Human Capital and Innovations

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June 26, 2017

Abstract

This paper explores the interaction between human capital and innovations in the process of economic growth. Using a model of endogenous economic growth, the focus of our analysis is on the incentives to acquire human capital and how they are affected by taxes on human capital and other policy instruments. In contrast to many other growth models we find that the taxation on human capital has a substantial negative effect on the accumulation of it. This in turn lowers the income growth rate. While subsidies to research and to intermediate inputs have positive effects on growth (and must be strictly positive in social optimum), they do not necessarily imply that there will be larger stock of human capital in the economy. If the elasticity of intertemporal substitution in consumption is sufficiently low, these policy instruments stimulate growth by inducing a reallocation of a shrinking stock of human capital in the direction of research.

Keywords: Technological Change, Human capital, Economic Growth, Taxation.

JEL classification: H2, I2, O3, O4.

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1 Introduction

The following kind of reasoning is often heard in the political debate: Innovations are important for the growth of GDP in an economy, and human capital is important for innovations. Consequently, taxes in general and in particular taxes that are detrimental to the formation of human capital, have negative effects on economic growth. Somewhat remarkably, however, there is not much theoretical work that supports precisely this kind of reasoning. The purpose of this paper is therefore to present and analyze a model that does have such effects.

Modern theories of economic growth of course emphasize the roles of innovation and human capital but these works seldom look at the combined effects of these two endogenous engines of growth.[1] One important line of research is found in the large literature with both real and human capital, for instance Lucas (1988), Lucas (1990), Rebelo (1991) and Stokey and Rebelo (1995). However, these works do not include innovations. Moreover, as discussed in detail in Stokey and Rebelo (1995), the effects of taxes on the growth rate are quite small for reasonable parameter values, with the tax on savings as an important exception. One reason for this is probably that both real and human capital are growing on the balanced growth path (BGP). The models therefore share some properties with the Jones (1995) model, which is famous for its lack of effects from taxes on the rate of growth.

The model in this paper is instead more like that in Romer (1990), where some of the human capital is used in research that increases aggregate productivity. The stock of human capital is constant on the BGP but, in contrast to Romer (1990), it is here endogenous. The constant stock of human capital therefore functions as a foothold for economic growth, and whatever makes it change will also have an effect on the growth rate. Since a tax on human capital has a negative effect on the incentives for schooling, that tax also influences the rate of economic growth in the negative direction. The model therefore reveals an important mechanism between the incentives for

[1] An interesting exception is Section 5.2 in Grossman and Helpman (1991), which presents a model with endogenous innovations and human capital formation. A tax on human capital would however not have an effect on the length of schooling in this model, since it would raise the cost and benefit of marginal education to the same extent. However, a higher productivity in schooling leads to an increased number of people acquire human capital, to the benefit of growth. It appears that a reduction in the tax on human capital could have a similar effect in their model.

schooling and the development of prosperity, thus (in a sense) 'confirming' the popular belief that was referred to above.

The standard formulation of the household's optimization problem in the earlier literature is responsible for the fact that the decision on the length of schooling is not responsive to the tax on human capital, and therefore does not have any effect on economic growth in most of the previous research. In the words of Jaimovich and Rebelo (2016), the reason is that the 'costs (foregone wages) and benefits (higher future wages) of this accumulation are affected by income taxes in the same proportion'. In this paper we propose an alternative formulation of the household's problem, which allows the tax on human capital to have an effect on the decision to accumulate human capital.[2]

Many researchers have attempted to find empirical evidence that human capital is important for income differences across countries and that taxes reduce growth. The results are mixed. For instance, Jones (2015) notes that, while some theoretical models show clear and strong negative effects from taxes on growth, correlations between tax revenues as a share of GDP and rates of growth are difficult to find in time series (US) and international cross-section data. Similar results are reported and reviewed in Jaimovich and Rebelo (2016).

On the other hand, there is the huge literature on cross-country regressions initiated by Barro (1991, 1998) and extended and updated in Barro and Sala-i-Martin (2004). A typical finding of these works is that (male) secondary and higher schooling has positive effects on the growth rate. This finding seems to be robust and the pattern becomes even clearer when the quality of human capital is incorporated[3]. Although this line of research is intensely debated (see for instance Acemoglu (2009), ch. 3) we find it convincing enough to make it worthwhile to develop a model where a tax on human capital has a negative effect on schooling and thus on the growth rate.

Our research questions and results can be summarized as follows.

Taxes on labor: First we examine the effects on the growth rate of dif-

[2]We include the foregone wage to raw labor. This is what makes the difference. Alternatively, one could include other costs of education, such as buildings, teachers and computers, as in Ben-Porath (1967).

[3]See Hanushek and Woessmann (2012). Note however that this particular result concerns primary schooling.

ferences in taxation between ordinary labor and human capital. We find that the decision on the length of schooling is indeed responsive to the tax on human capital. Our model thus shows that a distinction between taxes on unskilled and skilled labor has consequences: progressive taxes matter for educational choices, with direct consequences for the stock of potential researchers and therefore for economic growth. Our numerical simulations suggest that these effects may be substantial.

Saving tax: A noteworthy mechanism of the model is that an increased tax on savings has a positive effect on the accumulation of human capital, since future incomes then are discounted less heavily. However, our analysis shows that the total effect on growth is negative, as expected. Nevertheless, this means that the endogenization of human capital implies a considerable reduction of the usual negative effect of a tax on saving on the rate of economic growth.

Subsidies: A higher rate of subsidy to research will lead to a higher growth rate. While this result is expected, the mechanism behind it may be less so. The higher growth rate could be due to a mere reallocation of human capital toward research, while the total stock is decreasing (implying also lower schooling). This can happen if the intertemporal elasticity of substitution in consumption is low, so that the income effect dominates the household behavior.

Optimal policy: The decentralized equilibrium coincides with the social optimum if there are subsidies to research and to the use of the 'machines' that are developed through research. The former is due to positive externalities in research, while the latter is caused by mark-up pricing on 'machines'. Although human capital plays a key role in the growth process, there is no reason to subsidize schooling because there are no market failures involved in the formation of human capital in our model.

The paper that comes closest to our model is Jaimovich and Rebelo (2016), although they assume that the human capital is exogenous. They modify the Romer (1990) model by assuming heterogeneity in the ability to do research. Since the distribution of these abilities is skewed, the disincentives to do research become significant, with strongly negative effects on economic growth, only when taxes reach very high levels. Moreover, it is exceptional talents like Bill Gates and Steve Jobs that do the major part of research in their model. In this paper research is carried out by large groups of ‘everyday researchers’.

The rest of this paper is organized as follows. Section 2 presents the production structure for final output. Innovations are introduced in Section 3, while the household decisions are described in Section 4. Section 5 analyzes the Balanced Growth Path. Section 6 presents the comparative statics results, while some numerical results are found in Section 7.

2 Production

2.1 Production of Final Output

We assume that the final output is produced by a representative firm. The production function is

$$Y(t) = \frac{1}{1-\alpha-\beta} L^{\alpha} H_Y(t)^{\beta} \left[\int_0^{N(t)} x(\nu,t)^{1-\alpha-\beta} d\nu \right], \tag{1}$$

where L is unskilled labor. The amount of human capital used in this sector is H_Y. Instead of an aggregate stock of capital there is a range of machines. The quantity of machines of variety ν at time t is denoted by $x(\nu,t)$. The range of machines is $N(t)$, which grows over time due to costly research and thereby raises productivity. The constants α and β are positive and $\alpha + \beta < 1$. The price of final output is normalized to unity, while the wages are $w_L(t)$ and $w_H(t)$, respectively.

2.2 Producers of intermediate inputs

The monopolistically competitive producer of $x(\nu,t)$ holds a perpetual patent to it and charges the price $p^x(\nu,t)$. Due the market power the price will include a mark-up on the marginal cost. One unit of $x(\nu,t)$ costs ψ units of Y to produce. The profit of the holder to the patent for variety ν therefore is

$$\pi(\nu,t) = p^x(\nu,t)x(\nu,t) - \psi x(\nu,t). \tag{2}$$

Since the derivation of various optimality conditions for this part of the economy is by now very standard they are relegated to Appendix A.1. Maximization of the profit in the final output sector leads to a constant-elastic demand function for $x(\nu,t)$. Taking this into account when maximizing (2) with respect to $p^x(\nu, t)$ we get

$$p^x(\nu, t) = \frac{\psi}{1 - \alpha - \beta}.$$

Note that this price is the same for all ν and all t. Since $0 < \alpha + \beta < 1$, the price implies a mark-up on the marginal cost ψ. We assume that $\psi = 1 - \alpha - \beta$, which gives the constant price

$$p^x(\nu, t) = p^x = 1.$$

This assumption will simplify the subsequent exposition without any loss of generality.

2.3 Some Implications

As explained in Appendix A.1, the unitary price of machines implies that the demand for machines and the profits in the machine-producing sector are

$$x(t) = \left(\frac{L^\alpha H_Y^\beta}{1 - \varsigma_x} \right)^{\frac{1}{\alpha+\beta}}. \tag{3}$$

and

$$\pi(t) = (\alpha + \beta) \left(\frac{L^\alpha H_Y^\beta}{1 - \varsigma_x} \right)^{\frac{1}{\alpha+\beta}}, \tag{4}$$

respectively. The input x is subsidised at the rate ς_x, to correct for the markup. There is a symmetry in the use of the intermediate inputs, so both of these expressions are independent of ν.

Turning to the producers of final output, and using (3) in (1), the expression for production is simplified to

$$Y(t) = \frac{1}{1 - \alpha - \beta} \left(\frac{L^\alpha H_Y^\beta}{(1 - \varsigma_x)^{1-\alpha-\beta}} \right)^{\frac{1}{\alpha+\beta}} N(t). \tag{5}$$

By substitution of this into the optimality conditions with respect to H_Y and L in the final output sector, the wages of the two types of labor are

$$w_H(t) = \frac{\beta}{H_Y} \cdot Y(t) \qquad \text{and} \qquad w_L(t) = \frac{\alpha}{L} \cdot Y(t). \tag{6}$$

Both types of labor benefit from the expansion of the number of varieties of machines (N), via the growth of Y.

Let the total number of machines be denoted by $X(t) = x(t) \cdot N(t)$. Equation (3) and (5) then imply that

$$X = \frac{1-\alpha-\beta}{1-\varsigma_x} Y. \tag{7}$$

Moreover, the use of final output for consumption and investment is constrained by $Y \geq C + \psi X$. Assuming equality, due to unbounded utility of consumption, and using the expression for X above consumption can be written as proportional to production:

$$C = Y - \frac{(1-\alpha-\beta)^2}{(1-\varsigma_x)} Y, \tag{8}$$

where the assumption that $\psi = 1-\alpha-\beta$ has been used. In contrast to some other growth models, final output is here not used in the innovation sector. Instead, human capital is the central input there, as we will see now.

3 Innovations

The total amount of human capital active on the labor market is $H = H_Y + H_R$, where H_R is the human capital used in the research sector. Innovative efforts result in an expansion in the varieties of machines that can be produced. This increase in N is given by

$$\dot{N} = \eta N H_R, \tag{9}$$

where η is a productivity factor. The research process draws on the stock of previously generated knowledge, which is captured by the N on the right-hand side. Since H_R (and H) is constant in steady state the model becomes similar to the one in Romer (1990), with the important difference that H is endogenous here. Following the convention of using g_z to denote the proportional growth rate of any variable z, this implies that $\dot{N}/N \equiv g_N = \eta H_R$.

Let the value of an innovation be denoted $V(\nu, t)$. This value equals the discounted stream of profits that can be earned by producing and selling the machine that builds on this innovation. Using $r(t)$ to denote the interest rate, this can be expressed as

$$V(\nu, t) = \int_t^{\infty} \exp\left[-\int_t^s r(s')ds'\right] \pi(\nu, s) ds \tag{10}$$

Since we know that π is independent of ν, we can use $\pi(s)$ instead, which implies a symmetry over the values of the varieties.[4] Entry into the research sector is free, leading to the zero profit condition

$$(1-\varsigma_R)w_H = V\eta N,$$

where ς_R is the rate of subsidy to research. By use of equations (5) and (6) to eliminate the wage of human capital this equation is modified to

$$\frac{\beta}{H_Y}\cdot\frac{(1-\varsigma_R)}{1-\alpha-\beta}\left(\frac{L^\alpha H_Y^\beta}{(1-\varsigma_x)^{1-\alpha-\beta}}\right)^{\frac{1}{\alpha+\beta}} = V\eta. \tag{11}$$

This is the condition for equilibrium allocation of human capital between final goods production and research: the values of the marginal products should be equal in both activities.

4 Household

It is assumed that the household sector can be described as a representative household, or dynasty, with an infinite time horizon. It consists of a constant number of members, M, where each one has a constant-elastic felicity function of consumption. The utility of the representative household can therefore be written as

$$U = \int_0^\infty M\cdot e^{-\rho t}\frac{(C/M)^{1-\theta}-1}{1-\theta}dt, \tag{12}$$

where C/M is the consumption per member of the household and ρ and θ are non-negative constants.

[4]The profit at some time $s \geq t$ can be written as

$$\pi(\nu,s) = \pi(\nu,t)\exp\left[\int_t^s g_\pi(\nu,s')ds'\right],$$

where $g_\pi(\nu,s')$ is the growth rate of $\pi(\nu,s')$ at time s'. Using this in (10):

$$V(\nu,t) = \pi(\nu,t)\int_t^\infty \exp\left[-\int_t^s \left(r(s')-g_\pi(\nu,s')\right)ds'\right]ds$$

Assuming that the economy is on a balanced growth path such that r and g_π are constant:

$$V(\nu,t) = \pi(\nu,t)\int_t^\infty \exp\left[-\left(r-g_\pi\right)(s-t)\right]ds = \frac{\pi(\nu,t)}{r-g_\pi}.$$

Let $s \in [0, 1]$ be the share of time spent in schooling. The stock of human capital, h, then evolves according to the function[5]

$$\dot{h}(t) = \phi\left(h(t)s(t)\right) - \delta_h h(t), \tag{13}$$

where ϕ is an increasing and strictly concave function with Inada-like properties and δ_h is the depreciation rate of human capital[6].

In addition to the human capital, the household also has an amount of raw labor ω which is exogenous and constant. One possible formulation of the total net wage flow at time t is

$$W_1(t) = [1 - s(t)]\left[(1 - \tau_H)w_H(t)h(t) + (1 - \tau_L)w_L(t)\omega\right], \tag{14}$$

where τ_L and τ_H are taxes on w_L and w_H, respectively. This formulation means that the individual withdraws both human capital and raw labor from the labor market when he spends time in human capital accumulation. One possible interpretation of this case is that both types of skills are tied to the same individual. If he spends more time in education it will also imply that his less qualified abilities will be applied to work fewer hours. In this case the human capital supplied to the labor market is $H = (1 - s)h$ and the supply of raw labor is $L = (1 - s)\omega$.

An alternative formulation would interpret the household to consist of two categories of people. One part of them (ω) does not have the ability to acquire human capital, while the others do. This gives the modified household net wage flow:

$$W_2(t) = [1 - s(t)]\,(1 - \tau_H)w_H(t)h(t) + (1 - \tau_L)w_L(t)\omega. \tag{15}$$

This formulation means that no raw labor is involved in schooling and thus $L = \omega$.

