mind’s interaction with both the physical and social contexts as well as situational and
cultural contexts in activities such as perception, cognition and, of course, interpersonal
communication. In other words, dialogism stresses interactional and contextual factors
when analyzing discourses, activities and thinking. For instance, discourses are not solely
determined by the interlocutors’ own intention, but also developed in the interaction.
Moreover, communicative acts are both dependent on prior acts and constitute the basis
for further acts. One can therefore say that there is a strong reflexive relation between
the interlocutors as well as between the interlocutors and the activity, when creating
the discourse. Linell (1998) denotes these reflective/dialogical relations as the double
dialogicality and states “it (dialogue/discourse, my comment) is ‘dialogical’ both in the
contexts of in situ interaction and within the sociocultural practices” (p. 54). Notice here that a discourse is defined as a piece of “concrete, situated and connected verbal, esp. spoken, actions” (Linell, 1998, p. 6). Let us structure the above introduced ideas by introducing three basic principles of the dialogical approach (Linell, 1998; see also Markovà, 1990b).

Linell (1998) introduces three fundamental principles of dialogue/discourse. First, all discourses have a **sequential organization**. Thus, each discursive component’s meaning is to a high degree determined by its position in a sequence. This implies that the discursive components could not be fully understood outside of the sequences forming one of its contexts. Second, discourses are **jointly constructed**, even though they could be asymmetrical. Third, discursive components should be understood in relation to, and at the same time as constituting, the activity. Hence, their exists an **act-activity interdependence**.

In harmony with what is said above, **communication** and **cognition** are seen as interrelated, and simultaneously present **aspects** of intrapersonal and interpersonal processes and practices. According to Linell (1998):

> we define cognition, roughly, as **intelligent (non-random) behaviour and action**, often involving some aspect of problem-solving, whether in silent thinking (in-tramental processes) or overtly as mindful action publicly observable in the external world, and communication as **interaction between different individuals, minds, organisms or systems**, including subsystems of the individual mind (p. 17).

Thus, all types of communication include cognitive features, and conversely, cognition involves interpersonal and intrapersonal communication.

Above, some characteristics of both the monological and the dialogical approach have been outlined and I want to stress some important differences between these perspectives.
The monological approach is more focused on studying the structures of language, while the dialogical approach concentrates on the discourse itself. Further, while the monological approach takes the individual action as the unit of analysis, the units of analysis in the dialogical approach have an interactional character where turns and individuals are seen as interdependent (cf. Markovà, 1990a). There are major differences in how the relation between cognition and communication is treated. The monological approach views language as a tool for expressing thoughts, while the dialogical approach considers words as devices for producing thoughts. Finally, when studying communication the monological approach sees individuals exchanging messages. This stands in stark contrast to the dialogical approach which views communication as a highly context-sensitive interaction, mutually produced by individuals influenced by the social and cultural context as well as from each others.

From this short introduction of basic principles and definitions of the dialogical approach I now turn to the theoretical and methodological constructs used for analyzing talk-in-interaction.

5.3.3 Method of analysis - analytical constructs

Methodological constructs

Linell (1998) identifies two suggestions of building-blocks of a discourse, namely turns and idea units. A turn is basically a period of time when one speaker holds the floor while an idea unit refers to, as the name indicates, chunks of ideas. For instance, a turn could include several idea units. However, both turns and idea units could be discussed in dialogical terms and here I choose to use turns to introduce important aspects of these discourse contributions.

In line with the principle of sequentiality, each turn should be interpreted and understood in relation to the prior discourse as well as seen as creating conditions for the ongoing discourse. Expressed differently, each turn includes aspects of responsive and
initiative functions (cf. Bakhtin, 1986). The responsive aspects of turns could be of different types. For instance, turns could be **locally connected** or **non-locally connected**. Simply put, locally connected refers to the turn’s responsive character to an immediate prior turn (rather than some earlier turns). Moreover, a turn could be **focally connected** or **non-focally connected** to prior turns where focally connected concerns the turn’s topical relation to prior turns. That is, the turn could be seen as continuing the main topic of the discourse. Further, turns could be connected to other prior contributions or to the speaker’s own prior turns.

To conclude, idea units and turns are both reflexively related to the local and global contexts. In addition, a number of turns form larger units of discourse which, in turn, could be discussed in dialogical terms. Linell (1998) chooses to call these units **topical episodes**.

In creating a coherent discourse, the participants must be mutually attentive to each other, establishing a common focus and tying “their contributions together, thereby sustaining and developing foci of attention and discourse topic over sequences of interaction” (Linell, 1998, p. 179, italic added). Topical episodes should not be seen as something the discourse is about but rather as the activity of the discourse. The dynamic characters of topical episodes implies that they will not only be described in terms of what they are about, but also how the interlocutors shape the discourse. Hence, the interaction between the participants, how they create the discourse, becomes fundamentally important to study if we are interested in knowing what they are talking about. Further, topical episodes should be understood in relation to the principles of the dialogical approach. In other words, a topical episode should be analyzed as a part of a sequence, as jointly constructed, and as act-activity interdependent.