Let the asset of the household be denoted $\mathcal{A} \equiv VN$. It changes according to the equation

$$\dot{\mathcal{A}} = (1 - \tau_r)r\mathcal{A} + W_i - C, \qquad i = 1, 2$$

[5]This part of the model builds on Ben-Porath (1967) and Acemoglu (2009), ch. 10. Compared to Ben-Porath (1967) it is simplified. For instance, he assumes additional costly inputs in the production function for human capital.

[6]Acemoglu (2009) suggests the following interpretation of the depreciation rate: 'for example because new machines and techniques are being introduced, eroding the existing human capital of the worker'. Alternatively, the depreciation could arise from the turnover (deaths and births) in the members of the dynasty.

where W_i refers to (14) or (15), and τ_r is the tax on the interest rate. The utility in (12) is maximized subject to this constraint, equation (13) and some initial values $\mathcal{A}(0)$ and $h(0)$.

The necessary conditions for utility maximization are derived in Appendix A.2. First, we have

$$g_C = \frac{(1-\tau_r)r - \rho}{\theta}. \tag{16}$$

This is the familiar Euler equation for the case with a tax on the capital income.

If the wage flow in (14) applies, the conditions for optimal schooling imply that

$$\epsilon(z)g_z = (1-\tau_r)r + \delta_h - \frac{(1-\tau_H)w_H h g_{w_H} + (1-\tau_L)w_L\omega\left(g_{w_L} - g_h\right)}{(1-\tau_H)w_H h + (1-\tau_L)w_L\omega}$$
$$-\phi'(z)\left(\frac{(1-\tau_H)w_H h + s(1-\tau_L)w_L\omega}{(1-\tau_H)w_H h + (1-\tau_L)w_L\omega}\right), \tag{17}$$

where we have defined $z \equiv sh$ and $\epsilon(z) \equiv -\frac{\phi''(z)z}{\phi'(z)} > 0$. If instead the wage flow in (15) is regarded as more reasonable, the corresponding equation is

$$\epsilon(z)g_z = (1-\tau_r)r + \delta_h - g_{w_H} - \phi'(z). \tag{18}$$

An important difference between the two cases is that the taxes on human capital and on raw labor are absent in the latter equation. To see the reason for this, note that spending time in schooling has benefits as well as costs: foregone wage incomes now and higher wage incomes in the future, respectively. In (18) the income taxes affect benefits and costs in the same proportion, as noted by Lucas (1990) and Jaimovich and Rebelo (2016)[7].

In the former case, the derivatives of the Hamiltonian with respect to s and h are not affected by the taxes on wages to the same extent. On the one hand, the cost of marginal schooling is higher (compared to the case with (15)), because it implies a reduction of income from unskilled labor. On the other hand, the marginal effect of h does not depend on ω. Therefore the wage taxes are not cancelled out from (17).

[7]See also Ben-Porath (1967), p. 356, for a more explicit exposition of this neutrality result.

5 Balanced Growth

Because of the strict concavity of the function ϕ, h eventually stops growing. On a Balanced Growth Path (BGP) we therefore have $g_h = g_s = 0$. An immediate implication of this is that (13) can be solved for

$$h = \phi(z)/\delta_h. \tag{19}$$

By equations (5) and (6), we have $g_Y = g_N = g_{w_H} = g_{w_L}$ on the BGP. Moreover, by (8) Y and C grow at a common rate. Using (16) we therefore have

$$g_Y = g_N = g_{w_H} = g_{w_L} = g_C = \frac{(1-\tau_r)r - \rho}{\theta}. \tag{20}$$

The equilibrium interest rate, to be used in (20), is determined by equalling supply and demand for human capital. We first derive the demand and then the supply.

5.1 Demand for human capital

To obtain the demand for H_Y we first need an expression for V. On a BGP, where H_Y is constant, we have a constant π according to (4). Likewise, r is constant so (10) can be simplified to $V = \pi/r$. Using (4) and substituting the result into (11):

$$H_Y = \frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{\eta(1-\alpha-\beta)(\alpha+\beta)} \cdot r. \tag{21}$$

This is the equilibrium allocation of human capital to ordinary production. It can also be regarded as the demand for human capital in ordinary production.

Turning to the innovation sector, equations (9) and (20) can be combined to

$$H_R = \frac{(1-\tau_r)r - \rho}{\eta\theta}. \tag{22}$$

This is the demand for human capital in research, in the sense that it is the quantity required for a given growth rate of N.

Total demand for human capital is $H_D = H_R + H_Y$, which now can be written as

$$H_D = \left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)} + \frac{(1-\tau_r)}{\theta}\right) \cdot \frac{r}{\eta} - \frac{\rho}{\eta\theta} \tag{23}$$

This means that the demand for human capital is unambiguously increasing in the interest rate. The explanation for the positive relation between r and H_Y is that the higher interest lowers the present value of profits for innovators, V, thus shifting the allocation of human capital away from research to current production of output. A counteracting effect is that the higher growth rate of C and thus N, that follow from an increased interest rate, necessitates more human capital in research. This is a quite mechanical effect via the production function of blue-prints for new machines and less of a response to changed incentives.

5.2 Supply of human capital

We now turn to the supply of human capital, considering the two different wage cases in turn.

5.2.1 Case 1

Imposing the BGP condition $g_h = g_s = 0$ and using (20), equation (17) is modified to[8]

$$\phi'(sh)(1-\Psi) = \frac{(\theta-1)(1-\tau_r)r+\rho}{\theta} + \delta_h, \tag{24}$$

where

$$\Psi \equiv \frac{(1-s)(1-\tau_L)w_L\omega}{(1-\tau_H)w_H h + (1-\tau_L)w_L\omega} = \frac{(1-s)(1-\tau_L)\alpha H_Y}{(1-\tau_H)\beta(1-s)h + (1-\tau_L)\alpha H_Y}.$$

In the second equality we have used $w_H = w_L\beta(1-s)\omega/(\alpha H_Y)$ from equations (6) and $L = (1-s)\omega$. Note the $0 < \Psi < 1$.[9] Note also that Ψ includes general-equilibrium effects. The analysis here is thus not a pure analysis of the supply of human capital.

[8] An alternative version of (24) is

$$\phi'(hs)\left(1 - \frac{(1-s)(1-\tau_L)w_L\omega}{(1-\tau_H)w_H h + (1-\tau_L)w_L\omega}\right) + g_{w_H} = (1-\tau_r)r + \delta_h.$$

The left-hand side constitutes the gains of human capital accumulation while the costs are found on the right-hand side. In Case 2, when $\Psi = 0$, this is reduced to $\phi'(hs) + g_{w_H} = (1-\tau_r)r + \delta_h$.

[9] According to Appendix A.2.1 it is necessary, for fulfillment of the TVC, that $\delta_h - \phi'(hs)(1-\Psi) < 0$. This implies that $(1-\tau_r)r - g_{w_H} > 0$.

The household makes decisions about s and h based on (19) and (24). The former implies

$$\mathrm{d}s = \frac{1}{h}\left(\frac{\delta_h}{\phi'} - s\right)\mathrm{d}h. \tag{25}$$

Note that $\delta_h > \phi' s$, which means that s and h change in the same direction (when we compare 'steady-state' levels).[10] In Appendix A.3 (A.3.1) we compute the differential of (24). Then (in A.3.2) we compute the derivatives describing how the household's supply of h changes in response to changes in r and various parameters.

The signs of these derivatives are determined under the assumptions that $\theta > 1$ and $\psi'' < 0$.[11] Moreover, we must assume that

$$\frac{\phi'}{\phi''}\frac{1}{h}\frac{\phi'}{\delta_h}\left(\Psi\frac{1}{(1-s)^2}\left(1-\frac{\delta_h}{\phi'}\right)-1\right) < \frac{(1-\tau_H)\beta h}{(1-\tau_L)\alpha H_Y} + \frac{s}{1-s}. \tag{26}$$

It seems quite plausible that this inequality would hold.

The first results of these computations concern the effect on the human capital stock from changes in taxes on work. It turns out that

$$\frac{\partial h}{\partial \tau_H} < 0 \qquad \text{and} \qquad \frac{\partial h}{\partial \tau_L} > 0.$$

It is clearly expected that a higher tax on human capital decreases h and that a higher tax on raw labor has an opposite effect. We also find that

$$\frac{\partial h}{\partial r} < 0, \qquad \frac{\partial h}{\partial \tau_r} > 0 \qquad \text{and} \qquad \frac{\partial h}{\partial \rho} < 0.$$

The intuition for the negative effect of r on h is that the higher discount rate reduces the present value of future incomes from a large stock of human capital. It therefore becomes optimal to spend more of the current time at work and less time in schooling. The middle effect is particularly interesting: a higher tax on savings raises the stock of human capital, which should contribute to a higher growth rate. However, there is of course a counteracting effect that directly reduces the growth rate. We return to these two opposing effects below. Finally, a higher ρ means that the utility

[10] At the intersection the slope of the depreciation term, $\delta_h h$, is larger than the investment term, $\phi(sh)$. This means that $\delta_h > \phi' \cdot s$.

[11] Empirical research mostly points in the direction of θ being larger than 1, implying a low intertemporal elasticity of substitution in consumption. See for instance, Hall (1988), Attanasio and Weber (1993) and Hahm (1998).

of future consumption is discounted heavier, which lowers the value of the extra income that more human capital would generate in the future. Finally

$$\frac{\partial h}{\partial \varsigma_R} > 0, \qquad \frac{\partial h}{\partial \varsigma_x} > 0 \qquad \text{and} \qquad \frac{\partial h}{\partial \eta} > 0.$$

These three parameters improve the conditions for research, when they are increased. This raises the wage of human capital, which induces an increase in the stock.

The supply of human capital to the labor market is $H_S^1 = (1-s)h = h - sh$, where $h = h(r,\xi)$, ξ is a vector of parameters and $s = s(h(r,\xi))$. Thus

$$H_S^1(r,\xi) = h(r,\xi) - s(h(r,\xi))h(r,\xi). \tag{27}$$

Differentiating with respect to the interest rate and using (25)

$$\frac{\partial H_S^1(r,\xi)}{\partial r} = \left(1 - \frac{\delta_h}{\phi'}\right)\frac{\partial h}{\partial r} < 0.$$

By Appendix A.2.1 we know that it is necessary that $\delta_h + \phi'\Psi < \phi'$ to avoid overaccumulation of human capital, and thus to fulfill the transversality condition. It is therefore clear that $\phi'(z) > \delta_h$.[12] Thus H_S^1 changes in the same direction as h. Similarly,

$$\frac{\partial H_S^1(r,\xi)}{\partial \xi} = \left(1 - \frac{\delta_h}{\phi'}\right)\frac{\partial h}{\partial \xi}$$

for any parameter ξ. This means that H_S^1 changes in the same direction as h in response to a parameter change.[13]

[12]Intuitively, a kind of Golden rule is violated if $\phi'(z) > \delta_h$ is not fulfilled. To see this, note that H can be written as $H(z) = \frac{\phi(z)}{\delta_h} - z$. Differentiation gives $H'(z) = \frac{\phi'(z)}{\delta_h} - 1$. It would be wasteful to increase z (through s or h) so far that this expressions becomes negative, because H could then be increased through a reduction of z by lowering s, at no cost in terms of lost human capital.

[13]In the simpler case, when equation (18) replaces (17), the effect is that $\Psi = 0$ in (24). This in turn means that the supply of human capital is (on the BGP) neither influenced by the taxes on labor, nor of the subsidies to research and machines. See Appendix B.2.1 for details.

6 Comparative statics

6.1 Case 1

For an equation describing the equilibrium between supply and demand for human capital we now combine equations (23) and (27) and get

$$H_D(r(\xi), \xi) = H_S^1(r(\xi), \xi). \tag{28}$$

As argued above, the right-hand side is negatively sloping in r, while the left-hand side is positively sloping. Any equilibrium interest rate is therefore unique.

The interest rate is an implicit function of any parameter ξ. The effect of a change in ξ is

$$\frac{\partial r}{\partial \xi} = \Delta \left[\frac{\partial H_S^1}{\partial \xi} - \frac{\partial H_D}{\partial \xi} \right], \quad \text{where} \quad \Delta \equiv \left[\frac{\partial H_D}{\partial r} - \frac{\partial H_S^1}{\partial r} \right]^{-1} > 0. \tag{29}$$

The positive sign of Δ follows from the slopes of the supply and demand curves.

Below follows a discussion about the effects of changes in various policy parameters, on r and on other endogenous variables. Appendix A.4 shows the derivations.[14] We are primarily interested in the effects on the interest and growth rates but it is also interesting to track the effects on the quantities of human capital and on the relative wage. This is important because a policy change that raises growth does not necessarily lead to a larger stock of human capital. The higher growth rate may be due to a reallocation of human capital toward research, while the total stock is decreasing. This may happen if the intertemporal elasticity of substitution in consumption is low, so that the income effect dominates the household's schooling decision.

Labor taxes The policy parameters τ_H and τ_L are only found on the supply side of the market for human capital. We therefore have the simple derivatives

$$\frac{\partial r}{\partial \tau_H} = \Delta \left(1 - \frac{\delta_h}{\phi'} \right) \frac{\partial h}{\partial \tau_H} < 0 \qquad \text{and} \qquad \frac{\partial r}{\partial \tau_L} = \Delta \left(1 - \frac{\delta_h}{\phi'} \right) \frac{\partial h}{\partial \tau_L} > 0.$$

The higher tax on human capital lowers the supply of it, which means that the supply curve shifts in, as can be seen in Figure 1. This results in a lower

[14] The corresponding results for Case 2 (including comments) are found in Appendix B.2.2.

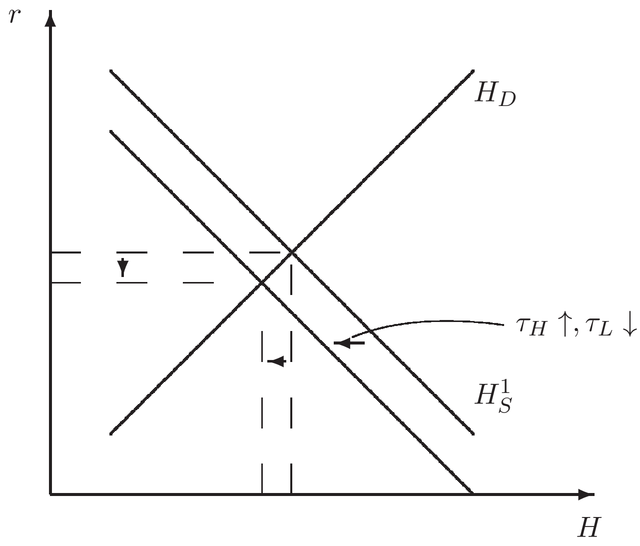

Figure 1: Labor taxes

equilibrium interest rate. According to (20) this higher τ_H therefore slows down the rate of growth. This finding confirms the hypothesis, set out in the introduction, that *worsened incentives to acquire human capital do indeed lower the rate of economic growth*.

The mechanism through which the tax on human capital is transmitted to the rate of growth is of course via the quantity and (relative) price of human capital. From equations (21) and (22) it directly follows that the lower r makes both H_Y and H_R decline. This is a movement along the human-capital demand curve in Figure 1. Furthermore, the Appendix shows that the higher τ_H raises its relative wage, w_H/w_L. It is of course entirely expected that the reduced supply of human capital raises its relative compensation since the demand curve is unchanged.