Turns, idea units, and topical episodes should be seen as **methodological constructs** used for structuring the data. We now address **theoretical constructs**, used for ascribing meanings to discourse.
Method of analysis - theoretical constructs

People do things with words in discourse, such as: to ask, to promise, to respond, to name, to accuse, to establish agreement, to initiate a discussion, to joke, to mention some examples. So if we are interested in which kinds of acts the interlocutors try to accomplish we must go beyond the discourse and ascribe meaning to it. By far the most influential theory to account for such acts is *speech acts theory* (Searle, 1969, 1975). However, researchers from the dialogical approach (e.g., Linell & Markovà, 1993) have criticized that theory. Below I will discuss some of that critique with the intention of introducing the dialogical counterpart to speech acts: *communicative projects*.

A communicative project should be understood in relation to a communicative goal (purpose, problem, task) that the interlocutors are about to accomplish. The communicative projects related to such goals could be of smaller size, such as reminding a prospective teacher that $40^2$ is not equal to $\pm 1600$, or on a larger scale, such as planning and carrying out a mathematical course for prospective teachers. Typically such bigger communicative projects consist of meso-level communicative projects, such as discussing a mathematical solution at the chalkboard, which in turn, consists of even smaller communicative projects (mini communicative projects). Typically, the researcher uses the construct of communicative project to account for communicative actions on a meso-level, roughly corresponding to pieces of discourse referred above to as topical episodes.

In further specifying aspects of communicative projects Linell (1998) contrasts it to some basic ideas of speech act theory. Within speech act theory *one act* is typically related to a discourse contribution. However, most discourse contributions fulfill many communicative functions, making it more appropriate to stress the multifunctionality of discourse. Within the dialogical approach this is accomplished by enabling the possibility to ascribe several communicative projects to the same piece of discourse. Second, in speech act theory the meaning of a discourse contribution is equated with the intention
of the speaker. However, in correspondence with the first two of the three guiding principles of the dialogical approach, communicative projects are seen as collective activities, partly dependent on individual speakers’ intentions, but also to a high degree developed through the in situ interaction. Finally, in speech act theory a number of elementary speech acts form the global activity. However, little is said about how the global activity affects the elementary speech acts. By using the construct of communicative projects, related to the third principle of act-activity interdependence, the analyst notifies that a number of smaller communicative projects together form larger communicative projects, but also, that smaller communicative projects must be understood in relation to larger communicative projects.

Even though Linell (1998) stresses that communicative projects are collective in nature, this does not mean that they necessarily are symmetrical. In fact, communicative projects are often asymmetrical, especially in institutional discourse. Further, in many interactions the agents pursue different purposes and one could therefore argue that the projects related to those purposes are individual, rather than collective. However, those individual projects must be “coordinated within an overarching collective project, such as having an argument” (Linell, 1998, p. 220). One may use the metaphor of a tennis match to clarify this, that is, the two tennis players definitely compete against each other, however, together they accomplishing the project of playing tennis.

Within the mini-theory of communicative project Linell (1998) introduces the construct of communicative strategy to refer to “a specific (more or less intentional or at least recurrently used) way, or method, of going about solving (trying to solve) the problem or task defining a communicative project or communicative activity” (p. 227). Without going into further details about communicative strategies I would like to stress that historically sedimented communicative strategies for solving large-scale communicative projects, referred to in terms of communicative activities, may be discussed in terms of communicative genres, to which I now turn.
A communicative activity aims at solving a communicative task, for instance, the communicative activity of classroom interactions usually aims at educating students. Many communicative activities have established fairly routinized ways of interacting with, for instance, certain historical and cultural norms, routines, and interactional patterns. Linell (1998), following Luckmann (1992), refers to these routinized ways of interacting in terms of *communicative genres*. A communicative genre is often seen as the most influential contextual resource for understanding communicative acts since within those genres not only have specific verbal and non-verbal means to solve the communicative task been established, but the participants typically engage in specific social roles (teacher-student, doctor-patient, judge-defendant) that, more or less, determine appropriate topics and turn-taking patterns.

Linell (1998) accentuates the homogeneity of communicative genres but also notices that interpretations of tasks and utterances could vary among participants and that certain communicative activities may be related to several communicative genres. In the latter case and expressed very broadly, one could think of mathematics teacher education as connected to a university mathematics genre, an educational genre, or a school genre.

This implies that different participants may talk about the same issue but from different perspectives leading to “contrasting and competing versions of the ‘same’ events in the world” (Linell, 1998, p. 256). In this thesis a specific theoretical framework is used to account for such different ways of interpreting the setting, namely the construct of *contextualization* (e.g., Halldén, 1999) which will be elaborated on below.

### 5.4 Contextualizations and intentional analysis

By using communicative projects the co-constructive nature of discourses is accentuated. However, Linell (1998) notices that in language games “participants may entertain divergent interests within this game and they may also understand or frame the common
project differently” (p. 214). When taking account for these divergences, the theoretical framework of *contextualization* is utilized.

On the one hand, the theoretical framework of contextualization could be seen as a development of constructivist theories of conceptual change, which traditionally assume that agents *replace* commonsense conceptions of phenomena with more scientifically valid conceptions. Within the theoretical framework of contextualizations, it is stressed that the development of new conceptions does not necessarily include replacement of prior conceptions but rather that the agents could hold different conceptions of the same phenomenon in different settings. For instance, in most everyday situations it is perfectly fine to equate the concepts of heat and temperature, while in other situations it is absolutely essential to distinguish between them. On the other hand, the theoretical framework of contextualization could be seen as a constructivist response to the critique directed from the sociocultural perspective that constructivist theories do not adequately take into account the situational character of cognition (e.g., Säljö, 1999). The theoretical framework of contextualization still claims that agents hold conceptions, but at the same time tell us that these conceptions are partly dependent upon the agent’s constructions of the situation. Below, I describe in more detail how the theoretical framework of contextualization is used to account for the situational character of students’ constructions.

The theoretical framework of contextualization stems from studies (e.g., Halldén, 1982; Wistedt, 1987, 1994) showing that students’ interpretations of a task could vary, both in relation to the teacher’s and to other students’ interpretations (cf. Cobb, 1986). The act of interpreting a task is discussed by means of the construct of contextualization. A contextualized task is called a *problem* (Halldén, 1988). Hence, students confronted with the same task could be working with different problems. These different ways of contextualizing the task are dependent on how the students frame the task. Within the theoretical framework of contextualization, three different kinds of contexts are introduced for understanding students’ contextualizations of tasks: *conceptual context*,...
Analytical approaches

situational context, and cultural context (Halldén, 1999). These three different contexts are interrelated, but separated for analytical purposes. Conceptual context refers to students’ personal constructs of the concepts embedded within the task which may or may not affect their way of contextualizing the task. The situational context is used for describing students’ interpretations, and consequent contextualizations, of the immediate setting while the cultural context denotes the constructions of the students with regard to the norms of the discursive practice. To take a concrete example, in Paper III prospective teachers presented solutions to mathematical assignments designed for school students at the chalkboard. The task of presenting these solutions could be contextualized in different ways dependent on, for instance, (1) the prospective teachers’ computational skills and conceptual understanding (conceptual context), (2) how other prospective teachers have presented their solutions at the chalkboard (situational context), (3) or how the prospective teachers’ relate the task of presenting the solution to the communicative genre of a mathematics department (cultural context).

One may ask the question: why not simply use the concept of interpretation instead of contextualization? First, the theoretical framework of contextualization supplies the researcher with an analytical construct to account for how students relate a specific task to different conceptual, situational, and cultural resources. Second, the theoretical framework of contextualization is part of a larger learning theory which talks about learning in terms of differentiation, hence the ability to contextualize a task in different ways dependent on the situation. Third, the theory of contextualization is closely related to analytical principles of intentional analysis, which creates the philosophical basis for the researcher to ascribe intentions to the agents. I now turn to these principles.

Mason (2002) deals with the problem of going beyond the data and punctuates the differences between, giving account of and giving account for on observed activity. An account of a phenomenon aims at describing it as objectively as possible by trying to minimize explanations, judgements and evaluations of what is happening. In contrast,
an account for a phenomenon entails interpretations of what is happening by introducing explanations, judgement, and values. Mason (2002) states:

If I mix up details of the event with judgements, with explanations and theorising, with value judgements and emotively evaluative terms, then I make it virtually impossible for others to challenge or discuss my analysis and decide whether to agree or disagree.

In approaching the discourse in order to offer an account of what is happening I have used several methodological constructs. Hence, the methodological constructs structure physical entities of the discourses and are not commensurable with the theoretical constructs which, in turn, are used when accounting for what is happening.

Along another dimension, and in relation to the study of discourses, Markovà (1990a) contrasts two types of unit of analysis: a physical type (cf. account of), and a conceptual type (cf. account for). While physical subdivisions of the discourse are appropriate when analyzing organization and patterns of the discourse, the conceptual unit of analysis is more appropriate if we are interested in, e.g., turns’ embeddedness in cultural settings. In the latter case, turns are generally seen as actions.

In being explicit about the process of going from the accounts of to the accounts for when approaching the data, I relate the analysis to the principles of intentional analysis (e.g., Booth, Wistedt, Halldén, Martinsson, & Marton, 1999; Scheja, 2002). The main assumption of intentional analysis is that all acts can be seen as intentional, whether that act is of a physical nature or a speech act. In order to interpret the interlocutors’ intentions, meaning is ascribed to their behaviors (Downes, 1998; von Wright, 1971). Furthermore, the “behavior gets its intentional character from being seen by the agent himself or by an outside observer in a wider perspective, from being set in a context of aims and cognitions” (von Wright, 1971, p. 115). Therefore, at this juncture I distinguish between behavior and act where the former could be described in terms of accounts of
while the latter includes interpretations, and thus is an account for an observed activity.

In sum, intentional analysis should be seen as a principle for structuring the researcher’s processes of going from accounts of to accounts for when analyzing the data. Intentional analysis should therefore be seen as a methodological tool which needs to be complemented by theoretical constructs used for ascribing interpretations to the observed behaviors.