The converse result arises if there is an increase in τ_L. Such a modification of the tax policy of course makes it more attractive to accumulate

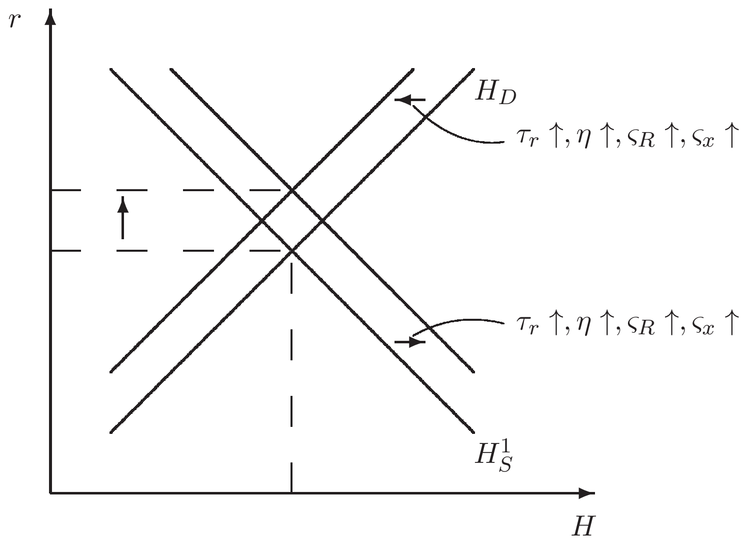

Figure 2: Both curves shifting

human capital. This results in higher equilibrium interest and growth rates, as well as larger quantities of H_Y and H_R.

Tax on savings The equilibrium interest rate increases as the tax on savings gets higher, i.e.

$$\frac{\partial r}{\partial \tau_r} > 0.$$

As noted above, a higher τ_r would induce the household to get more human capital, because it implies a lower net interest rate and thus raises the present value of (higher) future income. This in turn means that the supply curve shifts out. Meanwhile, there is a negative effect from τ_r on the growth rate (at given r) and thus on H_R. This shifts the demand curve in, as illustrated in Figure 2. Both these changes lead to an unambiguously higher equilibrium interest rate.

This raises the question whether the positive effect on r could be so strong that the higher τ_r even increases the growth rate (thus contradicting intuition). The result of differentiating the growth rate in (20) with respect to τ_r is

$$\frac{\partial g_Y}{\partial \tau_r} = \left[\frac{(1-\tau_r)}{r} \frac{\partial r}{\partial \tau_r} - 1 \right] \frac{r}{\theta} < 0.$$

If there were no effect on the interest rate via human capital, there would just be the familiar direct (and negative) effect $-r/\theta$. However, this is now counteracted by a significant effect in the opposite direction. As demonstrated in Appendix A.4.2 (A.4), however, the total effect is negative although smaller (in absolute terms) than in the basic model with exogenous human capital.

Concerning the equilibrium quantities of human capital, the fact that $H_R = g_Y/\eta$ first implies that the net effect on H_R is negative as well. On the other hand, the higher interest rate raises H_Y and it turns out that the total effect on H_D (and thus H_S^1) in equilibrium is ambiguous. (Therefore, the equilibrium H is (roughly) left unchanged in Figure 2.) It turns out that the magnitude of θ is important for the direction of change in total human capital. If $\theta = 1$ the derivative is negative. In this special case the supply of human capital does not change in response to changes in τ_r. The first impact, that reduces H_R, will then dominate the effect on w_H/w_L and reduce it, unless the elasticity of h with respect to r (on the supply side) is very high in absolute terms. If the latter holds the sign may be reversed, due to the reduction of human capital supply when the interest rate increases.

For $\theta > 1$ there is an addition of a positive effect on H_D (and H_S^1) from a higher τ_r. The important additional effect here is that the supply of human capital now increases when τ_r becomes higher. This tends to lower w_H/w_L and thus increase the demanded quantity. To explain why the households chooses more human capital when the relative wage falls, it should be recalled that the intertemporal elasticity of substitution is (very) low in this case, so that the income effect comes to dominate the decision on how much to invest in human capital. Because the (relative) wage of human capital gets lower the household decides that it cannot afford low human capital investments.

Higher subsidy to research If the rate of subsidy to research is increased, there will be a higher equilibrium interest rate in the economy:

$$\frac{\partial r}{\partial \varsigma_R} > 0,$$

and consequently the growth rate will be higher. The supply of human capital is again shifted out, because the improved conditions for research benefit the household as well.[15] On the demand side, H_Y clearly declines when the alternative use of human capital is given more support. At given r, H_R is unchanged and thus H_D definitely shifts in. These two shifts are illustrated in Figure 2, along with the higher equilibrium r.

Although H_R increases because of the higher r in equilibrium, the effect on the total equilibrium quantity of human capital on the labor market is again unclear. Appendix A.4.2 shows that $\partial H_D/\partial \varsigma_R > 0$ for $\theta \leq 1$ (and some range above). Then a higher subsidy to research actually leads to more human capital on the labor market. Again, however, a very high θ could reverse this result, implying that the higher growth rate comes about through a reallocation of a smaller stock towards research.

The derivative $\frac{\partial}{\partial \varsigma_R}\left(\frac{w_H}{w_L}\right)$ is ambiguous. The outward shift in the supply of human capital tends to lower $\frac{w_H}{w_L}$, while the shift in demand, away from H_Y and toward H_R, tends to increase it. It is possible that this derivative is positive even when $\theta = 1$ and a change in the positive direction gets more likely when θ increases. If this occurs in parallel with a lower total supply of human capital, it is again the income effect that dominates: the supply is reduced because it can be afforded when the relative wage of human capital

[15] On impact w_H/w_L increases.

increases.[16]

7 Numerically calculated effects of parameter changes on a steady state

In order to further investigate the properties of the model, especially its reaction to parameter changes, the analysis proceeds with numerical simulation. The model is calibrated and solved for the steady state. Parameter values are chosen so that they, as well as steady state values of endogenous variables, take values recognizable from macroeconomic data or from the literature of the field. Numerical derivatives of endogenous variables with respect to parameters are then calculated around the steady state.

7.1 Determination of the steady state

The steady state of the model can be calculated by solving a system of four simultaneous equations in four variables. The rest of the steady state variables then follow recursively, i.e. from one equation at the time.

The general function ϕ is specified to $\phi(z) = \phi_0 z^\gamma$ with the derivative $\phi'(z) = \phi_0 \gamma z^{\gamma-1}$, where ϕ_0 is a positive constant and γ is a constant between zero and one. Equation (19) then becomes:

$$\phi_0 (sh)^\gamma = \delta_h h$$

With this specification, the four steady state conditions (19), (21), (24) and (28) simultaneously determine four variables: the interest rate r, schooling s, total human capital h, and human capital in final goods production, H_Y.

From these four equations is derived a single equation of the form $f(s) = 0$ in only one variable, s. Since s is defined to lie in the interval $0 < s < 1$, search for a solution is limited to the unit interval. After determining s numerically, all other variables may be computed analytically in a recursive way. For numerical purposes, the advantage of this procedure is that, while methods searching for solutions to systems of equations in multidimensional

[16] The effect of a change in ς_x is left out because this parameter appears very much like ς_R in all expressions, with very similar effects.

space may sometimes miss the solution, with only one dimension, it's possible to bracket the solution within an interval that is gradually reduced by the numerical algorithm, so that convergence is assured.

The single equation in schooling of the form $f(s) = 0$ is derived from the four steady state equations above as follows.

Given s, equation (19) is solved for h:

$$h(s) = \left(\frac{\phi_0 s^\gamma}{\delta_h} \right)^{1/(1-\gamma)} \tag{30}$$

Then $h(s)$, along with (23) and (27), are substituted into equation (28), which is solved for r:

$$r(s) = \frac{(1-s)\eta h(s) + \rho/\theta}{\frac{\beta(1-\zeta_R)(1-\zeta_x)}{(1-\alpha-\beta)(\alpha+\beta)} + \frac{(1-\tau_r)}{\theta}} \tag{31}$$

Now, $r(s)$ is substituted into equation (21):

$$H_Y(s) = \frac{\beta(1-\zeta_R)(1-\zeta_x)}{\eta(1-\alpha-\beta)(\alpha+\beta)} r(s) \tag{32}$$

Finally, ϕ', $h(s)$, $r(s)$ and $H_Y(s)$ are substituted into equation (24), using the second expression for Ψ. Subtracting the right hand side from the left, noting that $\phi'(sh) = \gamma\delta_h/s$, yields an equation of the form $f(s) = 0$.

$$f(s) = \frac{\gamma\delta_h}{s} \left(1 - \frac{(1-s)(1-\tau_L)\alpha H_Y(s)}{(1-\tau_H)\beta(1-s)h(s) + (1-\tau_L)\alpha H_Y(s)} \right) - \frac{(\theta-1)(1-\tau_r)r(s) + \rho}{\theta} - \delta_h = 0 \tag{33}$$

Equation (33), is solved for schooling (s) by the single equation solver `fzero()` in the mathematical software package *Octave*. Given schooling, total human capital h, the interest rate r, and human capital in final goods production, H_Y, follow recursively from (30), (31), and (32) respectively.

Further endogenous variables, such as the growth rate of output, g_Y, human capital in research, H_R, total demand for human capital, H_D, the relative wage, w_H/w_L, the capital-output ratio, X/Y, and consumption share of output, C/Y, then follow from equations (20), (22), (23), (6), (7), and (8), respectively.

In addition to the conditions above, the two transversality conditions

$$(\theta - 1)g_Y + \rho > 0$$

and
$$\frac{\gamma\delta_h}{s}\left(1-\frac{(1-s)(1-\tau_L)\alpha H_Y}{(1-\tau_H)\beta(1-s)h+(1-\tau_L)\alpha H_Y}\right)-\delta_h>0$$
must also hold. The latter condition is implied by (24).

7.2 Calibration

Table 1 below shows the parameter values for the base line scenario. Most of the parameters were set to resemble corresponding values in macroeconomic data, or values of parameters that play a similar role in the literature of the field.

Parameter	Value	Description
α	0.33	income share of labor
β	0.33	income share of human capital
η	0.20	productivity in innovation
γ	0.80	elasticity of human capital production function
ϕ_0	0.14	productivity in human capital production
δ_h	0.08	depreciation rate of human capital
θ	1.50	household intertemporal elasticity of substitution
ρ	0.01	household subjective discount rate
ω	1.50	supply of raw labor
τ_r	0.20	tax rate on capital income
τ_L	0.30	tax rate on labor income
τ_H	0.50	tax rate on human capital
ζ_x	0.00	subsidy to capital
ζ_R	0.00	subsidy to research

Table 1: Parameters in the base line scenario

Mankiw, Romer and Weil (1992) estimate that about half of the total wage sum is due to human capital. Equal values for α and β yields this result in the model. Lucas (1990) uses a learning elasticity of $\gamma = 0.8$ with an interpretation similar, but not identical, to that of γ in this model. This value is used for γ in the base line scenario, in spite of the slight difference in context. The value of the human capital depreciation rate $\delta_h = 0.08$ is used by Stokey (1996) as well as by He and Liu (2008), and lies within the range 0.04 to 0.09 estimated by Heckman (1976).

Values for the intertemporal elasticity of substitution θ range between 1 and 2 in the literature. He and Liu (2008) use $\theta = 1.5$ (in their case γ), calling it "a standard value used in the literature". Another common value for θ is 2, used for example by Acemoglu et al. (2012), Jones (2016), and Jaimovich and Rebelo (2016). Also common is the logarithmic momentary utility $\theta = 1$, though this value may be used more for its simplicity than for its agreement with data. The base line scenario here uses the intermediate case $\theta = 1.50$. A subjective discount rate of $\rho = 1\%$ is used e.g. by Jaimovich and Rebelo (2016).

The tax rates $\tau_H = 50\%$, $\tau_L = 30\%$, and $\tau_r = 20\%$ are set to roughly correspond to Swedish marginal tax rates. The subsidies ζ_x and ζ_R are set to zero in the base line scenario.

The productivity in innovation η and in human capital production ϕ_0 are adjusted to obtain values for the growth rate g_Y and the real interest rate r that resemble those typically found in macroeconomic data. In other models, parameters corresponding to these may often be set arbitrarily without loss of generality. This is the case e.g. in Jaimovich and Rebelo (2016) and in Jones (2016).

The supply of raw labor ω influences only the wage payed to human capital relative to the wage to raw labor. The value $\omega = 1.50$ is chosen to set the relative wage w_H/w_L to somewhere midway between 2 and 3.

7.3 Results for the base line scenario

The model is solved for the steady state with the base line parameters. Some of the resulting steady state variables and ratios are listed in table 2.

Variable	Value	Description
g_Y	0.0170	growth rate of production
r	0.0443	interest rate
s	0.4674	schooling
h	0.7831	human capital
H_Y	0.3323	human capital in final goods production
H_R	0.0848	human capital in research
w_H/w_L	2.4044	relative wage
X/Y	0.3333	capital output ratio
C/Y	0.8889	consumption share of output

Table 2: Some endogenous variables in the steady state

The growth rate at 1.7% is at a level typical of industrialized economies today. It may be considered somewhat optimistic, considering it should be sustainable in the very long run. The real interest rate of 4.4% may seem a bit high if interpreted as the risk free real interest rate, but should rather be compared to the average return to capital according to the national accounts, whether risk free or not. Quite large a share of an individual's time, 47%, is devoted to schooling. Most of employed human capital, 0.33, is used in goods production, while considerably less, 0.085, is engaged in innovative activities. The consumption share of output is 89%. This is rather high if output is taken to be GDP, but the interpretation of production Y is somewhat open. It may, for example, be interpreted as NDP, in which case the consumption share wouldn't seem so high.

7.4 Comparative static analysis

The numerical derivatives of endogenous variables with respect to parameters are calculated at the steady state as quotients of differences, and presented in Table 3 as a transposed jacobian. The derivative of endogenous variable x with respect to parameter p is approximated by

$$\frac{dx}{dp} \approx \frac{x_1 - x_0}{p_1 - p_0}$$

where p_0 is the parameter value for the baseline scenario, x_0 is the corresponding steady state value of the endogenous variable, $p_1 = p_0 + \Delta p$ is the incremented parameter, and x_1 the steady state value obtained from the parameter value p_1 and all other parameters at their original values. An increment of $\Delta p = 0.00001$ is used.

	g_Y	r	s	h	H_Y	H_R	w_H/w_L
α	-0.0612	-0.1148	-0.2400	-1.6082	-0.3626	-0.3061	-3.5064
β	-0.0263	-0.0493	0.3568	2.3916	1.1258	-0.1314	-2.5438
η	0.0748	0.1402	-0.2368	-1.5868	-0.6097	-0.0501	5.4806
γ	0.0136	0.0256	0.8455	1.7314	0.1918	0.0682	-5.2051
ϕ_0	0.5342	1.0017	-1.6912	16.6326	7.5126	2.6712	-46.7173
δ_h	-0.7681	-1.4402	4.0992	-21.4643	-10.8013	-3.8405	59.6747
θ	-0.0192	-0.0148	-0.0744	-0.4986	-0.1112	-0.0961	1.1406
ρ	-1.1329	-0.8742	-4.3859	-29.3907	-6.5568	-5.6646	67.2581
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.6030
τ_r	-0.0182	0.0212	0.0243	0.1632	0.1590	-0.0911	-1.2605
τ_L	0.0187	0.0351	0.1282	0.8591	0.2635	0.0937	-2.4854
τ_H	-0.0262	-0.0492	-0.1795	-1.2027	-0.3689	-0.1312	3.4797
ζ_x	0.0146	0.0273	-0.0195	-0.1306	-0.1272	0.0729	1.0084
ζ_R	0.0146	0.0273	-0.0195	-0.1306	-0.1272	0.0729	1.0084

Table 3: Numerical derivatives around the steady state

Most of these derivatives have straightforward interpretations as partial derivatives, i.e. the effects on endogenous variables in the steady state from small changes in one parameter at the time. The parameters of the goods production function, α and β, however, also have effects on ψ, the marginal cost of the intermediate input, defined as $\psi = 1 - \alpha - \beta$. Therefore, there are no derivatives with respect to ψ in the table, and the derivatives with respect to α and β are combined effects from changes in the production technology for intermediate and final goods.