5.5 Concluding remarks

In this chapter I have introduced the analytical approaches as well as additional theoretical frameworks used for analyzing the mathematical discourses of this thesis. This presentation serves two purposes. First, to introduce the two analytical approaches of Sfard and Kieran (Kieran, 2001; Sfard, 2001; Sfard & Kieran, 2001a) and Linell (1998, 2005) that will be under scrutiny in the elaboration of the second aim of this thesis. Second, this chapter will hopefully help the reader to better understand how I arrived at the empirical findings presented in 6.1 and 7.2.
Chapter 6

Conclusions

6.1 First aim

The first aim of this thesis regards the characteristics of problem solving discourses in teacher programs. More precisely, the first aim of the thesis is to analyze and discuss the characteristics, and plausible reasons for those characteristics, of the discourse of two problem solving courses for prospective teachers.

As indicated earlier, the distribution of different kinds of discourses in the two courses studied, differs. That is, the discourse of both courses could be characterized in terms of subject oriented, didactically oriented, and problem oriented communicative projects. However, the distribution of the communicative projects in the three categories looks very different in each course. The table below indicates the distribution among the discourses of both courses, where, for instance, one could see that 23 out of 25 communicative projects in Course 1 are classified as subject oriented.

<table>
<thead>
<tr>
<th></th>
<th>Subject oriented</th>
<th>Didactically</th>
<th>Problem solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Course 1</td>
<td>92% (23 of 25)</td>
<td>4% (1 of 25)</td>
<td>4% (1 of 25)</td>
</tr>
<tr>
<td>Course 2</td>
<td>0% (0 of 18)</td>
<td>33% (6 of 18)</td>
<td>67% (12 of 18)</td>
</tr>
</tbody>
</table>
From this table we can conclude that, two apparently similar courses, differ substantially when it comes to the focus of the classroom discourses. In short, while the classroom discourse of Course 1 is mainly focused on pure mathematics, the discourse of Course 2 deals with mathematics in relation to secondary school students’ learning of mathematics or in relation to problem solving. Further, when going into detail about the nature of the co-construction of the discourses, we can conclude that the co-construction is fairly asymmetrical in both courses. More precisely, in Course 1 the discourse gradually became more asymmetrical in that the teacher educator mostly held the floor. In Course 2, the trend is the opposite: the prospective teachers became slightly more active as the course proceeded. But still, the teacher educator in Course 2 had a leading position in the discourses throughout the whole course.

The divergencies found could be given plausible explanations by referring to the principle of act-activity, that is by relating the communicative projects to the participants’ contextualization of the communicative activities of the two classrooms. That is, the analysis indicates that the nature of the communicative activities is to a high degree dependent on the teacher educators’ contextualization of the task of arranging a problem solving course for prospective teachers. Of course, the teacher educator’s contextualization of arranging a course always affects the communicative activity, but the ambiguity and complexity of the concept of problem solving imply that the teacher educators’ contextualizations are of particular importance. For instance, in Sweden at least, mathematics courses for prospective teachers are often developed in close connection with a textbook. However, when it comes to problem solving for prospective teachers there is no well established textbook\(^1\) so the teacher educators of this study used instructional material from a variety of sources such as curriculums, scientific articles, secondary school textbooks, national tests in mathematics, and tasks formulated by themselves. So, if we agree that

\(^1\)I refer to textbooks especially designed for teaching Swedish prospective teachers about problem solving. The data of this study were collected in 2004, and in 2005 an interesting candidate for such a book was published.
instructional materials are important, and that the teacher educators had little guidance in choosing the material, one may argue that the teacher educators’ contextualizations of problem solving are of particular importance when it comes to the content of the course. Let me elaborate further on this by focusing on the communicative activities of the two courses.

By analyzing the whole data material I came to the conclusion that it seemed reasonable to summarize the contextualization of TE1 as ‘let the prospective teachers present the problems at the chalkboard and see what comes up’. I drew this conclusion from the fact that no explicit instructions of how to present the tasks were introduced and also that no explicit frameworks were introduced or used for analyzing the solutions. This contextualization lead to instructions that gave the prospective teachers mandate to discuss different interpretations of the task, choose to present one or several solutions to the same task, discuss the characteristics of the solution, adapt the presentation to fictive school students. So it could be argued that there is a substantial latitude for the prospective teachers to contextualize the enterprise of presenting the solutions at the chalkboard. This seems to be the case in Course 1, as this relative freedom lead to more purely subject oriented presentations. After all, it is more complicated to, e.g., solve and present several solutions to the same task than it is to present one, so why bother about presenting several solutions if you pass the course anyway?