The derivatives of the ratios X/Y and C/Y do not appear in Table 3 since they only depend on the parameters α, β, and ζ_x, and their derivatives are easily obtained from equations (7) and (8) respectively.

The numerical derivatives in table 3 may be compared with the analytical results. Since the chosen parametrization satisfies condition (26), the numerical results presented here have the signs analytically determined in section 6.

The effects of policy parameters, i.e. taxes and subsidies, are of particular interest, and especially effects from τ_H, the tax on human capital income.

The direction of influence from labor taxes τ_H and τ_L on the total amount of human capital h and the growth rate g_Y are analytically determined, as is the effect from the capital income tax τ_r on growth. Thus $\partial h/\partial \tau_H < 0$, $\partial g_Y/\partial \tau_H < 0$, $\partial h/\partial \tau_L > 0$, $\partial g_Y/\partial \tau_L > 0$, and $\partial g_Y/\partial \tau_r < 0$.

As shown in section 6, the sign of the effect from the capital income tax τ_r on total human capital h depends on the value of θ. For $\theta = 1$ the effect is negative, but for higher values of θ, the sign may be reversed, which is the case here. For this parametrization, $\partial h/\partial \tau_r > 0$.

The subsidies to capital, ζ_x, and to research, ζ_R, enter symmetrically in the steady state conditions, so the derivatives of each endogenous variable in table 3 with respect to the two subsidies are equal. The sign of the effect from subsidies on growth is analytically determined to be positive: $\partial g_Y/\partial \zeta_R > 0$, while the effect on total human capital depends on the parameters. For this parametrization, $\partial h/\partial \zeta_R < 0$.

The effects from taxes and subsidies on schooling, s, all go in the same direction as those on human capital, h.

The effects on human capital accumulation and output growth, from an increase in the tax rate on human capital income, are quite substantial. For example, an increase in τ_H by 5 percentage points, from 0.50 to 0.55, reduces the total stock of human capital h by 7.7% and the growth rate g_Y by 0.13 percentage points, (a reduction by 7.7%).

This negative effect on growth, from an increase in the tax on human capital return, is stronger the higher the tax rate is to begin with. Figure 3 shows steady state values for the growth rate for different tax rates on human capital income. It is seen that the growth rate g_Y decreases in the tax τ_H at an increasing rate. This result is similar to that in Jaimovich and Rebelo (2016), though the non-linearity of their model is more pronounced. It is also seen that for very high tax rates, the growth rate turns negative, and for even higher tax rates, no steady state exists.[17]

[17] This will be explained further in Figure 6 below.

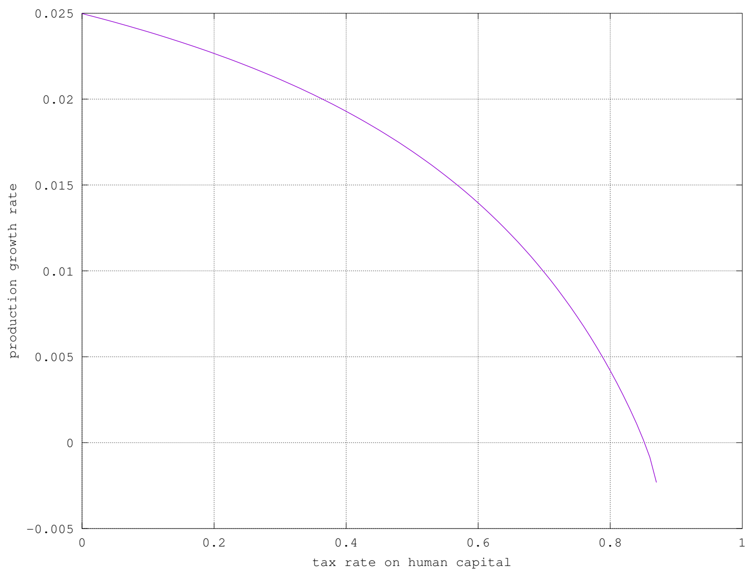

Figure 3: Growth as a function of the tax on human capital.

The effect that the tax on human capital return has on the growth rate depends on the parameters, of course, and is particularly sensitive to variations in the inverse of the intertemporal elasticity of substitution, θ. Figure 4 illustrates this dependence by showing how the partial derivative $\partial g_Y/\partial \tau_H$ decreases in absolute value with higher values of θ.

7.5 Existence and uniqueness of steady state solutions

As shown in section 6, the steady state of the model is unique provided that condition (26) is satisfied. It turns out, however, that the set of equations from which the steady state is calculated, typically have more than one solution. This does not mean that there are more than one steady state.

The function $f(s)$, defined on the unit interval, starts and ends on the same negative value. More precisely, the value $f(0)$ is not defined, since schooling s occur in the denominator, but $f(s)$ has a well defined limit as $s \to 0$ which is negative and equal to $f(1)$:

$$\lim_{s\to 0} f(s) = f(1) = -\frac{\rho}{\theta}\left(\frac{(\theta-1)}{\frac{\beta(1-\zeta_R)(1-\zeta_x)}{(1-\alpha-\beta)(\alpha+\beta)(1-\tau_r)}\theta+1}+1\right) - (1-\gamma)\delta_h < 0$$

Therefore, the number of zeros, if any, must be even. For reasonable sets of parameter values, there are typically two.

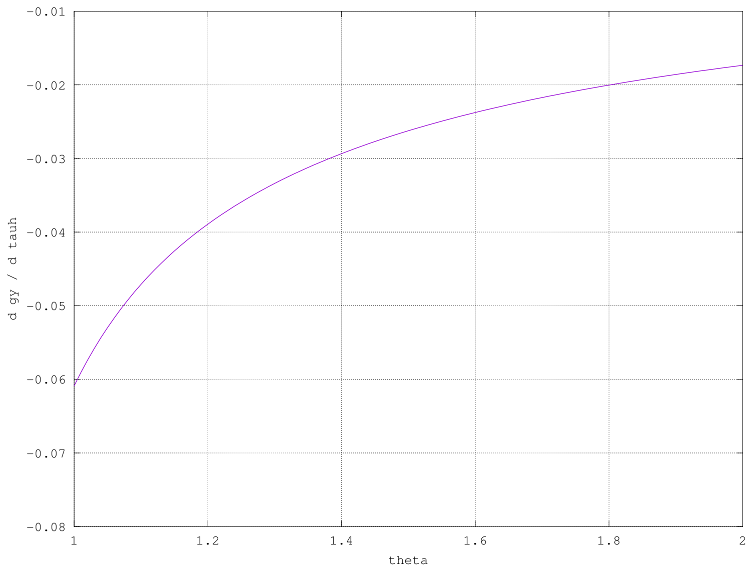

Figure 4: $\partial g_Y / \partial \tau_H$ as a function of θ

Figure 5 below shows a plot of the function $f(s)$ over the unit interval for the parameter values in the base line scenario. As seen in the figure, the function $f(s)$ has two zeros.

Both solutions satisfy the transversality conditions. The second zero, where f intercepts the horizontal axes from above, represents the steady state described above. The first zero, where f intercepts the horizontal axes from below, does not represent a valid steady state of the model.

The function $f(s)$ may be interpreted as the difference between the marginal benefits of schooling and the marginal costs. If $f(s) < 0$, the marginal costs of schooling outweighs its benefits, so schooling should be reduced, and vice versa. A solution where the function intercepts the horizontal axes from below is therefore unstable, since a deviation in either direction will induce further deviations away from the zero point.

Table 4 below give values of endogenous variables calculated from the first solution, where $f(s)$ intercepts the horizontal axes from below. The negative value of H_R, the amount of human capital in research, is not meaningful, therefore this solution cannot represent a valid steady state.

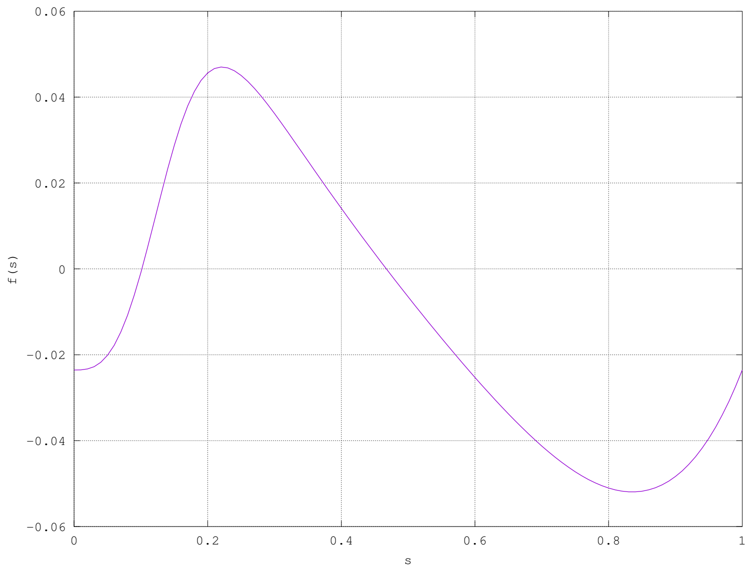

Figure 5: The function $f(s)$ on the interval $0 < s < 1$

Variable	**Value**	**Description**
g_Y	-0.0048	growth rate of y, c, n, wh, wl
r	0.0034	interest rate
s	0.1015	schooling
h	0.0017	human capital
H_Y	0.0257	human capital in final goods production
H_R	-0.0242	human capital in research
w_H/w_L	52.3495	relative wage
X/Y	0.3333	capital output ratio (?)
C/Y	0.8889	consumption share of output

Table 4: Some endogenous variables in the steady state

For extreme parameter values, there may be no solution to the steady state equations, and hence no steady state. An example is constructed from the base line scenario by raising the tax on return from human capital to $\tau_H = 90\%$. Figure 6 shows the function $f(s)$ in this case. Since a steady state requires that $f(s) = 0$, and the function $f(s)$ can be seen to be negative for all levels of schooling s in the unit interval $0 \leq s \leq 1$, there can be no steady state. This is the reason why the growth rate in Figure 3 is not plotted for tax rates $\tau_H > 88\%$.

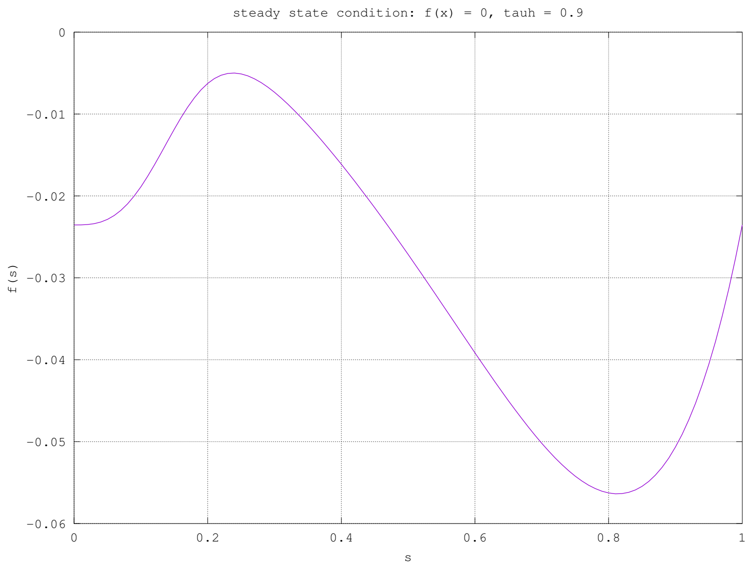

Figure 6: The function $f(s)$ on the interval $0 < s < 1$

8 Conclusions

Innovations and human capital are important for economic growth. This paper builds a growth model in which both the acquirement of human capital and innovations are endogenous, and where some of the human capital is used in research. We examine the incentives to do research and accumulate human capital. In particular, we are interested the role of economic policy for those activities and consequently for economic growth.

We formulate the model as a Romer model, with the important difference that human capital is endogenous. Although endogenous, the stock of human capital is constant on the balanced growth path.

In contrast to many other growth models we find that the taxation on human capital has a substantial negative effect on the accumulation of it. This in turn lowers the income growth rate. While subsidies to research and to intermediate inputs have positive effects on growth (and must be strictly positive in social optimum), they do not necessarily imply that there will be larger stock of human capital in the economy. If the elasticity of intertemporal substitution in consumption is sufficiently low, these policy instruments stimulate growth by inducing a reallocation of a *shrinking* stock of human capital in the direction of research.

References

Acemoglu, D. (1998), 'Why do new technologies complement skills? Directed technical change and wage inequality', *Quarterly Journal of Economics* 113(4): 1055–1089.

Acemoglu, D. (2009), *Introduction to Modern Economic Growth*, Princeton University Press, Princeton, NJ.

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous, "The environment and directed technical change," *American Economic Review*, 102 (2012), 131–66.

Philippe Aghion, Ufuk Akcigit and Jesús Fernández-Villaverdex (2013), 'Optimal Capital Versus Labor Taxation with Innovation-Led Growth', *Mimeo*, Penn State University.

Arnold, Lutz G. (1998), 'Growth, Welfare, and Trade in an Integrated Model of Human-Capital Accumulation and Research', *Journal of Macroeconomics* 20(): 81–105.

Attanasio, Orazio P. and Weber, Guglielmo, "Consumption Growth, the Interest Rate and Aggregation," *The Review of Economic Studies* 60 (1993), 631–649.

Barro, Robert J. (1991), 'Economic Growth in a Cross Section of Countries', *The Quarterly Journal of Economics* 106(2): 407–443.

Barro, Robert J. (1998), *Determinants of Economic Growth: A Cross-country Empirical Study*, MIT Press, Cambridge, MA.

Barro, R. J. and X. Sala-i-Martin (2004), *Economic Growth*, 2nd. edition, MIT Press, Cambridge, MA.

Ben-Porath, Yoram (1967), 'The production of human capital and the life cycle of earnings', *Journal of Political Economy* 75(4): 352–365.

Grossman, G. M. and E. Helpman (1991), *Innovation and Growth in the Global Economy*, MIT Press, Cambridge, MA.

Hahm, Joon-Ho, "Consumption adjustment to real interest rates: Intertemporal substitution revisited," *Journal of Economic Dynamics and Control*, 22 (1998), 293–320.

Hall, Robert E., "Intertemporal substitution in consumption," *Journal of Political Economy*, 96 (1988), 339–357.

Hanushek, Eric A. and Ludger Woessmann, "Do better schools lead to more growth? Cognitive skills, economic outcomes, and causation," *Journal of Economic Growth*, 17 (1988), 267–321.

He, Hui and Liu, Zheng, "Investment-specific technological change, skill accumulation, and wage inequality," *Review of Economic Dynamics*, 11(2) (2008), 314–334.

Heckman, J.J., (1976). "A life-cycle model of earnings, learning, and consumption", *Journal of Political Economy* 84(4) (1976), S11–S44.

Jaimovich, Nir and Sergio Rebelo (2016), 'Non-linear Effects of Taxation on Growth', *Journal of Political Economy*, (forthcoming).

Jones, C. I. (1995), 'R&D-based models of economic growth', *Journal of Political Economy*, 103(4): 759–784.

Jones, Charles I. (2016), 'Life and Growth,' *Journal of Political Economy*, 124(2): 539–578.

Jones, Charles I. (2015), 'The Facts of Economic Growth,' *Mimeo, Stanford GSB* (In preparation for the *Handbook of Macroeconomics*).

Lucas, R. E. (1988), 'On the Mechanics of Economic Development', *Journal of Monetary Economics*, 22: 3–42.

Lucas, R. E. (1990), 'Supply-Side Economics: An Analytical Review', *Oxford Economic Papers*, 42(2): 293–316.

Mankiw, N. Gregory, David Romer and David N. Weil (1992), "A contribution to the empirics of economic growth", The Quarterly Journal of Economics 107(2): 407–437

Manuelli, Rodolfo and Ananth Seshadri (2014), 'Human Capital and the Wealth of Nations', *American Economic Review*, 104(9): 2736-62.

Romer, P. M. (1990), 'Endogenous technological change', *Journal of Political Economy*, 98(5): S71–102.

Rebelo Sergio (1991), 'Long-Run Policy Analysis and Long-Run Growth', *Journal of Political Economy*, 99(3): 500-521.

Sala-i-Martin, Xavier, Gernot Doppelhofer and Ronald I. Miller (2004), 'Determinants of Long-Term Growth: A Bayesian Averaging of Classical Estimates (BACE) Approach', *American Economic Review*, 94(4): 813-835.

Stokey, Nancy L. and Sergio Rebelo (1995), 'Growth Effects of Flat-Rate Taxes', *Journal of Political Economy*, 103(3): 519-550.

Stokey, Nancy L., "Free trade, factor returns and factor accumulation", *Journal of economic growth* 1 (4), (1996), 421–447.

A Appendix

A.1 Necessary conditions

A.1.1 Production of Final Output

The profit function in final production is

$$\Pi_Y(t) = \frac{1}{1-\alpha-\beta} L^{\alpha} H_Y(t)^{\beta} \left[\int_0^{N(t)} x(\nu,t)^{1-\alpha-\beta} d\nu \right]$$

$$-w_L(t)L - w_H(t)H_Y - \int_0^{N(t)} (1-\varsigma_x) p^x(\nu,t) x(\nu,t) d\nu. \tag{34}$$

Here the subsidy of x is denoted by ς_x. This firm does not make any intertemporal decisions, so it just chooses the optimal quantities of $\{x(\nu,t)\}_{\nu \in [0,N(t)]}$ and $H_Y(t)$ at every point of time.

For maximization of profits with respect to $x(\nu,t)$ and $H_Y(t)$ it is necessary that

$$x(\nu,t) = \left(\frac{L^{\alpha} H_Y(t)^{\beta}}{(1-\varsigma_x) p^x(\nu,t)} \right)^{\frac{1}{\alpha+\beta}} \tag{35}$$

and

$$\beta \frac{Y(t)}{H_Y(t)} = w_H(t), \tag{36}$$

respectively. All raw labor is employed in this sector, which means that the condition

$$\alpha \frac{Y(t)}{L} = w_L(t) \tag{37}$$

determines the wage that makes supply equal to demand.

A.1.2 Producers of intermediate inputs

The optimal price of $x(\nu,t)$ is obtained by differentiation of (2) with respect to $p^x(\nu,t)$, taking equation (35) into account. This gives $p^x(\nu,t) = \frac{\psi}{1-\alpha-\beta}$, which was simplified to $p^x(\nu,t) = p^x = 1$ in the main text, by the assumption that $\psi = 1-\alpha-\beta$.

A.1.3 Some Implications

Using $p^x(\nu,t) = p^x = 1$ the demand function in (35) is simplified to (3). Similarly, equation (3) and $p^x = 1$ simplify the profits in (2) to (4).

Equations (36) and (37) imply that the wages of the two types of labor are as in equations (6).

A.2 Necessary conditions for the household

The current-value Hamiltonian for the household's optimization problem is

$$\mathcal{H} = M\frac{(C/M)^{1-\theta}-1}{1-\theta} + \lambda\left((1-\tau_r)r\mathcal{A} + W_i - C\right) + \mu\left(\phi\left(h(t)s(t)\right) - \delta_h h(t)\right),$$

where λ and μ are co-state variables. Maximizing with respect to the control variables C and s:

$$(C/M)^{-\theta} = \lambda \tag{38}$$

$$-\lambda\left[(1-\tau_H)w_H h + (1-\tau_L)w_L\omega\right] + \mu\phi'\left(hs\right)h = 0 \tag{39}$$

$$-\lambda(1-\tau_H)w_H + \mu\phi'\left(hs\right) = 0 \tag{40}$$

Equation (39) applies if the wage flow is given by (14), while equation (40) applies if the wage flow is given by (15). In the former case the cost of marginal schooling is higher, because it implies a reduction of income from unskilled labor. The costate variables must change according to

$$\mathcal{H}_{\mathcal{A}} = \lambda(1-\tau_r)r = \rho\lambda - \dot{\lambda} \tag{41}$$

and

$$\mathcal{H}_h = \lambda\left[1-s(t)\right](1-\tau_H)w_H + \mu\phi'\left(hs\right)s - \mu\delta_h = \rho\mu - \dot{\mu}. \tag{42}$$

Note that this latter equation is the same for both types of wage flow. (The marginal effect of h does not depend on ω.) Finally, the transversality conditions are

$$\lim_{t\to\infty} = e^{-\rho t}\mu(t)h(t) = 0. \tag{43}$$

and

$$\lim_{t\to\infty} = e^{-\rho t}\lambda(t)\mathcal{A}(t) = 0. \tag{44}$$

By (38) $g_\lambda = -\theta g_C$. Since $\mathcal{A} = VN$ and V is constant in the long run, $g_{\mathcal{A}} = g_N = g_C$. Thus it is necessary that $-\rho - \theta g_C + g_C < 0$, which clearly is fulfilled for $\theta > 1$.

To develop these expressions, we first note that equations (38) and (41) can be combined, along familiar lines, to equation (16).

A.2.1 Case 1

If the wage flow in (14) applies, equations (39) and (42) can be combined to

$$g_\mu = \rho + \delta_h - \phi'\left(hs\right)\left(1 - \frac{(1-s)(1-\tau_L)w_L\omega}{(1-\tau_H)w_H h + (1-\tau_L)w_L\omega}\right) \tag{45}$$

Differentiating (39) with respect to time and using (41):

$$g_\mu = -\frac{\phi''(hs)\,hs}{\phi'(hs)}(g_h+g_s)+\rho-(1-\tau_r)r+$$

$$+\frac{(1-\tau_H)w_H h g_{w_H}+(1-\tau_L)w_L\omega\,(g_{w_L}-g_h)}{(1-\tau_H)w_H h+(1-\tau_L)w_L\omega}$$

Combining these two equations:

$$-\frac{\phi''(hs)\,hs}{\phi'(hs)}(g_h+g_s)=$$

$$=(1-\tau_r)r+\delta_h-\frac{(1-\tau_H)w_H h g_{w_H}+(1-\tau_L)w_L\omega\,(g_{w_L}-g_h)}{(1-\tau_H)w_H h+(1-\tau_L)w_L\omega}$$

$$-\phi'(hs)\left(1-\frac{(1-s)(1-\tau_L)w_L\omega}{(1-\tau_H)w_H h+(1-\tau_L)w_L\omega}\right) \tag{46}$$

Defining $z\equiv hs$ and $\epsilon(z)\equiv-\frac{\phi''(z)z}{\phi'(z)}>0$, (46) becomes equation (17).

On the BGP h is constant. By (43) it must therefore be that $g_\mu-\rho<0$ asymptotically. Using and (45) it is therefore necessary that

$$\delta_h-\phi'(hs)\left(1-\frac{(1-s)(1-\tau_L)w_L\omega}{(1-\tau_H)w_H h+(1-\tau_L)w_L\omega}\right)<0$$

to avoid overaccumulation of human capital.

A.2.2 Case 2

If the wage flow in (15) applies, equations (40) and (42) are combined to

$$g_\mu=\rho+\delta_h-\phi'(hs)\,. \tag{47}$$

Differentiating (40) with respect to time and using (41):

$$g_\mu=\rho-(1-\tau_r)r+g_{w_H}-\frac{\phi''(hs)\,hs}{\phi'(hs)}(g_h+g_s)$$

We can now replace the rate of change of the costate variable in the above equation:

$$-\frac{\phi''(hs)\,hs}{\phi'(hs)}(g_h+g_s)=(1-\tau_r)r-g_{w_H}+\delta_h-\phi'(hs)\,. \tag{48}$$

The tax on the interest rate influences the schooling decision. Defining $z\equiv hs$ and $\epsilon(z)\equiv-\frac{\phi''(z)z}{\phi'(z)}>0$, (48) becomes (18).

On the BGP h is constant. By (43) it must therefore be that $g_\mu-\rho<0$ asymptotically. Using and (47) it is therefore necessary that $\delta_h-\phi'(hs)<0$ to avoid overaccumulation of human capital.

A.3 Derivatives Case 1

A.3.1 Computing the differential

The differential of (24) is

$$\frac{\phi''\delta_h}{\phi'}\left(1-\Psi\right)\mathrm{d}h-\phi'\mathrm{d}\Psi=\frac{(\theta-1)}{\theta}\left((1-\tau_r)\mathrm{d}r-r\mathrm{d}\tau_r\right), \tag{49}$$

where (25) has been used in the far-left term. The differential of Ψ is in turn

$$\mathrm{d}\Psi=\frac{\partial\Psi}{\partial s}\frac{\partial s}{\partial h}\mathrm{d}h+\frac{\partial\Psi}{\partial h}\mathrm{d}h+\frac{\partial\Psi}{\partial\tau_L}\mathrm{d}\tau_L+\frac{\partial\Psi}{\partial\tau_H}\mathrm{d}\tau_H+\frac{\partial\Psi}{\partial H_Y}\mathrm{d}H_Y.$$

We first focus on the dh terms. Writing $\Psi=N/D$, we first have[18]

$$\frac{\partial\Psi}{\partial s}\frac{\partial s}{\partial h}=\frac{-(1-\tau_L)\alpha H_Y\cdot D+N\cdot(1-\tau_H)\beta h}{D^2}\cdot\frac{1}{h}\left(\frac{\delta_h}{\phi'}-s\right),$$

where (25) has been used again. Next,

$$\frac{\partial\Psi}{\partial h}=\frac{-N\cdot(1-\tau_H)\beta(1-s)}{D^2}.$$

Combining these two terms, we have

$$\frac{\partial\Psi}{\partial s}\frac{\partial s}{\partial h}+\frac{\partial\Psi}{\partial h}=-\frac{(1-\tau_L)\alpha H_Y\cdot D}{D^2}\cdot\frac{1}{h}\left(\frac{\delta_h}{\phi'}-s\right)+$$

$$+\frac{N\cdot(1-\tau_H)\beta h}{D^2}\cdot\frac{1}{h}\left(\frac{\delta_h}{\phi'}-s\right)-\frac{N\cdot(1-\tau_H)\beta(1-s)}{D^2}$$

or

$$\frac{\partial\Psi}{\partial s}\frac{\partial s}{\partial h}+\frac{\partial\Psi}{\partial h}=-\Psi\cdot\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'}-s\right)+\Psi\cdot\frac{(1-\tau_H)\beta}{D}\cdot\left(\frac{\delta_h}{\phi'}-1\right).$$

This can be developed further to

$$\frac{\partial\Psi}{\partial s}\frac{\partial s}{\partial h}+\frac{\partial\Psi}{\partial h}=-\Psi\cdot\frac{1}{h}+\Psi\cdot\frac{D}{(1-s)hD}\left(1-\frac{\delta_h}{\phi'}\right)-\Psi\cdot\frac{(1-\tau_H)\beta(1-s)h}{(1-s)hD}\cdot\left(1-\frac{\delta_h}{\phi'}\right)$$

or

$$\frac{\partial\Psi}{\partial s}\frac{\partial s}{\partial h}+\frac{\partial\Psi}{\partial h}=\Psi\cdot\Psi\frac{1}{(1-s)^2h}\left(1-\frac{\delta_h}{\phi'}\right)-\Psi\cdot\frac{1}{h}.$$

Equation (49) can now be written as

$$\frac{\phi''\delta_h}{\phi'}\left(1-\Psi\right)\mathrm{d}h-\phi'\Psi\left(\Psi\frac{1}{(1-s)^2h}\left(1-\frac{\delta_h}{\phi'}\right)-\frac{1}{h}\right)\mathrm{d}h=$$

[18]This is not the N that has been used for the number of innovations.

$$= \frac{(\theta - 1)}{\theta}\left((1-\tau_r)\mathrm{d}r - r\mathrm{d}\tau_r\right) + \phi'\left(\frac{\partial \Psi}{\partial \tau_L}\mathrm{d}\tau_L + \frac{\partial \Psi}{\partial \tau_H}\mathrm{d}\tau_H + \frac{\partial \Psi}{\partial H_Y}\mathrm{d}H_Y\right).$$

The LHS can be rewritten as

$$\frac{\phi''\delta_h}{\phi'}\left[1 - \Psi \cdot \Xi\right]\mathrm{d}h,$$

where

$$\Xi \equiv \left(1 + \frac{\phi'}{\phi''}\cdot\Psi\frac{1}{(1-s)^2 h}\left(\frac{\phi'}{\delta_h} - 1\right) - \frac{\phi'}{\phi''}\cdot\frac{\phi'}{\delta_h}\frac{1}{h}\right).$$

Before substituting this back into the differential, we examine the conditions for having $\Psi \cdot \Xi < 1$. This inequality is equivalent to

$$1+\frac{\phi'}{\phi''}\cdot\Psi\frac{1}{(1-s)^2 h}\left(\frac{\phi'}{\delta_h} - 1\right) - \frac{\phi'}{\phi''}\cdot\frac{\phi'}{\delta_h}\frac{1}{h} < \frac{(1-\tau_H)\beta(1-s)h + (1-\tau_L)\alpha H_Y}{(1-s)(1-\tau_L)\alpha H_Y}$$

or

$$1+\frac{\phi'}{\phi''}\cdot\Psi\frac{1}{(1-s)^2 h}\left(\frac{\phi'}{\delta_h} - 1\right) - \frac{\phi'}{\phi''}\cdot\frac{\phi'}{\delta_h}\frac{1}{h} < \frac{(1-\tau_H)\beta h}{(1-\tau_L)\alpha H_Y} + \frac{1}{(1-s)}.$$

Since $\frac{1}{(1-s)} = 1 + \frac{s}{(1-s)}$:

$$\frac{\phi'}{\phi''}\frac{1}{h}\frac{\phi'}{\delta_h}\left(\Psi\frac{1}{(1-s)^2}\left(1 - \frac{\delta_h}{\phi'}\right) - 1\right) < \frac{(1-\tau_H)\beta h}{(1-\tau_L)\alpha H_Y} + \frac{s}{1-s}.$$

This is (26) in the main text, which we assume to hold.