In contrast, TE2 seems to contextualize the task of arranging a problem solving course as ‘to denote and contrast different solutions to the same task’. This interpretation seems reasonable for several reasons. First, several solutions to each task presented at the chalkboard are produced, either by letting all five groups present their solutions or by requesting that each group should present several solutions to the same task. This creates opportunities to contrast different solutions. Second, TE2 introduced and frequently used problem solving frameworks in commenting about the solutions at the chalkboard. One could therefore argue that the communicative activity established by the measures taken
by TE2, more or less, directed the prospective teachers into didactically and problem solving oriented discourses. Expressed differently, the instructions and the conceptual frameworks strongly encouraged the prospective teachers to initiate meta-shifts (Cobb, Boufi, McClain, & Whitenack, 1997) in the discourse, thus initiating didactically oriented and problem solving communicative projects.

6.2 Second aim

The second aim of this thesis is to elaborate on possibilities to develop two analytical approaches, the communicational approach to cognition (Kieran, 2001; Sfard, 2001; Sfard & Kieran, 2001a) and the dialogical approach to discourse, cognition, and communication (Linell, 1998, 2005), for studying mathematical discourse. As described above, both analytical approaches have been used for analyzing the empirical data of engineering students’ concept mapping in Linear Algebra and problem solving courses in teacher education, respectively.

6.2.1 Possible ways of developing the communicational approach

The theoretical perspective of the communicational approach creates opportunities to include individual, situational, and cultural aspects in studying mathematical discourse. However, it is one thing to create opportunities through theoretical foundations, and another to operationalize these principles into analytical tools and constructs. Although the methodological framework presented in Kieran (2001), Sfard (2001), and Sfard and Kieran (2001a) takes account for important features of mathematical discourse, my use and analysis of the methodological framework suggest possibilities to strengthen it, which will be discussed below.

On a general level I suggest basically two ways of strengthening the methodological strand of the analytical approach. First, there are possibilities to adjust the theoretical constructs in order to better harmonize with the principles of the analytical approach.
Second, there are possibilities, which are shown in Paper II, to add more detailed accounts of the mathematical characters of the discourses.

Let us start the first issue by considering the theoretical constructs used when carrying out the focal analysis. The constructs are; pronounced, attended, and intended foci. All three theoretical constructs have a cognitive/individualistic character. For instance, Sfard (2001) describes the process of developing the construct of pronounced focus as a search to account for “expression used by an interlocutor to identify the object of her or his attention” (p. 34, italic added) and for the attended focus since “it would be important to include some indication of what and how one is attending to ... when speaking and thinking” (p. 34, italic added). Furthermore, the intended focus “is mainly private” (Sfard & Kieran, 2001a, p. 53). My argument here is not that it is unproductive to stress cognitive aspects of those constructs but instead that aspects of co-construction and meta-discursive rules need to be included within these theoretical constructs in order to harmonize with the principles of the approach. Let me take a further example, if the intended focus “may be described as a cluster of experiences evoked by the other focal components plus all the statements a person would be able make on the entity in question” (Sfard & Kieran, 2001a, p. 53, italic added) one may argue that this definition indicates clear references to context-independent characters of the human mind. My suggestion then is to include and accentuate situational and cultural aspects of the different foci. The inclusions of situational and cultural aspects would make the focal analysis harmonize with the principles of the approach as well as tightening the interplay between the focal and preoccupational analyses.

More generally and in relation to this, Sfard (2001) argues that “since discourses are analyzed as acts of communication, anything that goes into communication and influences its effectiveness - body movements, situational clues, interlocutors’ histories, etc. - must be included in the analysis” (p. 28). I propose that the usefulness of the analytical approach will be strengthened if features such as body movements and interlocutor’s
histories are structured into a framework of contextual resources. Further, these features together with constructs such as meta-discursive rules and contextualization may serve as a starting point in building such a theory of contexts. We now turn to the discussion of how to complement the theoretical constructs of the preoccupational analysis.

As shown above, by constructing the interactive flowchart the researcher creates opportunities to consider whether the interlocutors are communicating with each other or themselves, and whether they communicate about mathematics or not. In relation to Kieran (2001), Hoyles (2001) asks “what were the meta-rules regulating the student work, in terms of what ways valued ... what was the influence of a school culture where questions were presented” (p. 282). As regards those questions of Hoyles (2001), one may argue that the distinctions/codings made within the preoccupational analysis do not fully capture the complexity of the meta-discursive rules. Therefore, I present several ways of widening the scope of the interactive flowchart in order to increase its usefulness in studying mathematical discourse. Paper II shows that the construction of the interactive flowchart could be improved by applying the theoretical construct of contextualization. The concept of contextualization enables the researcher to see the discourse contributions in a wider perspective and through that create more elaborated arguments for the interlocutors’ engagement in the discourse. For instance, by using the theory of contextualization on one of the group of engineering students, I discovered that the individual students’ way of acting in the discourse could be related to their way of contextualizing the enterprise of constructing a concept map. That is, one of students was interested in discussing the relation between the concepts while another student was more interested in just finishing the assignment. These divergent ways of viewing the task of constructing concepts maps influence how the students are interacting with each other. In Paper II, it is shown how this finding could be used for strengthening the arguments as to whether

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2Interesting discussions of the concept of context could be found in, e.g., van Dijk (1997a), Goodwin and Duranti (1992), and Linell (1998).
the students are communicating with each other or with themselves.