Before completing the differential, we compute the derivatives

$$\frac{\partial \Psi}{\partial \tau_L} = -\Psi\frac{(1-\tau_H)\beta(1-s)h}{(1-\tau_L)D} < 0, \qquad \frac{\partial \Psi}{\partial \tau_H} = \Psi\frac{\beta(1-s)h}{D} > 0$$

and

$$\frac{\partial \Psi}{\partial H_Y} = \Psi\frac{(1-\tau_H)\beta(1-s)h}{H_Y D} > 0.$$

We also note that

$$\mathrm{d}H_Y = \frac{H_Y}{r}\mathrm{d}r - \frac{H_Y}{(1-\varsigma_R)}\mathrm{d}\varsigma_R - \frac{H_Y}{(1-\varsigma_x)}\mathrm{d}\varsigma_x - \frac{H_Y}{\eta}\mathrm{d}\eta.$$

Thus, finally, the total differential is

$$\frac{\phi''\delta_h}{\phi'}\left[1 - \Psi\cdot\Xi\right]\mathrm{d}h = \frac{(\theta-1)}{\theta}(1-\tau_r)\mathrm{d}r - \frac{(\theta-1)}{\theta}r\mathrm{d}\tau_r +$$

$$+\phi'\Psi\frac{\beta(1-s)h}{D}\mathrm{d}\tau_H - \phi'\Psi\frac{(1-\tau_H)\beta(1-s)h}{(1-\tau_L)D}\mathrm{d}\tau_L +$$

$$+\phi'\Psi\frac{(1-\tau_H)\beta(1-s)h}{H_Y D}\left(\frac{H_Y}{r}\mathrm{d}r - \frac{H_Y}{(1-\varsigma_R)}\mathrm{d}\varsigma_R - \frac{H_Y}{(1-\varsigma_x)}\mathrm{d}\varsigma_x - \frac{H_Y}{\eta}\mathrm{d}\eta\right).$$

or

$$\frac{\phi''\delta_h}{\phi'}\left[1-\Psi\cdot\Xi\right]\mathrm{d}h=\frac{(\theta-1)(1-\tau_r)}{\theta}\mathrm{d}r+\phi'\Psi\frac{(1-\tau_H)\beta(1-s)h}{rD}\mathrm{d}r+$$

$$+\phi'\Psi\frac{\beta(1-s)h}{D}\mathrm{d}\tau_H-\phi'\Psi\frac{(1-\tau_H)\beta(1-s)h}{(1-\tau_L)D}\mathrm{d}\tau_L-\frac{(\theta-1)}{\theta}r\mathrm{d}\tau_r+$$

$$-\phi'\Psi\frac{(1-\tau_H)\beta(1-s)h}{(1-\varsigma_R)D}\mathrm{d}\varsigma_R-\phi'\Psi\frac{(1-\tau_H)\beta(1-s)h}{(1-\varsigma_x)D}\mathrm{d}\varsigma_x-\phi'\Psi\frac{(1-\tau_H)\beta(1-s)h}{\eta D}\mathrm{d}\eta. \tag{50}$$

A.3.2 Computing the derivatives

From equation (50) we now compute various derivatives of h. The signs of these derivatives are determined under the assumptions that equation (26) holds, $\theta > 1$ and $\psi'' < 0$. First,

$$\frac{\partial h}{\partial r}=\frac{\phi'}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right]}\left(\frac{(\theta-1)(1-\tau_r)}{\theta}+\phi'\Psi\frac{(1-\tau_H)\beta(1-s)h}{rD}\right)<0.$$

Next,

$$\frac{\partial h}{\partial\tau_H}=\Psi\frac{(\phi')^2\beta(1-s)h}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right]D}<0,\qquad\frac{\partial h}{\partial\tau_L}=-\Psi\frac{(\phi')^2(1-\tau_H)\beta(1-s)h}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right](1-\tau_L)D}>0$$

$$\text{and}\qquad\frac{\partial h}{\partial\tau_r}=-\frac{\phi'(\theta-1)r}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right]\theta}>0.$$

Finally,

$$\frac{\partial h}{\partial\varsigma_R}=-\Psi\frac{(\phi')^2(1-\tau_H)\beta(1-s)h}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right](1-\varsigma_R)D}>0,\qquad\frac{\partial h}{\partial\varsigma_x}=-\Psi\frac{(\phi')^2(1-\tau_H)\beta(1-s)h}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right](1-\varsigma_x)D}>0$$

$$\text{and}\qquad\frac{\partial h}{\partial\eta}=-\Psi\frac{(\phi')^2(1-\tau_H)\beta(1-s)h}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right]\eta D}>0.$$

A.4 Comparative statics: Case 1

A.4.1 Effects on the interest rate

To compute the derivatives of the equilibrium interest rate with respect to various parameters we use the details in (23) and (27) and also write r as a function of any parameter ξ. Thus (28) is modified to

$$\left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)}+\frac{(1-\tau_r)}{\theta}\right)\cdot\frac{r(\xi)}{\eta}-\frac{\rho}{\eta\theta}=h(r(\xi),\xi)-s(h(r(\xi),\xi))h(r(\xi),\xi).$$

Implicit differentiation with respect to any parameter ξ gives

$$\left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)}+\frac{(1-\tau_r)}{\theta}\right)\cdot\frac{1}{\eta}\cdot\frac{\partial r}{\partial\xi}+\frac{\partial \text{LHS}}{\partial\xi}=\frac{\partial \text{RHS}}{\partial r}\frac{\partial r}{\partial\xi}+\frac{\partial \text{RHS}}{\partial\xi},$$

where LHS $= H_D(r,\xi)$ is the left-hand side and RHS $= H_S^1(r,\xi)$ is the right-hand side. It follows from (27) and the derivatives thereafter that

$$\left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)}+\frac{(1-\tau_r)}{\theta}\right)\cdot\frac{1}{\eta}\cdot\frac{\partial r}{\partial\xi}+\frac{\partial \text{LHS}}{\partial\xi}=\left(1-\frac{\delta_h}{\phi'}\right)\frac{\partial h}{\partial r}\frac{\partial r}{\partial\xi}+\left(1-\frac{\delta_h}{\phi'}\right)\frac{\partial h}{\partial\xi}$$

Collecting terms, we get

$$\frac{\partial r}{\partial\xi}=\Delta\left[\left(1-\frac{\delta_h}{\phi'}\right)\frac{\partial h}{\partial\xi}-\frac{\partial \text{LHS}}{\partial\xi}\right],$$

where

$$\Delta\equiv\left[\left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)}+\frac{(1-\tau_r)}{\theta}\right)\frac{1}{\eta}-\left(1-\frac{\delta_h}{\phi'}\right)\frac{\partial h}{\partial r}\right]^{-1}>0.$$

This is a somewhat more detailed version of equation (29).

The following derivatives emerge:

$$\frac{\partial r}{\partial\tau_r}=\Delta\left[\frac{1}{\eta}-\left(\frac{\phi'}{\delta_h}-1\right)\frac{(\theta-1)}{\phi''\left[1-\Psi\cdot\Xi\right]}\right]\frac{r}{\theta}>0.$$

$$\frac{\partial r}{\partial\eta}=\Delta\left[\left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)}r+\frac{(1-\tau_r)r-\rho}{\theta}\right)\frac{1}{\eta^2}-\left(1-\frac{\delta_h}{\phi'}\right)\Psi\frac{(\phi')^2(1-\tau_H)\beta(1-s)h}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right]\eta D}\right]>0$$

$$\frac{\partial r}{\partial\varsigma_R}=\Delta\left[\frac{\beta(1-\varsigma_x)r}{(1-\alpha-\beta)(\alpha+\beta)\eta}-\left(1-\frac{\delta_h}{\phi'}\right)\Psi\frac{(\phi')^2(1-\tau_H)\beta(1-s)h}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right](1-\varsigma_R)D}\right]>0$$

$$\frac{\partial r}{\partial\varsigma_x}=\Delta\left[\frac{\beta(1-\varsigma_R)r}{(1-\alpha-\beta)(\alpha+\beta)\eta}-\left(1-\frac{\delta_h}{\phi'}\right)\Psi\frac{(\phi')^2(1-\tau_H)\beta(1-s)h}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right](1-\varsigma_x)D}\right]>0$$

A.4.2 Effects on other variables

Tax on savings To get the total effect of the saving tax on the growth rate, we differentiate (20) wrt τ_r:

$$\frac{\partial g_Y}{\partial\tau_r}=\left[\frac{(1-\tau_r)}{r}\frac{\partial r}{\partial\tau_r}-1\right]\frac{r}{\theta}$$

Substituting the above expression for $\frac{\partial r}{\partial\tau_r}$:

$$\frac{\partial g_Y}{\partial\tau_r}=\Delta\left[\frac{(1-\tau_r)}{\theta}\left[\frac{1}{\eta}-\left(\frac{\phi'}{\delta_h}-1\right)\frac{(\theta-1)}{\phi''\left[1-\Psi\cdot\Xi\right]}\right]-\frac{1}{\Delta}\right]\frac{r}{\theta}$$

Now, the expression for Δ can be developed, by use of the above expression for $\frac{\partial h}{\partial r}$, to

$$\frac{1}{\Delta} \equiv \left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)} + \frac{(1-\tau_r)}{\theta} \right) \frac{1}{\eta}$$
$$- \left(\frac{\phi'}{\delta_h} - 1 \right) \frac{1}{\phi'' \left[1 - \Psi \cdot \Xi\right]} \left(\frac{(\theta-1)(1-\tau_r)}{\theta} + \phi' \Psi \frac{(1-\tau_H)\beta(1-s)h}{rD} \right) > 0$$

Using this in the above derivative:

$$\frac{\partial g_Y}{\partial \tau_r} = \Delta \left[\frac{(1-\tau_r)}{\theta} \left[\frac{1}{\eta} - \left(\frac{\phi'}{\delta_h} - 1 \right) \frac{(\theta-1)}{\phi'' \left[1 - \Psi \cdot \Xi\right]} \right] \right] \frac{r}{\theta}$$
$$-\Delta \left[\left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)} + \frac{(1-\tau_r)}{\theta} \right) \frac{1}{\eta} \right] \frac{r}{\theta}$$
$$-\Delta \left[- \left(\frac{\phi'}{\delta_h} - 1 \right) \frac{1}{\phi'' \left[1 - \Psi \cdot \Xi\right]} \left(\frac{(\theta-1)(1-\tau_r)}{\theta} + \phi' \Psi \frac{(1-\tau_H)\beta(1-s)h}{rD} \right) \right] \frac{r}{\theta}$$

or

$$\frac{\partial g_Y}{\partial \tau_r} = -\Delta \left[\left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)} \right) \frac{1}{\eta} - \left(\frac{\phi'}{\delta_h} - 1 \right) \frac{\phi' \Psi (1-\tau_H)\beta(1-s)h}{\phi'' \left[1 - \Psi \cdot \Xi\right] rD} \right] \frac{r}{\theta} < 0.$$

The coefficient in front of $\frac{r}{\theta}$ is clearly between 0 and 1 (in absolute terms).

The total effect of an increase in τ_r on H_D (and thus on the equilibrium quantity of human capital on the labor market) is

$$\frac{\partial H_D(r(\tau_r), \tau_r)}{\partial \tau_r} = \left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)} + \frac{(1-\tau_r)}{\theta} \right) \cdot \frac{1}{\eta} \frac{\partial r}{\partial \tau_r} - \frac{1}{\theta} \cdot \frac{r}{\eta}.$$

The final term is the direct effect that lowers H_R (shifting the curve), whereas the first term represents the movement along the demand curve to a higher quantity. Using the expressions for $\frac{\partial r}{\partial \tau_r}$ and Δ above:

$$\frac{\partial H_D(r(\tau_r), \tau_r)}{\partial \tau_r} = \frac{\Delta}{\theta \eta} \left(\frac{\phi'}{\delta_h} - 1 \right) \frac{1}{\phi'' \left[1 - \Psi \cdot \Xi\right]} \cdot$$
$$\left[\phi' \Psi \frac{(1-\tau_H)\beta(1-s)h}{D} - (\theta-1) \frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)} \cdot r \right]$$

If $\theta = 1$ the derivative is clearly negative.[19]

Productivity in research From the main text we have (after slight modification)

$$\frac{\partial H_R}{\partial \eta} = \frac{(1-\tau_r)}{\theta \eta} \frac{\partial r}{\partial \eta} - H_R \eta^{-1}.$$

[19] To determine the sign of this, one could use the definition of H_Y. One could also use (22) to rewrite (24) as $\phi'(sh)(1-\Psi) = (1-\tau_r)r - \eta H_R + \delta_h$.

Substituting for $\frac{\partial r}{\partial \eta}$:

$$\frac{\partial H_R}{\partial \eta} = \frac{\Delta}{\eta}\frac{(1-\tau_r)}{\theta\eta}\left[H_D - \left(\frac{\phi'}{\delta_h}-1\right)\Psi\frac{\phi'(1-\tau_H)\beta(1-s)h}{\phi''\left[1-\Psi\cdot\Xi\right]D}\right] - \frac{\Delta}{\eta}\frac{1}{\Delta}H_R.$$

Substituting for $\frac{1}{\Delta}$:

$$\frac{\partial H_R}{\partial \eta} = \frac{\Delta}{\eta r}\frac{\rho}{\theta\eta}H_Y$$

$$+\frac{\Delta}{\eta r}\left(\frac{\phi'}{\delta_h}-1\right)\frac{1}{\phi''\left[1-\Psi\cdot\Xi\right]}\left(\frac{(\theta-1)(1-\tau_r)r}{\theta}H_R - \frac{\rho}{\theta\eta}\Psi\frac{\phi'(1-\tau_H)\beta(1-s)h}{D}\right)$$

This effect is ambiguous, but for $\theta \leq 1$ (and some range above) the effect is positive: a higher productivity in research leads to more research. A very high θ could reverse this.

Furthermore,

$$\frac{\partial H_Y}{\partial \eta} = H_Y\left(\frac{1}{r}\frac{\partial r}{\partial \eta} - \frac{1}{\eta}\right)$$

Substituting for $\frac{\partial r}{\partial \eta}$:

$$\frac{\partial H_Y}{\partial \eta} = \frac{H_Y\Delta}{\eta r}\left(H_D - \left(\frac{\phi'}{\delta_h}-1\right)\Psi\frac{\phi'(1-\tau_H)\beta(1-s)h}{\phi''\left[1-\Psi\cdot\Xi\right]D} - \frac{r}{\Delta}\right)$$

Substituting for $\frac{1}{\Delta}$:

$$\frac{\partial H_Y}{\partial \eta} = \frac{H_Y\Delta}{\eta r}\left(\frac{\phi'}{\delta_h}-1\right)\frac{1}{\phi''\left[1-\Psi\cdot\Xi\right]}\left(\frac{(\theta-1)(1-\tau_r)r}{\theta}\right) - \frac{H_Y\Delta}{\eta r}\frac{\rho}{\theta\eta} < 0$$

Finally

$$\frac{\partial H_D}{\partial \eta} = \frac{\partial H_Y}{\partial \eta} + \frac{\partial H_R}{\partial \eta} = H_D\frac{\Delta}{\eta r}\left(\frac{\phi'}{\delta_h}-1\right)\frac{1}{\phi''\left[1-\Psi\cdot\Xi\right]}\left(\frac{(\theta-1)(1-\tau_r)r}{\theta}\right)$$

$$-\frac{\Delta}{\eta r}\left(\frac{\phi'}{\delta_h}-1\right)\frac{1}{\phi''\left[1-\Psi\cdot\Xi\right]}\left(\frac{\rho}{\theta\eta}\Psi\frac{\phi'(1-\tau_H)\beta(1-s)h}{D}\right)$$

This effect is ambiguous, but for $\theta \leq 1$ (and some range above) the effect is positive: a higher productivity in research leads to more research. A very high θ could reverse this.