The second issue, that of adding mathematical aspects to the analysis has also been analyzed in Paper II. By introducing three different types of mathematical utterances in the construction of the interactive flowchart, Paper II shows that the picture of whether the discourses could be seen as mathematically productive or not is changed. One may therefore argue that such a complement is necessary.

6.2.2 Possible ways of developing the dialogical approach for studying mathematical discourse

The dialogical approach, as presented by Linell (1998, 2005) is a coherent analytical approach. That is, the methodological approach harmonizes with the theoretical perspective. Moreover, these methodological and theoretical constructs are relatively straightforward to use. Finally, the approach offers constructs that stress situational and cultural aspects in the study of discourses, which helps the researcher to create an elaborated analysis of how teachers and students interact in mathematical discourses. However, the individual aspects of the discourses are not emphasized even though Linell (2005) states “individuals are still to a large extent the origins of personal initiatives, and are accountable for their actions” (p. 22). Below, I will elaborate further on the relation between individual agency and the dialogical approach in order suggest ways of developing the approach in relation to the study of mathematical discourses.

Through the use of the dialogical approach in Paper III, I found three ways in which the dialogical approach needs to be complemented for studying mathematical discourses. First, the dialogical approach does not include a theory of learning. Therefore, a conceptual framework aimed at accounting for learning, and especially mathematical learning, has to be related to this analytical approach3. Second, Linell (2005) states that “a full-blown

3From the analysis it seems natural to connect the dialogical approach to a theory of learning that is in tune with the metaphor “learning as becoming a participant of a certain discourse”. One may find inspiration of how to connect a linguistic theory to a theory of learning by studying how Wells (1999) relates the theories of Vygotsky and Halliday to each other. An alternative way might be to use the theory of contextualization and the theory of learning connected to it.
dialogism would assign important functions to all four kinds - intrapersonal and interpersonal, present and sociohistorical - of construction” (p. 22). However, while the dialogical approach has developed theoretical constructs accounting for the last three constructions, the approach seems to lack adequate concepts for analyzing the first. Therefore, in Paper III the analytical approach was complemented by the theoretical framework of contextualization. The combination of the constructs from the dialogical approach and the construct of contextualization created an arsenal of theoretical constructs used to account for the complexity of the problem solving discourses. Third, the dialogical approach provides the researcher with constructs, such as communicative projects, that create possibilities to ascribe meaning to discourses. However, the actual meanings ascribed must be situational in character, making it necessary to add theories of, e.g., meta-mathematical rules or, in the case of Paper III, theories of mathematical problem solving.

6.3 Concluding remarks

In this chapter I have presented the most important conclusions of the thesis. In the next chapter the conclusions of this thesis will be set in a wider perspective by discussing the relevance and the practical pedagogical implications of this thesis.
Chapter 7

Discussions

7.1 Theoretical and methodological contributions

In discussing the contribution or relevance of this thesis one could ask the questions: ‘What is the use of it?’ and ‘For whom?’ (cf. Kilpatrick, 1993, 1995). Roughly speaking, the elaboration of the first aim, and the related discussion of practical pedagogical implications performed below, are of relevance for researchers, teacher educators, and teachers while the elaboration of the second aim is of relevance for researchers interested in approaching mathematical discourse. Firstly, let me concentrate on the theoretical relevance of the thesis since the practical relevance is explicitly elaborated on in 7.2. The theoretical relevance of this thesis has three main strands: The framework of different kinds of discourses derived from the problem solving course for prospective teachers, the elaboration on the two analytical approaches, and the discussions of the relation between the theory of contextualizations and intentional analysis.

The division of discourses into subject oriented, didactically oriented, and problem solving oriented, together with the subcategories belonging to each of them, could be of relevance to researchers interested in studying problem solving courses for prospective teachers. That is, when I was approaching the data collected in the problem solving courses I realized that the cognitively oriented problem solving work of, e.g., Pòlya (1945),
Schoenfeld (1985, 1992), and De Corte et al. (2000) were of limited help when accounting for what was happening in the classroom discourse. Therefore propose that the framework of the different kinds mathematics problem solving discourse derived in relation to the data could be used to complement the more cognitively oriented frameworks when approaching classrooms discourses.

The contribution of the elaboration on the second aim is fourfold. First, it is of direct relevance for researchers performing in-depth analysis of mathematical discourses, especially when we consider the substantial need for well-developed analytical approaches for studying mathematical discourses (cf. Kieren et al., 2001). Second, the elaborations of the analytical approaches are performed along mathematical as well as discourse analytical dimensions, which increases the relevance of the analytical approaches for studying mathematical discourse. Third, as shown in the thesis, the elaboration of the analytical approaches is performed in relation to the individual and sociocultural perspective and therefore contributes to the ongoing discussion of how to relate those perspectives to each other. Fourth, I should stress that the choice of these two analytical approaches for scrutiny has to be based on their relevance for the study of mathematical discourses. The relevance of the original approaches for studying mathematical discourse is shown in Sfard (2001), Kieran (2001), and Sfard and Kieran (2001a, 2001b) as well as in Cestari (1997) and Bjuland (2002, 2004), respectively.

The final theoretical contribution of this thesis, related to the theory of contextualization and intentional analysis, has until now not been explicitly discussed. To understand this contribution it is of importance to notice that it is not entirely clear whether or not contextualization should be seen as a theory or an analytical construct and how to relate contextualization and intentional analysis to each other (cf. Scheja, 2002). Expressed differently, contextualization as used in, e.g., Halldén (1999), Nilsson (2004), Ryve (in

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press), Scheja (2002), and Wistedt and Brattström (2005) should be seen as an evolving theory where some work has to be done before contextualization could be seen as a full-fledged theory. In relation to this short background I want to accentuate two aspects that this thesis contributes to. First, in this thesis I not only show how contextualizations and intentional analysis could be used on new empirical data such as collaborative concept mapping and mathematical problem solving discourses but also how they could be related to other analytical approaches such as the communicational approach and the dialogical approach. Expressed in more detail, I show how the more cognitively oriented theory of contextualization could be related to more collaborative constructs such as communicative projects. Second, the status and function of intentional analysis in relation to contextualization have been clarified in this thesis by means of Mason (2002). That is, this thesis makes clear that intentional analysis should be seen as a method of analysis of how to structure the analyst’s process of going from accounts of (descriptions) to accounts for (interpretations). In addition, it is important to accentuate that intentional analysis is a methodological device which has to be related to theoretical constructs used for ascribing meaning to the data.

7.2 Practical pedagogical contributions

As clarified above, the driving force behind conducting the studies reported on in this thesis is examining ways of designing instructions that could help students develop different kinds of mathematical competencies. Below I introduce a number of aspects to consider in developing such instructions.

In relation to the first study of engineering students constructing concept maps in Linear Algebra I would like to highlight aspects to take into considerations when teaching. The study shows that the students’ interpretation of the task of constructing concept maps strongly affects the nature of their group discussions. Therefore, I claim that mathematics
teachers aiming at using concept mapping in their teaching need to reflect upon and, if necessary, change the meta-discursive rules of the mathematical classroom practice in order to create better opportunities for in-depth discourses of mathematical concepts and their relations. This could, for instance, be accomplished by stressing that it is not only the number of connections that are of importance when constructing concept maps but also the quality of the connections. Further, and in relation to what Mason writes in Sfard, Nesher, Streefland, Cobb, and Mason (1998), the presence of a mathematical expert seems to be of importance. That is, the presence of a teacher guiding the students when concept mapping may help the students in developing their conceptual understanding.

The findings presented in relation to the first aim and in Paper III are here used for deriving several practical pedagogical implications. First, at least in Sweden, there are numerous discussions, in quantitative terms, of how many credit points of mathematics prospective teachers should have. This thesis shows that two on the surface very similar courses substantially differed regarding the content of the course. Therefore, it is absolutely necessary to complement the quantitative discussions by qualitative discussions focusing on what to include in such courses (cf. Monk, 1994; Begle, 1979). I suggest that these discussions should be a part of a larger discussion of what knowledge in mathematics is. Such a discussion could, for instance, take its starting point from within the framework presented in Ball et al. (2005). The framework is developed by both mathematicians and mathematics educators and aims at presenting principles of school mathematics and mathematics learning that could function as a common ground for discussions and developments of mathematics teaching. The findings of Grant, Hiebert, and Wearne (1998) show that discussions about competencies in mathematics are of particular importance both for in-service teachers and prospective teachers. For deepening these discussions, mathematicians, teacher educators, and teachers may also focus discussions around documents such as Kilpatrick et al. (2001), Niss and Jensen (2002), and Schoenfeld (1992).

Second, the results may be interesting for educators arranging, or planning to arrange,
problem solving courses for prospective teachers. I use the results of the study to discuss a number of, partly interrelated, issues to consider when designing such a course. If we assume that learning about problem solving is best accomplished through problem solving, we must carefully consider which tasks the prospective teachers should be working on.

Expressed differently, as a parallel to learning mathematics through problem solving, what we want the prospective teachers to learn should be embedded within the tasks (Lester & Lambdin, 2004). For instance, if the tasks are designed in relation to the level the prospective teachers will teach at, there will be better opportunities to introduce didactically oriented discourses or if we want them to discuss university mathematics such aspects should be embedded in the tasks. In relation to didactically oriented discourses, Cooney (2001) states that “the best entry into their (prospective teachers, my comment) belief systems about mathematics and the teaching of mathematics is through the study of school mathematics” (pp. 26-27). That is, tasks taken from school mathematics may not only be used for discussing students’ learning of mathematics but also the prospective teachers’ beliefs about mathematics (cf. Ball & Bass, 2000). One may therefore assume that the tasks to some extent regulate the relation between subject oriented, didactically oriented, and problem solving oriented discourses. Nevertheless, as shown in relation to Course 1, the tasks chosen just create opportunities and the teacher educator needs to consider ways of highlighting the embedded properties. This could be accomplished by “initiating shifts in the discourse such that what was previously done in action can become an explicit topic of conversation” (Cobb et al., 1997, p. 269). To be able to make such shifts, and not just solve the problems, may be seen as particularly important for prospective teachers since they must be able to discuss and understand different ways of solving problems. Paper III shows that the explicit introduction of conceptual frameworks in Course 2 is one way of enabling such shifts. One may argue that without such explicit frameworks the educators’ questions may instead “degenerate into a social guessing game in which students try to infer what the teacher wants them to say and do” (Cobb et al.,
1997, p. 269). This is actually what happened on a number of occasions in Course 1. Therefore, one should carefully consider the aims of the course and design conceptual frameworks that facilitate the shifts needed to fulfill these aims.