Higher subsidy to research Differentiating (23) with respect to ς_R:

$$\frac{\partial H_D}{\partial \varsigma_R} = \left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)} + \frac{(1-\tau_r)}{\theta}\right)\cdot\frac{1}{\eta}\frac{\partial r}{\partial \varsigma_R} - \left(\frac{\beta(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)}\right)\cdot\frac{r}{\eta}$$

Substituting for $\frac{\partial r}{\partial \varsigma_R}$:

$$\frac{\partial H_D}{\partial \varsigma_R} = \frac{1}{r}\left(H_D + \frac{\rho}{\eta\theta}\right)\frac{\Delta}{(1-\varsigma_R)}\left[H_Y - \left(\frac{\phi'}{\delta_h} - 1\right)\Psi\frac{\phi'(1-\tau_H)\beta(1-s)h}{\phi''\left[1-\Psi\cdot\Xi\right]D}\right]$$
$$-\frac{H_Y}{(1-\varsigma_R)}\frac{\Delta}{\Delta}$$

Substituting for $\frac{1}{\Delta}$:

$$\frac{\partial H_D}{\partial \varsigma_R} = \frac{\Delta}{(1-\varsigma_R)r}\left(\frac{\phi'}{\delta_h} - 1\right)\frac{1}{\phi''\left[1-\Psi\cdot\Xi\right]}\cdot$$
$$\cdot\left[H_Y\frac{(\theta-1)(1-\tau_r)r}{\theta} - \phi'\Psi\frac{(1-\tau_H)\beta(1-s)h}{D}\left[H_R + \frac{\rho}{\eta\theta}\right]\right]$$

For $\theta \leq 1$ (and some range above) the effect is positive: a higher subsidy to research leads to more human capital on the labor market. However, a very high θ could reverse this result.)

Higher subsidy to x Differentiating (23) with respect to ς_x:

$$\frac{\partial H_D}{\partial \varsigma_x} = \left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)} + \frac{(1-\tau_r)}{\theta}\right)\cdot\frac{1}{\eta}\frac{\partial r}{\partial \varsigma_x} - \left(\frac{\beta(1-\varsigma_R)}{(1-\alpha-\beta)(\alpha+\beta)}\right)\cdot\frac{r}{\eta}$$

Substituting for $\frac{\partial r}{\partial \varsigma_x}$:

$$\frac{\partial H_D}{\partial \varsigma_x} = \frac{1}{r}\left(H_D + \frac{\rho}{\eta\theta}\right)\frac{\Delta}{(1-\varsigma_x)}\left[H_Y - \left(\frac{\phi'}{\delta_h} - 1\right)\Psi\frac{\phi'(1-\tau_H)\beta(1-s)h}{\phi''\left[1-\Psi\cdot\Xi\right]D}\right]$$
$$-\frac{H_Y}{(1-\varsigma_x)}\frac{\Delta}{\Delta}$$

Substituting for $\frac{1}{\Delta}$:

$$\frac{\partial H_D}{\partial \varsigma_x} = \frac{\Delta}{(1-\varsigma_x)r}\left(\frac{\phi'}{\delta_h} - 1\right)\frac{1}{\phi''\left[1-\Psi\cdot\Xi\right]}\cdot$$
$$\cdot\left(H_Y\frac{(\theta-1)(1-\tau_r)r}{\theta} - \phi'\Psi\frac{(1-\tau_H)\beta(1-s)h}{D}\left[H_R + \frac{\rho}{\eta\theta}\right]\right)$$

This is the same as for ς_R, but with ς_R replaced by ς_x.

The relative wage From (6) we have

$$\frac{w_H}{w_L} = \frac{\beta(1-s)\omega}{\alpha H_Y}.$$

The derivative with respect to any parameter ξ (using (25)):

$$\frac{\partial}{\partial \xi}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right) = -\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'} - s\right)\frac{\partial h}{\partial \xi} - \frac{1}{H_Y}\frac{\partial H_Y}{\partial \xi}.$$

Now note that $h = h(r(\xi), \xi)$ and $H_Y = H_Y(r(\xi), \xi)$, which means that

$$\frac{\partial h}{\partial \xi} = \frac{\partial h}{\partial r}\frac{\partial r}{\partial \xi} + \frac{\partial h}{\partial \xi} \qquad \text{and} \qquad \frac{\partial H_Y}{\partial \xi} = \frac{\partial H_Y}{\partial r}\frac{\partial r}{\partial \xi} + \frac{\partial H_Y}{\partial \xi} = \frac{H_Y}{r}\frac{\partial r}{\partial \xi} + \frac{\partial H_Y}{\partial \xi}.$$

Substituting this into the above equation:

$$\frac{\partial}{\partial \xi}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right) = -\left(\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'} - s\right)\frac{\partial h}{\partial r} + \frac{1}{r}\right)\frac{\partial r}{\partial \xi}$$

$$-\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'} - s\right)\frac{\partial h}{\partial \xi} - \frac{1}{H_Y}\frac{\partial H_Y}{\partial \xi}. \tag{51}$$

Recall from the discussion after equation (25) that $\delta_h > \phi' s$. We now use this formula for a number of parameters.

First, let us examine the effect of an increase in τ_H. We first note that $\frac{\partial H_Y}{\partial \tau_H} = 0$. Thus (51) becomes

$$\frac{\partial}{\partial \tau_H}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right) = -\left(\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'} - s\right)\frac{\partial h}{\partial r} + \frac{1}{r}\right)\frac{\partial r}{\partial \tau_H}$$

$$-\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'} - s\right)\frac{\partial h}{\partial \tau_H}.$$

Substituting for $\frac{\partial r}{\partial \tau_H}$:

$$\frac{\partial}{\partial \tau_H}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right) =$$

$$-\Delta\left[\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'} - s\right)\left(\left(1 - \frac{\delta_h}{\phi'}\right)\frac{\partial h}{\partial r} + \frac{1}{\Delta}\right) + \left(\frac{1}{r}\right)\left(1 - \frac{\delta_h}{\phi'}\right)\right]\frac{\partial h}{\partial \tau_H}$$

Now,

$$\frac{1}{\Delta} \equiv \left[\frac{1}{r}\left(H_D + \frac{\rho}{\theta\eta}\right) - \left(1 - \frac{\delta_h}{\phi'}\right)\frac{\partial h}{\partial r}\right] > 0,$$

so

$$\frac{\partial}{\partial \tau_H}\left(\frac{w_H}{w_L}\right) = -\frac{\Delta}{r}\cdot\frac{w_H}{w_L}\left[\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'} - s\right)\left(H_D + \frac{\rho}{\theta\eta}\right) + \left(1 - \frac{\delta_h}{\phi'}\right)\right]\frac{\partial h}{\partial \tau_H} > 0.$$

(The derivative with respect to τ_H is very similar.)

Next, the effect of a higher subsidy to research is

$$\frac{\partial}{\partial \varsigma_R}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right) = -\left(\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'} - s\right)\frac{\partial h}{\partial r} + \frac{1}{r}\right)\frac{\partial r}{\partial \varsigma_R}$$

$$-\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'}-s\right)\frac{\partial h}{\partial\varsigma_R}-\frac{1}{H_Y}\frac{\partial H_Y}{\partial\varsigma_R}.$$

Substituting for $\frac{\partial r}{\partial\varsigma_R}$:

$$\frac{\partial}{\partial\varsigma_R}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right)=-\left[\left(\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'}-s\right)\frac{\partial h}{\partial r}+\frac{1}{r}\right)\Delta\left(1-\frac{\delta_h}{\phi'}\right)\right]\frac{\partial h}{\partial\varsigma_R}$$

$$+\left(\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'}-s\right)\frac{\partial h}{\partial r}+\frac{1}{r}\right)\Delta\left[\frac{\partial\text{LHS}}{\partial\varsigma_R}\right]-\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'}-s\right)\frac{\partial h}{\partial\varsigma_R}-\frac{1}{H_Y}\frac{\partial H_Y}{\partial\varsigma_R}.$$

Noting that $\frac{\partial H_Y}{\partial\varsigma_R}=\frac{\partial\text{LHS}}{\partial\varsigma_R}$:

$$\frac{\partial}{\partial\varsigma_R}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right)=$$

$$-\Delta\left[\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'}-s\right)\left(\frac{\partial h}{\partial r}\left(1-\frac{\delta_h}{\phi'}\right)+\frac{1}{\Delta}\right)+\frac{1}{r}\left(1-\frac{\delta_h}{\phi'}\right)\right]\frac{\partial h}{\partial\varsigma_R}$$

$$+\Delta\left[\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'}-s\right)\frac{\partial h}{\partial r}+\frac{1}{r}-\frac{1}{H_Y}\frac{1}{\Delta}\right]\frac{\partial H_Y}{\partial\varsigma_R}$$

Using the above expression for $\frac{1}{\Delta}$:

$$\frac{\partial}{\partial\varsigma_R}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right)=-\Delta\left[\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'}-s\right)\frac{1}{r}\left(H_D+\frac{\rho}{\theta\eta}\right)+\frac{1}{r}\left(1-\frac{\delta_h}{\phi'}\right)\right]\frac{\partial h}{\partial\varsigma_R}$$

$$+\Delta\left[\left(\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'}-s\right)+\frac{1}{H_Y}\left(1-\frac{\delta_h}{\phi'}\right)\right)\frac{\partial h}{\partial r}-\frac{1}{r}\frac{1}{H_Y}\left[(H_D-H_Y)+\frac{\rho}{\theta\eta}\right]\right]\frac{\partial H_Y}{\partial\varsigma_R}$$

Since $\frac{\partial h}{\partial\varsigma_R}>0$ the first term is negative. Since $\frac{\partial h}{\partial r}<0$ and $\frac{\partial H_Y}{\partial\varsigma_R}<0$ the second term is positive. Now,

$$\frac{\partial h}{\partial r}=\frac{\phi'(\theta-1)(1-\tau_r)}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right]\theta}-\frac{\partial h}{\partial\varsigma_R}\cdot\frac{(1-\varsigma_R)}{r}\qquad\text{and}\qquad\frac{\partial H_Y}{\partial\varsigma_R}=-\frac{H_Y}{(1-\varsigma_R)}.$$

Thus

$$\frac{\partial}{\partial\varsigma_R}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right)\frac{r(1-\varsigma_R)}{\Delta}=\left(H_R+\frac{\rho}{\theta\eta}\right)-\frac{(1-\varsigma_R)}{(1-s)h}\left(\frac{\delta_h}{\phi'}-s\right)\left(H_R+\frac{\rho}{\theta\eta}\right)\frac{\partial h}{\partial\varsigma_R}$$

$$-\left(\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'}-s\right)H_Y+\left(1-\frac{\delta_h}{\phi'}\right)\right)\frac{\phi'(\theta-1)(1-\tau_r)r}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right]\theta}$$

Use the equilibrium condition $H_D=(1-s)h$ and $\frac{(1-\tau_r)r}{\theta}=\eta\left(H_R+\frac{\rho}{\eta\theta}\right)$:

$$\frac{\partial}{\partial\varsigma_R}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right)\frac{r(1-\varsigma_R)H_D}{\Delta}\left(H_R+\frac{\rho}{\eta\theta}\right)^{-1}=$$

$$H_D - \frac{(1-\varsigma_R)}{1}\left(\frac{\delta_h}{\phi'} - s\right)\frac{\partial h}{\partial \varsigma_R}$$
$$-\left(\left(\frac{\delta_h}{\phi'} - s\right)H_Y + H_D\left(1 - \frac{\delta_h}{\phi'}\right)\right)\frac{\phi'(\theta-1)\eta}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right]}$$

When $\theta = 1$:

$$\frac{\partial}{\partial \varsigma_R}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right)\frac{r(1-\varsigma_R)H_D}{\Delta}\left(H_R + \frac{\rho}{\eta\theta}\right)^{-1} = (1-s)h\left(1 - \frac{(1-\varsigma_R)}{(1-s)\varsigma_R}\left(\frac{\delta_h}{\phi'} - s\right)\frac{\partial h}{\partial \varsigma_R}\frac{\varsigma_R}{h}\right)$$

Thus

$$\frac{\partial}{\partial \varsigma_R}\left(\frac{w_H}{w_L}\right) > 0 \qquad \Leftrightarrow \qquad 1 - \frac{(1-\varsigma_R)}{(1-s)\varsigma_R}\left(\frac{\delta_h}{\phi'} - s\right)\frac{\partial h}{\partial \varsigma_R}\frac{\varsigma_R}{h} > 0$$

In the general case, using the expression for $\frac{\partial h}{\partial \varsigma_R}$:

$$\frac{\partial}{\partial \varsigma_R}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right)\frac{r(1-\varsigma_R)H_D}{\Delta}\left(H_R + \frac{\rho}{\eta\theta}\right)^{-1} = H_D + \frac{\phi'}{\phi''\delta_h\left[1-\Psi\cdot\Xi\right]}\cdot$$
$$\cdot\left[\left(\frac{\delta_h}{\phi'} - s\right)\cdot\Psi\phi'\frac{(1-\tau_H)\beta(1-s)h}{D} - \left(\left(\frac{\delta_h}{\phi'} - s\right)H_Y + H_D\left(1 - \frac{\delta_h}{\phi'}\right)\right)\cdot(\theta-1)\eta\right]$$

To sum up: a positive derivative possible already when $\theta = 1$ and gets more likely when θ increases.

Finally, the effect of a higher tax on savings is

$$\frac{\partial}{\partial \tau_r}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right) = -\left(\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'} - s\right)\frac{\partial h}{\partial r} + \frac{1}{r}\right)\frac{\partial r}{\partial \tau_r}$$
$$-\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'} - s\right)\frac{\partial h}{\partial \tau_r} - \frac{1}{H_Y}\frac{\partial H_Y}{\partial \tau_r}.$$

Noting that $\frac{\partial H_Y}{\partial \tau_r} = 0$ and substituting for $\frac{\partial r}{\partial \tau_r}$:

$$\frac{\partial}{\partial \tau_r}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right) =$$
$$-\Delta\left[\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'} - s\right)\left[\left(1 - \frac{\delta_h}{\phi'}\right)\frac{\partial h}{\partial r} + \frac{1}{\Delta}\right] + \frac{1}{r}\left(1 - \frac{\delta_h}{\phi'}\right)\right]\frac{\partial h}{\partial \tau_r}$$
$$-\left(\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'} - s\right)\frac{\partial h}{\partial r} + \frac{1}{r}\right)\Delta\left[\frac{r}{\theta\eta}\right]$$

Using the above expression for $\frac{1}{\Delta}$:

$$\frac{\partial}{\partial \tau_r}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right) =$$

$$-\frac{\Delta}{r}\left[\frac{1}{(1-s)h}\left(\frac{\delta_h}{\phi'}-s\right)\left(H_D+\frac{\rho}{\theta\eta}\right)+\left(1-\frac{\delta_h}{\phi'}\right)\right]\frac{\partial h}{\partial \tau_r}$$
$$-\frac{\Delta}{r}\left(\frac{1}{(1-s)}\left(\frac{\delta_h}{\phi'}-s\right)\frac{\partial h}{\partial r}\frac{r}{h}+1\right)\frac{r}{\theta\eta}$$

When $\theta = 1$, $\frac{\partial h}{\partial \tau_r} = 0$. Then

$$\frac{\partial}{\partial \tau_r}\left(\frac{w_H}{w_L}\right)\cdot\left(\frac{w_L}{w_H}\right) = -\frac{\Delta}{r}\left(\frac{1}{(1-s)}\left(\frac{\delta_h}{\phi'}-s\right)\frac{\partial h}{\partial r}\frac{r}{h}+1\right)\frac{r}{\theta\eta}$$

This means that

$$\frac{\partial}{\partial \tau_r}\left(\frac{w_H}{w_L}\right) < 0 \qquad \Leftrightarrow \qquad (1-s)\left(\frac{\delta_h}{\phi'}-s\right)^{-1} > -\frac{\partial h}{\partial r}\frac{r}{h}$$

In the general case, with $\theta > 1$, there is an additional negative term.