Moreover, if we view learning as becoming a participant in a specific discourse, it is also important that the prospective teachers become active in the co-construction of that discourse. Here, once again, I claim, by means of the empirical findings of this study, that explicit frameworks facilitate the engagement of prospective teachers in the discourse. From a theoretical perspective this claim also makes sense. That is, if language is seen as a tool for thinking rather than something used for expressing one’s thoughts (cf., e.g., Sfard, 2001), one may argue that language, in the form of conceptual frameworks, helps students to think about, and engage in, mathematical classroom discourses. Or as Säljö (1999) puts it “human knowledge is largely conceptual in nature” (p. 81).

### 7.3 Further research

In discussing possible continuations of the research conducted in this thesis I choose to focus on three themes; design research in mathematics teacher education, mathematics teacher knowledge, and analytical approaches for studying mathematical discourses. First, in relation to the findings related to the first aim, it would be interesting to carry out a design research project in connection to a course focused on mathematical problem solving in the teacher education (cf. Wood & Berry, 2003). This thesis suggests that the choice of tasks, ways of presenting the solutions, and conceptual framework for discussing the solutions are of importance not only for the topics of the discourse but also for how the discourses are co-constructed. However, in the thesis I just discuss those issues in general and I suggest that carefully planned and implemented design research projects could furnish teacher educators with more specific guidelines regarding the choice of, for instance, which conceptual frameworks to introduce and use. Further, it is absolutely
necessary to get the prospective teachers engaged in the construction of the discourse and especially facilitating the possibilities for them to initiate meta-shifts in the discourses. As discussed above, the thesis suggests that one way to accomplish this is through the introduction of conceptual frameworks. Nevertheless, this is just one suggestion and a design research project may help us develop other ways of getting the prospective teachers engaged in the construction of the discourse.

Second, if we view problem solving as a process goal (cf. National Council of Teachers of Mathematics, 2000), there are also other process goals that are closely related to the goals of the Swedish curriculums in mathematics. My research shows that the concept of problem solving is used in different ways in Swedish teacher education and it would be interesting to see how other process goals come into play in teacher education and how they are related to the mathematical courses given in teacher education. If one is widening the scope a little bit, further research might want to study the discourse about teacher knowledge in (Swedish) mathematics teacher education. More precisely, I suggest a research project consisting of three studies. The aim of the first study would be to examine the directors of studies discourse about the courses given to prospective teachers in mathematics. The second study would be aimed at examining the classroom discourse of courses that seem to be of particular interest, in relation to the first study, for understanding how teacher educators work with, e.g., process goals. The third study would be focused on the prospective teachers. What do the prospective teachers regard as mathematics teacher knowledge? One could for instance follow prospective teachers through their teacher training to be able to examine possible changes in their views of mathematics teacher knowledge.

Several of the studies suggested above are focused on mathematical classroom discourses. As argued elsewhere in this thesis, the development of analytical approaches for studying mathematical discourse needs much more attention (cf. Kieran et al., 2001). This thesis contributes to such a development by scrutinizing the communicational and
the dialogical approach. However, there are important questions that need to be examined further. For instance, if we want to analyze discourses in quantitative terms some kind of coding needs to be conducted. In the thesis I mention that the coding scheme of interactive flowcharts may not fully capture the complexity of the mathematical meta-rules and I therefore suggest that further research continues into the discussion of how to develop coding schemes that are in harmony with the principles of the communicational approach of cognition. For instance, instead of viewing utterances as either proactive or reactive, or both, one might want to examine those features as aspects of utterances. This implies, in line with Initiative-Response analysis (Linell & Markovà, 1993; Markovà & Linell, 1996), that each utterance contains both proactive and reactive aspects. One may here ask: what difference does this make? First, instead of focusing on whether an utterance is proactive or reactive, the researcher can focus on the nature of how the utterances are connected to each other. I propose that such a shift of focus, from whether to how, will provide us with good ideas “about what could be done in order to make mathematical communication, and thus mathematical learning more effective” (Sfard, 2001, p. 44).

A related issue, but with a slightly different focus, is the discourse about discourses in mathematics education. That is, it would be interesting to study how the concept of discourse is used in scientific communications in the research field of mathematics education since “the communication will not be regarded as effective unless, at any given moment, all the participants seem to know what they are talking about and feel confident that all the parties involved refer to the same things when using the same word” (Sfard, 2001, p. 34). I doubt that the scientific communication about mathematical discourse could be regarded as effective.


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