B Additional computations (not for publication)

B.1 Social optimum

B.1.1 Static allocation

Output net of investment is

$$\tilde{Y} = \frac{1}{1-\alpha-\beta} L^{\alpha} H_Y(t)^{\beta} \left[\int_0^{N(t)} x(\nu,t)^{1-\alpha-\beta} d\nu \right] - (1-\alpha-\beta) \int_0^{N(t)} x(\nu,t) d\nu.$$

Maximizing this (at given N) with respect to an arbitrary $x(\nu,t)$, $\nu \in (0,N]$:

$$x(\nu,t) = \left(\frac{L^{\alpha} H_Y^{\beta}}{1-\alpha-\beta} \right)^{\frac{1}{\alpha+\beta}}.$$

Substituting this into the production function:

$$Y = \left(\frac{L^{\alpha} H_Y^{\beta}}{1-\alpha-\beta} \right)^{\frac{1}{\alpha+\beta}} N.$$

The corresponding expressions for the decentralized equilibrium, (3) and (5), are identical to these if the subsidy to intermediate inputs is $\varsigma_x = \alpha+\beta$. Such a subsidy would also modify (8) to

$$C = (\alpha+\beta)Y.$$

Finally, putting $\varsigma_x = \alpha+\beta$ into (4) we have

$$\pi(t) = (\alpha+\beta) \left(\frac{L^{\alpha} H_Y^{\beta}}{1-\alpha-\beta} \right)^{\frac{1}{\alpha+\beta}} = (\alpha+\beta)\frac{Y}{N}.$$

These expressions will be used repeatedly below.

B.1.2 Conditions for Optimal Growth

Turning to the dynamic optimization problem, note first that the supplies of human capital and raw labor are $H = (1-s)h$ and $L = (1-s)\omega$, respectively.[20] This means that the human capital constraint $H = H_Y + H_R$ can be written as $(1-s)h = H_Y + H_R$ and that consumption is

$$C = (\alpha+\beta) \left(\frac{((1-s)\omega)^{\alpha} H_Y^{\beta}}{1-\alpha-\beta} \right)^{\frac{1}{\alpha+\beta}} N. \tag{52}$$

[20]This holds for Case 1; for Case 2 we have $L = \omega$.

The problem of the social planner is then to maximize

$$U = \int_0^\infty M \cdot e^{-\rho t} \frac{(C/M)^{1-\theta} - 1}{1-\theta} dt \tag{53}$$

subject to

$$\dot{h} = \phi(sh) - \delta_h h \tag{54}$$

and

$$\dot{N} = \eta N ((1-s)h - H_Y). \tag{55}$$

The current-value Hamiltonian for the planner's problem is[21]

$$\mathcal{H} = M\frac{(C/M)^{1-\theta} - 1}{1-\theta} + \chi\eta N ((1-s)h - H_Y) + \upsilon(\phi(sh) - \delta_h h),$$

where χ and υ are co-state variables for N and h respectively. Maximizing with respect to the control variables H_Y and s:

$$(C/M)^{-\theta} \cdot C \cdot \frac{\beta}{\alpha+\beta}\frac{1}{H_Y} = \chi\eta N \tag{56}$$

and[22]

$$-(C/M)^{-\theta} \cdot C \cdot \frac{\alpha}{\alpha+\beta}\frac{1}{(1-s)} = \chi\eta N h - \upsilon\phi'(sh) h. \tag{57}$$

Moreover, the costate variables change according to

$$\mathcal{H}_N = (C/M)^{-\theta} \cdot C \cdot \frac{1}{N} + \chi\eta ((1-s)h - H_Y) = \rho\chi - \dot{\chi} \tag{58}$$

and

$$\mathcal{H}_h = \chi\eta N(1-s) + \upsilon\left(\phi'(sh)s - \delta_h\right) = \rho\upsilon - \dot{\upsilon} \tag{59}$$

There are also two transversality conditions which we will not analyze here.

It is useful to combine some of these equations into some simpler expressions. First equations (56) and (58) can be combined to

$$\eta\left((1-s)h + \frac{\alpha}{\beta}H_Y\right) = \rho - g_\chi \tag{60}$$

Equations (56) and (57) can be combined to

$$\upsilon\phi'(sh)(1-s)h = \chi N\eta\left((1-s)h + \frac{\alpha}{\beta}H_Y\right) \tag{61}$$

[21]Here λ and μ have been replaced by χ and υ, respectively.

[22]In Case 2 the LHS of (57) would be equal to zero.

Using this to eliminate χ from (59):

$$\frac{\phi'(sh)(1-s)h}{((1-s)h+(\alpha/\beta)H_Y)}(1-s)+\phi'(sh)\,s-\delta_h=\rho-g_\upsilon$$

or

$$\phi'(sh)\left[1-\frac{(1-s)(\alpha/\beta)H_Y}{(1-s)h+(\alpha/\beta)H_Y}\right]-\delta_h=\rho-g_\upsilon \tag{62}$$

B.1.3 Comparing with the decentralized equilibrium

Here we use the necessary conditions above and see how they match necessary conditions of the representative household, invoking also some conditions from the firms. The necessary conditions for the household are found in Appendix A.2.

To make the the two sets of conditions comparable, we replace prices with expressions in terms of quantities in the equations for the decentralized equilibrium. There are four pairs of equations to consider.

1. The condition for optimal schooling

To compare (39) and (61), multiply first (39) through by $(1-s)$:

$$\mu\phi'(hs)(1-s)h=\lambda\left[(1-\tau_H)w_H(1-s)h+(1-\tau_L)w_L(1-s)\omega\right]$$

Use (6) to eliminate the wages:

$$\mu\phi'(hs)(1-s)h=\lambda Y\frac{\beta}{H_Y}\left[(1-\tau_H)(1-s)h+(1-\tau_L)\frac{\alpha}{\beta}H_Y\right]$$

Use (38) to eliminate λ:

$$\mu\phi'(hs)(1-s)h=(C/M)^{-\theta}Y\frac{\beta}{H_Y}\left[(1-\tau_H)(1-s)h+(1-\tau_L)\frac{\alpha}{\beta}H_Y\right] \tag{D1}$$

Similarly, use (56) to eliminate χ from (61):

$$\upsilon\phi'(sh)(1-s)h=(C/M)^{-\theta}Y\frac{\beta}{H_Y}\left((1-s)h+\frac{\alpha}{\beta}H_Y\right) \tag{S1}$$

Comparing the first equation for the decentralized equilibrium (D1) with the corresponding expression for the social optimum (S1), we see that these two expressions are equivalent if $\tau_H=\tau_L=0$ and $\mu=\upsilon$.

2. The costate for human capital

Here we compare (42) and (59). First, use (6) and (38) in (42):

$$(C/M)^{-\theta}\left[1-s\right](1-\tau_H)\frac{\beta}{H_Y}\cdot Y+\mu\left(\phi'\left(hs\right)s-\delta_h-\rho\right)=-\dot{\mu} \qquad \text{(D2)}$$

Similarly, use (56) in (59):

$$(C/M)^{-\theta}Y\frac{\beta}{H_Y}\left(1-s\right)+\upsilon\left(\phi'\left(sh\right)s-\delta_h-\rho\right)=-\dot{\upsilon} \qquad \text{(S2)}$$

These two expressions are equivalent if $\tau_H=0$ and $\mu=\upsilon$.

3. The condition for optimal saving

Next we compare (38) and (56). When the optimal subsidy of x is chosen, i.e. $\varsigma_x=\alpha+\beta$, then (11) is simplified to (using the above expression for Y)

$$\frac{\beta(1-\varsigma_R)}{H_Y}\frac{Y}{N}=V\eta$$

Combine this with (38)

$$(C/M)^{-\theta}Y\frac{\beta}{H_Y}(1-\varsigma_R)=VN\eta\lambda \qquad \text{(D3)}$$

Compare this to (56)

$$(C/M)^{-\theta}Y\frac{\beta}{H_Y}=\chi\eta N \qquad \text{(S3)}$$

These two expressions are equivalent if $1-\varsigma_R=V\lambda/\chi$. This would require that the composite variable $V\lambda$ grows at the same rate as χ. We now turn to this.

4. The costates of N and $\mathcal{A}$

To compare (41) and (58) (or rather (60)) we first need an expression for $\dot{V}/V$. For this we use the Bellman equation $rV=\dot{V}+\pi$, which can be derived from (10). Using also

$$\pi(t)=(\alpha+\beta)\frac{Y}{N} \qquad \text{and} \qquad \frac{H_Y\eta}{\beta(1-\varsigma_R)}\frac{N}{Y}=\frac{1}{V}$$

we have

$$\frac{\dot{V}}{V}=r-\frac{\eta}{(1-\varsigma_R)}\cdot\frac{(\alpha+\beta)}{\beta}H_Y$$

Combine this with (41):

$$\frac{\dot{\lambda}}{\lambda}+\frac{\dot{V}}{V}=\rho+\tau_r r-\frac{\eta}{(1-\varsigma_R)}\cdot\frac{(\alpha+\beta)}{\beta}H_Y \qquad \text{(D4)}$$

Compare this to (60):

$$\frac{\dot{\chi}}{\chi} = \rho - \eta\left((1-s)h + \frac{\alpha}{\beta}H_Y\right) \tag{S4}$$

These expressions are equivalent if $\tau_r = 0$ and (if we use $(1-s)h = H_Y + H_R$)

$$\varsigma_R = \frac{\beta H_R}{\beta H_R + (\alpha+\beta)\, H_Y}$$

B.2 Case 2

B.2.1 Supply of human capital

In this simpler case equation (18) applies instead of (17). On the BGP, it again holds that $g_h = g_s = 0$, which means that (18) can be written as

$$\phi'(sh) = \frac{(1-\tau_r)(\theta-1)r + \rho}{\theta} + \delta_h,$$

where (20) has been used to eliminate g_{w_L}. Differentiation and use of (25) gives[23]

$$\frac{\partial h}{\partial r} = \frac{\phi'}{\phi''\delta_h}\cdot\frac{(1-\tau_r)(\theta-1)}{\theta} < 0 \qquad \text{and} \qquad \frac{\partial h}{\partial \tau_r} = -\frac{\phi'}{\phi''\delta_h}\cdot\frac{(\theta-1)r}{\theta} > 0,$$

where the assumptions that $\phi'' < 0$ and $\theta > 1$ have been used to determine the signs. The intuition for this is simple: a higher (net) interest rate discounts future incomes from saving heavier, making it more beneficial to work more earlier. It is worth noting that an increase in the tax on saving has an opposite effect.[24]

Again, the supply of human capital to the labor market is $H = (1-s)h = h - sh$, where $h = h(r,\tau_r)$ and $s = s(h(r,\tau_r))$. Thus

$$H_S^2(r,\tau_r) = h(r,\tau_r) - s(h(r,\tau_r))h(r,\tau_r). \tag{63}$$

[23]The differential is

$$\phi''(sh)h\mathrm{d}s + \phi''(sh)s\mathrm{d}h = \frac{-(\theta-1)r\mathrm{d}\tau_r}{\theta} + \frac{(1-\tau_r)(\theta-1)\mathrm{d}r}{\theta}.$$

Using (25):

$$\frac{\phi''\delta_h}{\phi'}\mathrm{d}h = \frac{-(\theta-1)r\mathrm{d}\tau_r}{\theta} + \frac{(1-\tau_r)(\theta-1)\mathrm{d}r}{\theta}.$$

[24]In the logarithmic case ($\theta = 1$) the time in schooling is independent of the interest rate. This is the well-known case when the substitution and income effects exactly cancel each other.

Using (25), we then have

$$\frac{\partial H_S^2}{\partial r} = \left(\frac{\phi'}{\delta_h} - 1\right) \frac{(1-\tau_r)(\theta-1)}{\phi''\theta} < 0 \qquad \text{and} \qquad \frac{\partial H_S^2}{\partial \tau_r} = -\left(\frac{\phi'}{\delta_h} - 1\right) \frac{(\theta-1)r}{\phi''\theta} > 0.$$

According to Appendix A.2.2 it is necessary for the fulfillment of the household's transversality condition that $\delta_h < \phi'(sh)$, otherwise there is overaccumulation of human capital. This helps us to determine the signs of the above expressions.

We thus find that the supply of human capital decreases if the interest rate gets higher. That is, the supply curve is negatively sloping. Conversely, the supply increase if there is a higher tax on savings.

B.2.2 Case 2: Comparative Statics

Combining the supply of human capital in (63) with the demand in (23), we have the equilibrium condition

$$\left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)} + \frac{(1-\tau_r)}{\theta}\right) \cdot \frac{r}{\eta} - \frac{\rho}{\eta\theta} = h(r,\tau_r) - s(h(r,\tau_r))h(r,\tau_r). \tag{64}$$

This equation determines the equilibrium interest rate on the BGP. The LHS is clearly monotonously increasing in r. As discussed above, it follows from our assumptions that the RHS is monotonously decreasing in r. The equilibrium interest rate is then unique.

Implicit differentiation of r with respect to τ_r and with respect to ς_R gives the derivatives

$$\frac{\partial r}{\partial \tau_r} = \Theta \frac{r}{\theta}\left[\frac{1}{\eta} - \left(\frac{\phi'}{\delta_h} - 1\right)\frac{(\theta-1)}{\phi''}\right] > 0 \quad \text{and} \quad \frac{\partial r}{\partial \varsigma_R} = \Theta \frac{\beta(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)} \cdot \frac{r}{\eta} > 0,$$

where

$$\Theta \equiv \left[\left(\frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)} + \frac{(1-\tau_r)}{\theta}\right) \cdot \frac{1}{\eta} - \left(\frac{\phi'}{\delta_h} - 1\right)\frac{(1-\tau_r)(\theta-1)}{\phi''\theta}\right]^{-1} > 0.$$

The first result appears quite surprising, but recall that the higher τ_r makes the net returns to savings lower. The household is therefore more willing to postpone some income and spend some more time in education. The larger supply of human capital increases the growth rate and the interest rate. On the other hand, the result that a higher subsidy leads to a higher interest rate and then to a higher growth rate is quite expected.

What is the total effect on the growth rate in (20) from an increase in the tax on saving? There are two effects from such a change, which can be seen in

$$\frac{\partial g_Y}{\partial \tau_r} = \left[\frac{(1-\tau_r)}{r} \frac{\partial r}{\partial \tau_r} - 1 \right] \frac{r}{\theta}$$

Absent the effect via the change in human capital, this would collapse to the usual effect $-r/\theta$. Now, however, there is a considerable effect in the positive direction. Using the above expression for $\frac{\partial r}{\partial \tau_r}$, we find that

$$\frac{\partial g_Y}{\partial \tau_r} = -\Theta \frac{\beta(1-\varsigma_R)(1-\varsigma_x)}{(1-\alpha-\beta)(\alpha+\beta)\eta} \frac{r}{\theta} < 0.$$

As in the standard model, the tax on saving does indeed have a negative effect on economic growth. In this case the effect is however somewhat diminished.